ICS 6B	**Due: Wednesday, October 14, 2015, 5:30PM**

Homework 2

Instructor: Sandy Irani

Covers Sections 1.7-1.12. There are no challenge activities for this material.

Please make sure to staple together all the pages of your written homework before putting it in the ICS 6B slot. Also write your student ID number and your name very clearly in the upper right corner of every page. The written portion should be turned into the dropbox labeled "ICS 6B" on the first floor of Bren Hall.

1. Apply De Morgan's Law to each English statement below to give another English sentence with the same meaning. For the first four questions, the domain is a set of people who came to take a test, so you can assume that everyone did actually showed up for the test. For example, if the sentence in part (a) were to be expressed in logic, it would be $\neg\forall x S(x)$, where $S(x)$ is the predicate that x had sharpened pencils at the test.

   (a) It is not true that everyone showed up to the test with sharpened penciles.

   (b) It's not true that everyone showed up for the test with sharpened penciles and a calculator.

   (c) No one showed up for the test with sharpened pencils.

   (d) No one showed up to the test with sharpened penciles and a calculator.

   (e) It's not true that everyone knows someone's phone number.

   (f) It's not true that someone sent everyone an email.

2. Apply De Morgan's Law and other laws of propositional logic to give an equivalent expression so that every negation symbol applied directly to a predicate. For example $\neg P(x)$ is OK, but $\neg(P(x) \vee Q(x))$, $\neg(P(x) \to Q(x))$ and $\neg\forall x P(x)$ are all not acceptable.

   (a) $\neg\forall x(P(x) \vee Q(x))$.

   (b) $\neg\forall x \exists y T(x, y)$.

   (c) $\neg\exists x(P(x) \leftrightarrow Q(x))$.

   (d) $\neg\forall x \forall y(P(x) \to T(x, y))$.

   (e) $\neg\exists x \forall y(P(x) \wedge (Q(x) \to T(x, y)))$.

3. The domain of discourse is a set of girls in a girl scout troop. The scouts are selling boxes of cookies for their annual fund raiser. Nancy is one of the girls in the troop. A *big seller* is a girl scout who has sold at least 50 boxes. Define the following predicates.

   - $C(x, y)$: x bought a box of cookies from y.
   - $S(x)$: x is a first year girl scout.
   - $B(x)$: x is a big seller.

   Give a qunatified statement that is equivalent to each of the English statements below:

   (a) At least one girl scout bought a box of cookies from everyone.

   (b) All the girl scouts bought a box of cookies from someone.

   (c) Somone bough a box of cookies from a big seller.

(d) Everyone bought a box of cookies from a big seller.

(e) Every first year scout bought a box of cookies from every big seller.

(f) There is exactly one big seller.

(g) Everyone bough a box of cookies from everyone else.

(h) Nancy bought a box of cookies from everyone.

(i) There is a first year scout who bought a box of cookies from everyone.

4. Determine the truth value of each expression below. The domain is the set of all real numbers.

   (a) $\forall x \exists y (xy > 0)$.

   (b) $\exists x \forall y (xy = 0)$

   (c) $\forall x \exists y (y^2 = x)$.

   (d) $\forall x \forall y \exists z (z = (x - y)/3)$.

   (e) $\forall x \exists y \forall z (z = (x - y)/3)$.

   (f) $\forall x \forall y (xy = yx)$.

5. The domain of discourse for this problem is a group of three people who are working on a project. To make notation easier, the people are numbered 1, 2, 3. The predicate $M(x, y)$ indicates whether x has sent an email to y, so $M(2, 3)$ is read "Person 2 has sent an email to person 3." The table below shows the value of the predicate $M(x, y)$ for each (x, y) pair. The truth value in row x and column y gives the truth value for $M(x, y)$.

M	1	2	3
1	T	T	T
2	T	F	T
3	T	T	F

   Give the truth value for each expression:

   (a) $\forall x \forall y M(x, y)$.

   (b) $\forall x \forall y ((x \neq y) \rightarrow M(x, y))$.

   (c) $\exists x \exists y \neg M(x, y)$.

   (d) $\forall x \exists y M(x, y)$

   (e) $\forall x \exists y \neg M(x, y)$.

   (f) $\exists x \forall y M(x, y)$.

   (g) $\exists x \forall y \neg M(x, y)$.

6. Indicate whether the argument is valid or invalid. For valid arguments, prove that the argument is valid using a truth table. For invalid arguments give truth values for the variables showing that the argument is not valid.

   (a)

$$\begin{array}{l} p \lor q \\ p \\ \hline \therefore \neg q \end{array}$$

(b)

$$\begin{array}{l} p \leftrightarrow q \\ p \vee q \\ \hline \therefore p \end{array}$$

(c)

$$\begin{array}{l} p \\ q \\ \hline \therefore p \leftrightarrow q \end{array}$$

(d)

$$\begin{array}{l} p \vee q \\ \neg q \\ \hline \therefore p \leftrightarrow q \end{array}$$

(e)

$$\begin{array}{l} (p \vee q) \rightarrow r \\ \hline \therefore (p \wedge q) \rightarrow r \end{array}$$

(f)

$$\begin{array}{l} (p \wedge q) \rightarrow r \\ \hline \therefore (p \vee q) \rightarrow r \end{array}$$

7. Use the rules of inference and the laws of propositional logic to prove that each argument is valid. Number each line of your argument and label each line of your proof ”Hypothesis” or with the name of the rule of inference used at that line. If a rule of inference is used, then include the numbers of the previous lines to which the rule is applied.

   (a)

$$\begin{array}{l} (p \vee q) \rightarrow r \\ \neg r \\ \hline \therefore \neg q \end{array}$$

   (b)

$$\begin{array}{l} (p \vee q) \rightarrow r \\ \text{p} \\ \hline \therefore r \end{array}$$

   (c)

$$\begin{array}{l} p \vee q \\ \neg p \vee r \\ \neg q \\ \hline \therefore r \end{array}$$

8. Prove that each argument is valid by replacing each proposition with a variable to obtain the form of the argument. Then use the rules of inference to prove that the form is valid.

   (a)

If I drive on the freeway, I will see the fire.
I will drive on the freeway or take surface streets (or both).
I am not going to take surface streets.
$\therefore$ I will see the fire.

(b)

If I work out hard, then I am sore.
If I am sore, I take an aspirin.
I did not take an aspirin.
$\therefore$ I did not work out hard.

9. Which of the following arguments are valid? Explain your reasoning.
   (a) I have a student in my class who is getting an A. Therefore, John, a student in my class is getting an A.
   (b) Every girl scouts who sells at least 50 boxes of cookies will get a prize. Suzy, a girl scout, got a prize. Therefore Suzy sold 50 boxes of cookies.

10. Prove that the given argument is valid by replacing each predicate with a variable to obtain the form of the argument. Then use the rules of inference to prove that the form is valid.
    (a) The domain of discourse is the set of students at a university. Larry and Hubert are students in the domain.

    Larry and Hubert are taking Boolean Logic.
    Any student who takes Boolean Logic can take Algorithms.
    $\therefore$ Larry and Hubert can take Algorithms.

    (b) The domain of discourse is the set of musicians in an orchestra.

    Everyone practices hard or plays badly (or both).
    Someone does not practice hard.
    $\therefore$ Someone plays badly.

    (c) The domain is the set of people who work at a company. Linda is an employee of the company.

    Linda owns a Ferrari.
    Everyone who owns a Ferrari has gotten a speeding ticket.
    $\therefore$ Linda has gotten a speeding ticket.