# Homework 7

Instructor: Sandy Irani

Covers Sections 6.7-6.10
There are no challenge activities for this material.
Please make sure to staple together all the pages of your written homework before putting it in the ICS 6B slot. Also write your student ID number and your name very clearly in the upper right corner of every page. The written portion should be turned into the dropbox labeled "ICS 6B" on the first floor of Bren Hall.

1. Below is a Hasse diagram for a partial order:
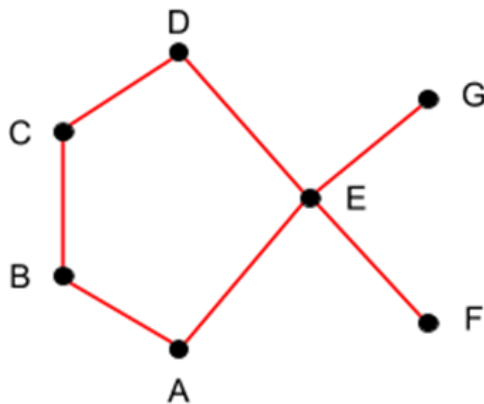
   (a) Name the minimal elements.

   (b) Name the maximal elements.

   (c) Which of the following pairs are comparable? $(A, D), (B, E), (G, F), (D, B), (C, F), (C, E)$

2. Draw a Hasse diagram for the following relation on the set $A = \{a, b, c, d, e, f\}$.

$$R = \{(b, e), (b, d), (c, a), (c, f), (a, f), (a, a), (b, b), (c, c), (d, d), (e, e), (f, f)\}$$

3. For each relation defined below, indicate whether the relation is a partial order or strict order or neither. If it is neither, indicate which property is violated. In the case that it is a partial order or a strict order, indicate whether it is also a total order. Note that when the domain is a set of people, "not necessarily" can be a correct answer.

   (a) The domain is the set of all positive integers. a is related to b if $b = a3n$, for some positive integer n.

   (b) The domain is the power set of a finite set. X is related to Y if $X - Y$ is not empty.

   (c) The domain is the set of all words in the Engilsh language (as defined by, say, Webster's dictionary). Word x is related to word y if x appears before y in alphabetical order.

   (d) The domain is the set of all words in the Engilsh language (as defined by, say, Webster's dictionary). Word x is related to word y if x appears as a substring of y. For example, "ion" is related to the word "companions" because the letters i-o-n appear in order in the word "companions".

(e) The domain is the set of all students in a class. Student x is related to student y if x knows student y's phone number.

(f) The domain is the set of all cell phone towers in a network. Two towers can communicate if they are within a distance of three miles from each other. x is related to y, if x can send information to y through a path of communication links.

(g) The domain is the set of members of a chess club. x is related to y if x has ever beaten y in a game of chess.

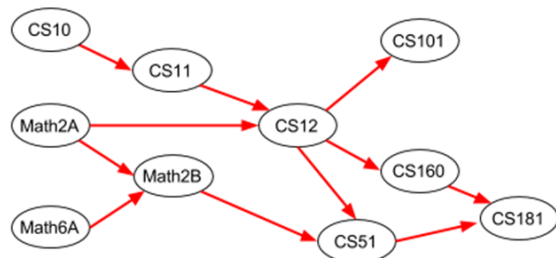4. Give two different topological sorts for the digraph below:
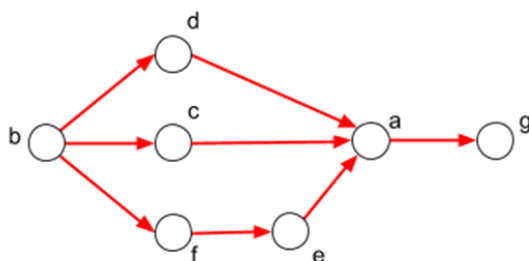
5. Given the graph below:
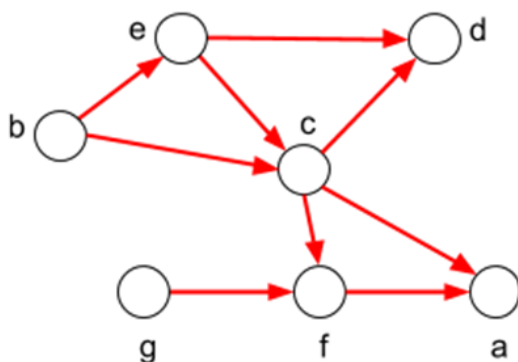
Which orderings of the vertices are topological sorts for the graph?

(a) b, e, c, g, f, a, d

(b) b, g, f, c, e, a, d

(c) b, e, g, c, f, d, a

(d) b, e, c, g, a, f, d

6. Below is a set of required courses for a degree. The directed acyclic graph shows the prerequisite structure for the courses.


Your job is to devise an academic plan for a student. An academic plan is a set of courses for the student to take in each quarter. In each case, you need to devise an academic plan which respects the

prerequisites and takes the fewest number of quarters. You can assume that every course is offered in every quarter.

