Recap: Selecting \( n \) items from a set of \( m \) varieties.

- Items of the same variety are indistinguishable.
- The order in which the items are selected does not matter.
- Unlimited quantity of each item to select from.

Picking a dozen donuts from 4 varieties (choc, glazed, jelly, plain).

Bijection between binary code words with \( m \) 1's and \( n \) 0's.

\[
\binom{n+m-1}{m-1}
\]

Added constraint: at least x of some variety (at least 4 chocolate).

\[
\binom{8+4-1}{4-1} = \binom{11}{3}
\]
Selecting 12 donuts of 4 varieties.

Bakery has short supply of chocolate (only 7 left).

By complement,

\[
\binom{12 + 4 - 1}{4 - 1} - \binom{4 + 4 - 1}{4 - 1} = 12 - r
\]

Number of solutions to the equation:

\[x_1 + x_2 + x_3 + x_4 = 12\]

Each \(x_i\) must be a non-negative integer.

Bijecton: Ways to select 12 donuts from 4 varieties

\[X_i = \# \text{ selected from } i^{th} \text{ variety}\]
\[ X_1 = X_2 = X_3 = X_4 = \]
\[ \# \text{ choc} \quad \# \text{ glazed} \quad \# \text{ jelly} \quad \# \text{ plain}, \]

\[ \begin{cases} 
4 \text{ choc} \\
0 \text{ glazed} \ \\
3 \text{ jelly} \\
5 \text{ plain} \\
\end{cases} \quad \rightarrow \quad \begin{cases} 
X_1 = 4 \\
X_2 = 0 \\
X_3 = 3 \\
X_4 = 5 \\
\end{cases} \frac{12}{12}.
\]

\[ \begin{align*}
X_1 &= 2 \\
X_2 &= 5 \\
X_3 &= 1 \\
X_4 &= 4 \\
\end{align*} \quad \rightarrow \quad \begin{align*}
2 \text{ choc} \\
5 \text{ glazed} \\
1 \text{ jelly} \\
4 \text{ plain}. \\
\end{align*} \]

\# solutions to:

\[ X_1 + X_2 + X_3 + \ldots + X_m = n \quad \# \text{ selected.} \]

Each \( X_i \) is a non-negative integer.

\[ \rightarrow \binom{n+m-1}{m-1} \]
Permutations w/ Repetition.

# permutations of $S = \{ H, E, L, P \}$.

HELP, HPEL, EHELP
HEPL, EHLPL, etc.
HLEP, etc.
HLPE, etc.

What about permutations of $\{ B, A, L, \tilde{L} \}$? 

(\# permutations of BALL) = \left( \frac{\# permutations of B\tilde{A}L}{2} \right) = \frac{4!}{2}.
What about: \( \exists A, L, L, L^3 \) ?

\[
\text{ALLL} \Rightarrow \# \text{ permutations.}
\]

\[
\begin{array}{c}
\text{LALL}\\
\text{LALL}\\
\text{LALL}\\
\text{LALL}\\
\text{LALL}\\
\text{LALL}\\
\end{array}
\]

\[
\frac{4!}{6} = \left( \frac{\# \text{ permutations of } \text{ALLL, LLL}}{6} \right) = \frac{\# \text{ permutations of } \exists A, L, L, L^3}{9}
\]

\[
6-4-1
\]
What about: \( \exists A, L, L, L^3 \) ?

\[
\text{ALLL} \Rightarrow \# \text{ permutations.}
\]

\[
\begin{align*}
\text{LALL} \\
\text{LALL} \\
\text{LLAL} \\
\text{LALL} \\
\text{LALL} \\
\text{LALL} \\
\end{align*}
\]

# permutations of \( \exists A, L, L, L^3 \) 

# permutations of \( \exists A, L, L, L^3 \)
\[
\begin{aligned}
\sum \frac{x_1^r}{r!} & \quad \text{if } r_1 + r_2 + \cdots + r_k = n.
\end{aligned}
\]
\[ \frac{n!}{4! \cdot 4! \cdot 2! \cdot 2!} \]