6.2.

Note Title 11/22/2013

A set of vectors $\{\vec{u}_1, \ldots, \vec{u}_p\}$ in $\mathbb{R}^n$

is an orthogonal set if whenever $i \neq j$

$$\vec{u}_i \cdot \vec{u}_j = 0.$$

$$\vec{u}_i \cdot \vec{u}_j = \delta_{ij}$$

$$\vec{u}_i \neq \vec{0}$$

Theorem An orthogonal set is linearly indep.

If holds & $\vec{u}_1 \cdots \vec{u}_r$ is orthogonal then $c_1 = c_2 = c_3 = \cdots = c_p = 0$.

$$(c_1\vec{u}_1 + c_2\vec{u}_2 + \cdots + c_p\vec{u}_p) \cdot \vec{u}_i = (\vec{0}) \cdot \vec{u}_i$$

$$\Downarrow$$

$$c_1 \vec{u}_1 \cdot \vec{u}_i + c_2 \vec{u}_2 \cdot \vec{u}_i \cdots + c_i \vec{u}_i \cdot \vec{u}_i + \cdots + c_p \vec{u}_p \vec{u}_i = 0$$

$$\Downarrow$$

$$\vec{u}_i \cdot \vec{u}_i = (u_{i1} \cdot u_{i1} + u_{i2} \cdot u_{i2} + \cdots + u_{in} u_{in}) \qquad c_i \|\vec{u}_i\|^2 = 0.$$

$$\|\vec{u}_i\|^2 > 0$$

$$\vec{v}_1 = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix} \quad \vec{v}_2 = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} \quad \vec{v}_3 = \begin{bmatrix} -1 \\ -4 \\ 7 \end{bmatrix}$$

$$\vec{v}_1 \cdot \vec{v}_3 = 3(-1) + 1 \cdot (-4) + 1 \cdot 7 = 0.$$

$\mathcal{B} = \{\vec{b}_1, \ldots, \vec{b}_n\}$ of $\mathbb{R}^n$

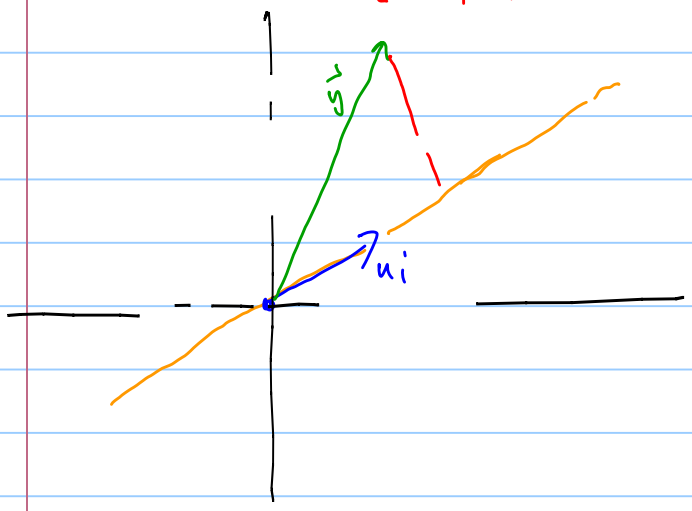
$$[\vec{x}]_{\mathcal{B}} = \begin{bmatrix} c_1 \\ \vdots \\ c_n \end{bmatrix} \qquad c_1\vec{b}_1 + \cdots + c_n\vec{b}_n = \vec{x}.$$

Theorem: If $\{\vec{u}_1, \ldots, \vec{u}_p\}$ is an orthogonal basis for some W subspace of $\mathbb{R}^n$.

$$\vec{y} = c_1\vec{u}_1 + \cdots + c_p\vec{u}_p$$

$$c_j = \frac{\vec{y}\cdot\vec{u}_j}{\vec{u}_j\cdot\vec{u}_j}$$

$$\vec{y} = \left[\frac{\vec{y}\cdot\vec{u}_1}{\vec{u}_1\cdot\vec{u}_1}\right]\vec{u}_1 + \cdots + \left[\frac{\vec{y}\cdot\vec{u}_p}{\vec{u}_p\cdot\vec{u}_p}\right]\vec{u}_p$$

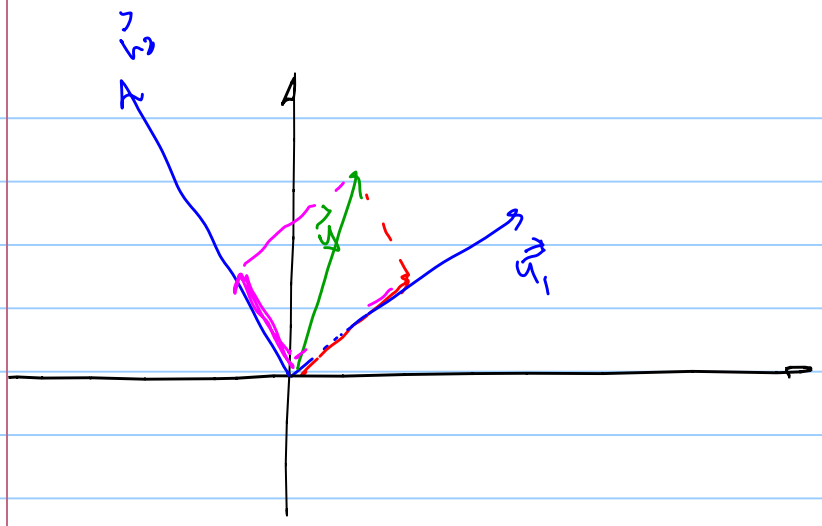


replace $\vec{u}_i$ by $k\vec{u}_i$

$$\frac{\vec{y}\cdot k\vec{u}_i}{k\vec{u}_i\cdot k\vec{u}_i}\,k\vec{u}_i$$

$$= \frac{\vec{y}\vec{u}_i}{\vec{u}_i\cdot\vec{u}_i}\cdot\vec{u}_i$$

orthogonal projection of $\vec{y}$ onto the subspace spanned by $\vec{u}_i$



$\vec{u} \in \mathbb{R}^n$ $\qquad \vec{y} = \hat{y} + \vec{z}$ → orthogonal to $\vec{u}$.

↳ proj of $\vec{y}$ onto $\vec{u}$

$$\hat{y} = \frac{\vec{y} \cdot \vec{u}}{\vec{u} \cdot \vec{u}} \vec{u}.$$

$$\vec{v}_1 = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix} \quad \vec{v}_2 = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} \quad \vec{v}_3 = \begin{bmatrix} -1 \\ -4 \\ 7 \end{bmatrix}$$

$$\vec{y} = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix} \qquad [\vec{y}]_B = \frac{5}{11}\vec{v}_1 + \frac{-1}{6}\vec{v}_2 + \frac{-13}{66}\vec{v}_3$$

$$\begin{bmatrix} 5/11 \\ -1/6 \\ -13/60 \end{bmatrix}$$