ICS 6D **Due: Wednesday, November 19, 2014, 3:00PM**

Homework 6

Instructor: Sandy Irani

Leave your answer for the questions below as an arithmetic expression, including the $P(n, k)$ or $\binom{n}{k}$ notation. You do not have to compute a final numeric value.

1. For this problem you will count the number of binary strings of length 10 such that each string to be counted satisfies the following constraints:
   (a) At least one 1.
   (b) At least one 1 and at least one 0.
   (c) Exactly five 1's or begins with a 0.
   (d) The string begins or ends with a 0.
   (e) The string 01010101 appears somewhere in the string.
   (f) The number of 1's is five or it has an even number of 1's.
2. A five card hand is dealt from a standard playing deck.
   (a) How many hands have at least one 7?
   (b) How many hands have at least one Jack, Queen, or King?
   (c) How many hands have exactly two 7's or exactly two 8's?
3. How many integers between 1 and 140 are integer multiples of 2, 5, or 7?
4. Ten identical prizes are given to a class with 100 students. Each student can receive at most one prize. There are two particular students in the class named Alice and Bob.
   (a) How many ways are there to distribute the prizes so that either Alice or Bob (or both) receive a prize?
   (b) Solve the same problem, except with different prizes.
5. A family with two parents, three daughters and four sons line up for a photograph. How many ways are there for the family to line up so that the mother is next to at least one of her three daughters?
6. A university offers 3 calculus classes: Math 2A, 2B and 2C. This years freshmen have all taken at least one of the three classes. In particular, 28 ofthem have taken Math 2A, 28 have taken Math 2B, and 25 have taken Math 2C. 11 students have taken Math 2A and 2B, 9 have taken Math 2A and 2C, and 10 have taken Math 2B and 2C. Only 3 have taken all three classes. How many freshmen were there this year?
7. The seventh line of Pascal's triangle is: 1-7-21-35-35-21-7-1.
   (a) What is the 8^{th} line of Pascal's triangle?
   (b) Use your answer to show the expanded form of $(x + y)^8$.
8. What is the coefficient of x^5y^3 in $(3x - 4y)^8$?
9. What is the coefficient of x^6y^3 in $(-2x + 5y)^9$?

10. Use the Binomial Theorem to find the closed form for:

$$\sum_{k=0}^{n} \binom{n}{k} 3^k (-1)^{n-k}.$$

11. For this question, your answers to the first three parts should just be a single "n choose k" term, not a sum of such terms.

    (a) How many 7-subsets are there of the set $\{x_1, x_2, ..., x_n\}$?

    (b) How many of those subsets include x_1?

    (c) How many do not include x_1?

    (d) Use a combinatorial argument to establish an identity involving the three answers above.

12. 50 students take an exam that is scored out of 50 points. No partial credit is given, so every student's score is an integer. Can you conclude that there are two students with the same score?

13. A round-robin tournament is one where each player plays each of the other players exactly once. Prove that if no person loses all his games, then there must be two players with the same number of wins.

14. There are 14 3-digit numbers in a list.

    (a) Can you conclude that there are two distinct subsets of the 14 numbers that have the same sum?

    (b) What about if there are 13 numbers?

15. Seven members of the track team run the mile. Their times are all faster than 7 minutes but not faster than 6 minutes.

    (a) Can you conclude that there are two runners whose times are less than nine seconds apart?

    (b) What if there are eight runners?

16. A team wishes to purchase 12 shirts of the same color. A store sells shirts in 4 different colors. What must the inventory of the store be in order to conclude that there at least 12 shirts in one of the four colors?