\[ \exists x \exists y (C(x) \land A(y) \land B(x,y)) \]

**Company 1**

<table>
<thead>
<tr>
<th>Name</th>
<th>( T(x) )</th>
<th>( B(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Joe</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>Belinda</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>Lucy</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>Brad</td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>

There is an executive who got a large bonus. Not true for Company 1

\[ \exists x \ (T(x) \land B(x)) \neq \]

True for Company 1

For example:

\[ T(\text{Brad}) \rightarrow B(\text{Brad}) \]

\[ \text{F} \rightarrow \text{T} \]

\[ T(\text{Lucy}) \rightarrow B(\text{Lucy}) \]

\[ \text{F} \rightarrow \text{T} \]

**Company 2**

<table>
<thead>
<tr>
<th>Name</th>
<th>( T(x) )</th>
<th>( B(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nancy</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>Ingrid</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>Jose</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>Ralph</td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>

Every executive got a large bonus.

\[ \forall x \ (T(x) \rightarrow B(x)) \neq \]

\[ \forall x \ (T(x) \land B(x)) \neq \]

Not true for Company 2.

Counterexamples:

\[ T(\text{Jose}) \land B(\text{Jose}) = \text{F} \]

\[ T(\text{Ralph}) \land B(\text{Ralph}) = \text{T} \]
De Morgan’s law for quantified statements.

- \( \neg \exists x \ T(x) = \forall x \ \neg T(x) \).

There does not exist a student in the class who is a transfer student.

Every student in the class is not a transfer student.

Consistent w/ DM’s law on a finite domain:

\[ 7 \left( T(a_1) \lor T(a_2) \cdots \lor T(a_n) \right) \equiv T(x) \land T(a_1) \land \cdots \land T(a_n) . \]

- \( \exists x \ T(x) = \forall x \ \neg T(x) \).

It’s not true that all the students are transfer students.

There is at least one student who is not a transfer student.
What is \( \forall x (x^2 > x) \equiv \exists x \neg (x^2 \leq x) \)?

\( \exists x (x^2 = 2) \equiv \forall x \neg (x^2 = 2) \)

Show: \( \forall x (P(x) \rightarrow Q(x)) \equiv \exists x (P(x) \land \neg Q(x)) \)

True for any domain & blank values.

\[ \forall x (P(x) \rightarrow Q(x)) \]
\[ \exists x \neg (P(x) \rightarrow Q(x)) \]

**DH.**
\[ \exists x (\neg P(x) \lor Q(x)) \]

**Comp. Id.**
\[ \exists x (\neg P(x) \land \neg Q(x)) \]

**DM.**
\[ \exists x (P(x) \land \neg Q(x)) \]
De Morgan's Law with nested quantifiers:

When the negation moves from the left to the right of a quantifier, the quantifier changes:

- $\forall \rightarrow \exists$
- $\exists \rightarrow \forall$

Find a logically equivalent quantified statement in which negation signs immediately precede a predicate:

\[
\neg \forall x \exists y \forall z \ P(x,y,z).
\]

\[
\equiv \exists x \neg \forall y \exists z \ P(x,y,z).
\]

\[
\equiv \exists x \forall y \exists z \neg P(x,y,z).
\]

\[
\neg \forall x \exists y \left( D(x) \rightarrow P(x,y) \right).
\]

\[
\equiv \exists x \forall y \neg \left( D(x) \rightarrow P(x,y) \right)
\]

\[
\equiv \exists x \forall y \neg \left( \neg D(x) \lor P(x,y) \right)
\]

\[
\equiv \exists x \forall y \left( \neg D(x) \land \neg P(x,y) \right)
\]

\[
\neg \exists x \forall y \left( T(x) \land S(x,y) \right).
\]

\[
\equiv \forall x \exists y \left( \neg T(x) \lor \neg S(x,y) \right)
\]

\[
\equiv \forall x \exists y \left( \neg T(x) \lor \neg S(x,y) \right)
\]
Argument: \[ p_1 \]
\[ p_2 \]
\[ \vdots \]
\[ p_n \]
\[ \therefore c \]

\[ \therefore \text{conclusion}. \]

An argument is valid if the conclusion is true whenever all the hypotheses are true.

Argument is valid if \((p_1 \land p_2 \land \ldots \land p_n) \rightarrow c\) is a tautology.

