ICS 6D **Due: Wednesday, October 29, 2014, 3:00PM**

Homework 3

Instructor: Sandy Irani

1. Prove that any postage of 8 cents or more can be made from 5-cent or 3-cent stamps.

2. Problem 1 from homework 2 ask you to define the set of properly nested sequences that have parentheses and curly braces. Give an inductive proof that for sequences in this set, the number of left regular parens (is the same as the number of right regular parens) and the number of left curly braces $\{$ is equal to the number of right curly braces $\}$.

3. The sequence $\{g_n\}$ is defined recursively as follows:

   - $g_0 = 51$
   - $g_1 = 348$.
   - $g_n = 5 \cdot g_{n-1} - 6 \cdot g_{n-2} + 20 \cdot 7^n$

   Use induction to show that $g_n = 3^n + 2^n + 49 \cdot 7^n$.

4. - $f_0 = \frac{5}{3}$
   - $f_1 = \frac{11}{3}$.
   - $f_n = 3 \cdot f_{n-1} + 4 \cdot f_{n-2} + 6n$

   Use induction to show that $f_n = 2 \cdot 4^n + \frac{3}{2}(-1)^n - n - \frac{11}{6}$.

5. Compute the value of the following expressions:

   (a) 344 mod 5.

   (b) 344 div 5.

   (c) -344 mod 5.

   (d) -344 div 5.

   (e) 38^7 mod 3.

   (f) $(72 \cdot (-65) + 211)$ mod 7.

   (g) $(77 \cdot (-65) + 147)$ mod 7.

   (h) (44^{12}) mod 6.

   (i) (17^{12}) mod 3.

6. Give the multiplication table for $\mathbb{Z}_7$.

7. Some numbers and their prime factorizations are given below.

$$\begin{aligned} 140 &= 2^2 \cdot 5 \cdot 7 \\ 1078 &= 2 \cdot 7^2 \cdot 11 \\ 175 &= 5^2 \cdot 7 \\ 25480 &= 2^3 \cdot 5 \cdot 7^2 \cdot 13 \end{aligned}$$

   Express each of the quantities below as a product of primes:

(a) $gcd(1078, 140)$

(b) $gcd(1078, 25480)$

(c) $lcm(1078, 140)$

(d) $lcm(175, 25480)$

(e) $lcm(140, 25480)$

(f) $175 \cdot 25480$

(g) $25480/140$

8. Suppose that two positive integers x and y are expressed using a common set of primes as follows:

$$\begin{aligned} x &= p_1^{\alpha_1} \cdot p_2^{\alpha_2} \cdot p_3^{\alpha_3} \cdots p_r^{\alpha_r} \\ y &= p_1^{\beta_1} \cdot p_2^{\beta_2} \cdot p_3^{\beta_3} \cdots p_r^{\beta_r} \end{aligned}$$

   (a) What is the prime factorization for $x \cdot y$?

   (b) If $y|x$, give the prime factorization for x/y. (It's OK to have some p^0's in your prime factorization).

9. Let a, b and c be positive integers. Consider the statement:

   If $a|b$ or $a|c$, then $a|bc$.

   (a) Show that the statement is true.

   (b) Show that the converse of the statement is not true by giving a counter-example. (The converse of the statement says that if $a|bc$, then $a|b$ or $a|c$.)

   (c) Is the converse of the statement true if a is a prime number?

10. Group the following numbers according to equivalence mod 13. That is, put two numbers in the same group if they are equivalent mod 13.

$$\{-63, -54, -41, 11, 13, 76, 80, 130, 132, 137\}$$

11. Use the prime number theorem to give an approximation for the number of prime numbers in the range 2 throught $10,000,000$.