    (a) Devise an academic plan for the student if he can only take one course per quarter.

    (b) Devise an academic plan for the student if he can take up to two courses per quarter.

    (c) Devise an academic plan for the student if he can take up to three courses per quarter.

    (d) What's the fewest number of quarters the degree will take if the student can take an unlimited number of courses per quarter?
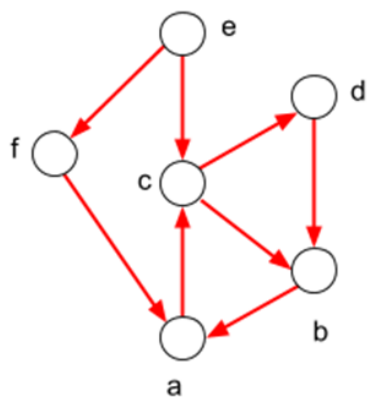
7. Which of the following graphs are acyclic?

Graph F:
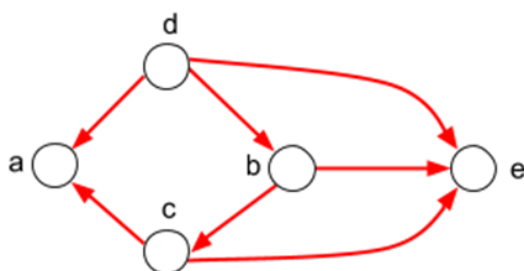
Graph G:
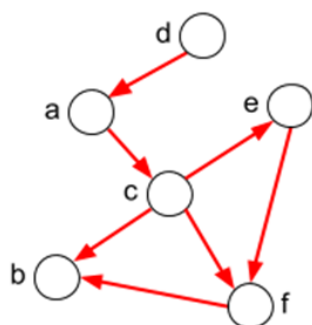
Graph H:

8. The relations below are all defined on a set of people. Which ones are equivalence relations. In the case that the relation is not an equivalence relation, give the condition that is violated.

    (a) (x, y) is in the relation if person x has the same biological mother as person y.

(b) (x, y) is in the relation if person x and person y have at least one common biological parent.

(c) (x, y) is in the relation if x is currently the spouse of y. (Assume that there is no polygamy).

(d) Is the transitive closure of any of the relations above an equivalence relation?

9. Consider the equivalence relation on the set $S = \{7, 2, 13, 44, 56, 34, 99, 31, 4, 17\}$ such that two numbers are equivalent if they have the same remainder when divided by 4. Group the elements of S together that are in the same equivalence class.

10. Which 4-tuples are in the relation $\{(a, b, c, d) : a, b, c, d \text{ are positive integers and } a + b + c + d = 6\}$?

11. List the tuples in the relation $\{(a, b, c) : a, b, c \text{ are positive integers and } 0 < a < b < c < 5\}$.

12. Consider the following small database whose records correspond to flights departing from an airport.

Airline	Flight number	Gate	Destination	Departure time
FlyRight	122	34	Detroit	08:10
JetGreen	221	22	Denver	08:17
JetGreen	122	33	Anchorage	08:22
JetGreen	323	34	Honolulu	08:30
FlyRight	199	13	Detroit	08:47
JetGreen	222	22	Denver	09:10
FlyRight	322	34	Detroit	09:44

(a) Are the attributes **Airline** and **Destination** a key for this database?

(b) Give examples of two separate sets of attributes that are keys for this database

(c) Show the results of the operation SELECT[Airline="JetGreen" and Destination="Denver"]

(d) Show the results of the operation PROJECT[Airline, Gate]

(e) Show the results of the operation SELECT[Destination="Denver"] followed by Project[Gate]

(f) What operation or operations should be performed if you wanted to know which airlines use gate 22?

(g) What operation or operations should be performed if you wanted to know whether there are any flights to Detroit departing before 9:10? A flight is identified by the airline and the flight number.

13. The table below shows the database of course offerings for the Computer Science Department for the current academic year. It's actually just a portion of the course offerings, but for the purposes of this question, assume it lists all the courses.

Course number	Course title	Instructor	Quarter
CS 111	Digital Image Processing	Majumder	Spring
CS 112	Computer Graphics	Majumder	Spring
CS 117	Project in Computer Vision	Fowlkes	Fall
CS 116	Computational Photography Vision	Fowlkes	Winter
CS 122A	Introduction to Data Management	Carey	Spring
CS 122A	Introduction to Data Management	Li	Winter
CS 122C	Principles of Data Management	Carey	Winter

(a) Give a key for this database.

(b) Show the results of the operation SELECT[Course number = "CS122A" $\vee$ Course number = "CS122C"]

(c) Show the results of the operation PROJECT[Instructor]

(d) Show the results of the operation SELECT[Quarter="Spring"] followed by Project[Course number]. Express in English what question this combination of queries is asking.

(e) What operation or operations should be performed if you wanted to know which quarters CS 122A is being offered?

(f) What operation or operations should be performed if you wanted to know which courses are being taught by Prof. Majumder? Assume that a course is identified by its course title.