For example:

\[ \frac{P \rightarrow q}{\therefore q} \]

\[
\begin{array}{c|c|c}
P & q & P \rightarrow q \\
T & T & T \\
T & F & F \\
F & T & F \\
F & F & T \\
\end{array}
\]

Here's an example of an argument that is not valid:

\[ \rightarrow P \rightarrow q \equiv T \]
\[ \rightarrow \neg P \equiv T \]
\[ \rightarrow \therefore \neg q \equiv F \]

\[ (p \rightarrow q) \land \neg q \rightarrow \neg q \]

is not a tautology.

\[ (p = F, q = T) \]
Arguments are often expressed in English as in:

\[ \Rightarrow \text{If I will study for my exam, then I will pass my exam.} \]
\[ \text{I will study for my exam.} \]
\[ \therefore \text{I will pass my exam.} \]

The form of an argument expressed in English is obtained by replacing each distinct proposition with a propositional variable:

\[ p : \text{I will study for my exam.} \]
\[ q : \text{I will pass my exam.} \]

\[ p \Rightarrow q \]
\[ p \]
\[ \therefore q \]

An argument can be invalid even if the hypotheses and the conclusion are true:

If 5 is even, then 7 is even.
\[ 5 \text{ is not even} \]
\[ \therefore 7 \text{ is not even.} \]

\[ p \Rightarrow q \]
\[ p \]
\[ \therefore q \]

In order for an argument to be valid, it must be true regardless of the actual truth values of the propositional variables.
This same argument could be used to conclude something that is not true:

\[ p \implies q \]

\[ \begin{array}{c|c|c}
  p & q & p \implies q \\
  \hline
  T & T & T \\
  T & F & F \\
  F & T & T \\
  F & F & T \\
\end{array} \]

Another example:

\[ (p \lor q) \implies q \]

\[ \begin{array}{c|c|c|c}
  p & q & p \lor q & (p \lor q) \implies q \\
  \hline
  T & T & T & T \\
  T & F & T & T \\
  F & T & T & T \\
  F & F & F & T \\
\end{array} \]

An alternative way to show an argument is valid:

apply arguments that have already been shown to be valid.

(Much like showing logical equivalences using laws of propositional logic which are themselves logical equivalences already established via truth tables.)
There is a set of rules of inference that are useful for putting together valid arguments. (6 things then):

\[ p \rightarrow q \]
\[ q \rightarrow r \]
\[ \therefore p \rightarrow r \]

1. \( p \rightarrow q \) Hyp.
2. \( q \rightarrow r \) Hyp.
3. \( p \rightarrow r \) Hyp. Syl., 1, 2.
4. \( \neg r \) Hyp.
5. \( \therefore \neg p \) Mod. Toll 3, 4.

Sometimes in reasoning it is clearer to translate English sentences into the language of logic and use rules of inference to reason.

If I have a head wound, I can be sure. If I am sure I take aspirin. I am not taking aspirin.

\[ w: \text{head wound} \]
\[ s: \text{sure.} \]
\[ a: \text{aspirin} \]
\[ w \rightarrow s \]
\[ s \rightarrow a \]
\[ \neg a \]
\[ \therefore \neg w. \]
If it's not foggy or it doesn't rain then the race will be held and we will go see the race.
If the race is held, there will be a trophy ceremony.
The trophy ceremony was not held.

\[
(\neg f \vee \neg r) \rightarrow (\neg c \wedge \neg g)
\]
\[
c \rightarrow t
\]
\[
\neg t
\]
\[
\neg c \vee \neg g
\]

1. \(c \rightarrow t\) Hyp.
2. \(\neg t\) Hyp.
3. \(\neg c\) Mod. Toll 1 \& 2.
4. \(\neg c \vee \neg g\) Addition 3.
5. \(\neg (c \wedge g)\) DM 4.
6. \((\neg f \vee \neg r) \rightarrow (c \wedge g)\) Hyp.
7. \(\neg f \wedge (\neg f \vee \neg r)\) Mod. Toll 5, 6.
8. \((\neg f \wedge \neg f)\) DM 7.
9. \((\neg f \wedge \neg f)\) Double Neg. 8.
10. \(r\) Subj. 9.

\[
(\neg f \vee \neg r) \rightarrow (c \wedge g)
\]
\[
\neg f \leftarrow g\]
\[
\neg f \rightarrow \neg g\]