ICS 6B	**Due: Wednesday, October 21, 2015, 5:30PM**

Homework 3

Instructor: Sandy Irani

Covers Sections 2.1-2.5. There are no challenge activities for this material.

Please make sure to staple together all the pages of your written homework before putting it in the ICS 6B slot. Also write your student ID number and your name very clearly in the upper right corner of every page. The written portion should be turned into the dropbox labeled "ICS 6B" on the first floor of Bren Hall.

If a problem requires a proof, then your solution should clearly state the beginning and end of the proof. Your proof should state what proof technique is being used, as well as an initial statement about what you are assuming and what is being proven.

1. Disprove each of the following statements by giving a counter-example.
   (a) If n is an integer and n^2 is divisible by 4, then n is divisible by 4.
   (b) For every positive integer x, $x^3 \leq 2^x$.
   (c) Every real number has a multiplicative inverse. (The multiplicative inverse of a real number x, is a real number y such that $xy = 1$.)
2. Determine whether the statement is true or false. If the statement is true, give a proof. If the statement is false, give a counter-example.
   (a) If x and y are both even integers, then $x + y$ is also an even integer.
   (b) x and y are integers and $x + y$ is even, then x and y are both even.

Prove each of the statements below:

3. If x is a real number and $x < 3$, then $12 - 7x + x^2 > 0$.
4. The sum of two odd integers is an even integer.
5. If x is an integer and $x^2 - 2x + 7$ is even, then x is an odd integer.
6. If x and y are positive real numbers and $xy > 400$, then $x > 20$ or $y > 20$.
7. If x is an irrational number, then $-x$ is also an irrational number.
8. If n is an integer and n^3 is even, then n is even.
9. The average of three real numbers is greater than or equal to at least one of the numbers.
10. $\sqrt[3]{2}$ is irrational.
11. For any real number z, $|z| \geq z$ and $|z| \geq -z$.
12. For real numbers x and y, $|x + y| \leq |x| + |y|$.
    Hint: You can use the fact proven in the previous problem that for any real number z, $z \leq |z|$ and $-z \leq |z|$. Try the following two cases: $x + y < 0$ and $x + y \geq 0$.

13. Let x and y be two integers. If xy is not an integer multiple of 5, then neither x nor y is an integer multiple of 5.

    Hint: your proof should be by contrapositive and will involve cases.