7.1

Note Title 12/2/2013

Diagonalize A: $\quad A = P\underbrace{D}_{\text{diagonal matrix}}P^{-1}$

P invertible

Cols of P: eigenvectors of A

Diag elts of D: corresponding eigenvalues.

Matrix U is orthonormal if the cols of U are mutually orthogonal & normalized to 1:

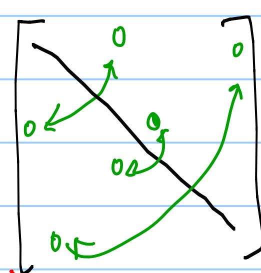
$$\delta_{ij} = \begin{cases} 0 & \text{if } i \neq j \\ 1 & \text{if } i = j. \end{cases}$$

$$U = [\vec{u}_1 \cdots \vec{u}_m] \qquad \vec{u}_i \cdot \vec{u}_j = \delta_{ij}$$

$$U^T U = I. \qquad U^{-1} = U^T$$

Def: A is symmetric if $A^T = A$. (A is square)

row i = col i



$$\begin{bmatrix} 1 & 4 & 6 \\ 4 & 2 & 5 \\ 6 & 5 & 3 \end{bmatrix}$$

A is symmetric if & only if

$A = PDP^{-1}$ $\quad P$ is an orthonormal matrix.

$$\vec{u}\cdot\vec{w} = \vec{u}^T \cdot \vec{w}$$

$$[\;][\;] \qquad \underset{1\times n}{[\;\vec{u}\;]}\underset{n\times 1}{\begin{bmatrix}\vec{w}\end{bmatrix}} \Rightarrow \underset{1\times 1}{[\;1\;]}$$

If A is a symmetric matrix any two eigenvectors w/ different eigenvalues are orthogonal.

$$\lambda_1 \longleftrightarrow \vec{v}_1 \qquad \lambda_1 \neq \lambda_2 \quad \Longrightarrow \quad \vec{v}_1\cdot\vec{v}_2 = 0.$$
$$\lambda_2 \longleftrightarrow \vec{v}_2$$

$$\lambda_1 \vec{v}_1\cdot\vec{v}_2 = (\underline{\lambda v_1})^T \vec{v}_2 = (A\vec{v}_1)^T \vec{v}_2$$
$$= (v_1)^T A^T \vec{v}_2$$
$$= (v_1)^T \underline{A\vec{v}_2}$$
$$= (v_1)^T \lambda_2 \vec{v}_2$$
$$= \lambda_2 \vec{v}_1\cdot\vec{v}_2.$$

$$\vec{v}_1\cdot\vec{v}_2 = 0$$

$A = PDP^{-1}$ $\quad$ P is orthonormal matrix $\quad P^{-1} = P^T$.

$$A^T = (PDP^T)^T = P^{TT} D^T P^T = PDP^T = A$$

A is orthogonally diagonalizable

$\equiv \quad A = PDP^T$

P orthonormal

D diagonal.

Start w/ Symmetric matrix A

Find char eqn
Solve for all roots. $\rightarrow$ eigenvalues are all real!
$\lambda_1 \cdots \lambda_k$.

For each λ

Solve $\underline{(A - I\lambda)} \vec{x} = 0$

$\Downarrow$

Give m lin indep solutions if multiplicity of λ is m. $\vec{x}_1 \cdots \vec{x}_m$

Same span.

Apply Gram-Schmidt to get orthonormal $\vec{v}_1 \cdots \vec{v}_m$

$D = \begin{bmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_k \end{bmatrix}$ $\quad P = \begin{bmatrix} \vec{v}_1 & \cdots & \vec{v}_n \end{bmatrix}$.

<u>The Spectral Theorem</u> An $n \times n$ Symmetric matrix has:

a) n real eigenvalues counting multiplicities.

b) The dimension of the eigenspace for each eigenvalue λ = the multiplicity of the root λ in the char. eqn.

c) Eigenspaces are mutually orthogonal.

d) A is orthogonally diagonalizable.