Student ID Number:

Name:____________________________________

Final Exam

Version A

ICS 6D
Fall 2016
Dec 5, 2016
Instructor: Sandy Irani

Instructions

- Wait until instructed to turn over the cover page.
- The total number of points on the test is 50.
- There are questions on both sides of the page. The back of the last page is for scratch work.
- Special Instructions for Part I of this exam. **Please read carefully:**
  - Each of the questions in Part I counts 1 point. There is no partial credit.
  - You can use any standard notation in your final answer, including $P(n,k)$, $C(n,k)$, $\binom{n}{k}$, the factorial function ($n!$), or standard arithmetic. You will get full credit as long as the expression evaluates to the correct answer.
  - Circle your answer to each question. You should have only one expression inside the circle for each question. We will score the question based only on the value of the expression inside the circle.
  - In all of the questions, the word ”or” means the inclusive ”or”.

Part I

1. A PIN for a bank card consists of four digits. Any of the 10 standard digits can be used anywhere in the PIN, and there are no restrictions on the number of times a digit can be used. How many different PIN numbers are there?

2. How many ways are there to permute the letters in the word ”PAPPARAZZI”?

3. If 100 applicants apply for a job, how many ways are there to select a subset of 9 to be interviewed?

4. How many binary strings of length 8 do **not** begin with 000?

5. What is the coefficient of the a^7b^5 term in $(4a - 3b)^{12}$?

6. There are 20 different servers in a network. 5 idential copies of a movie will be placed on servers in the network with at most one copy per server. How many ways are there to select where the copies will be placed?

7. There are 20 different servers in a network. 5 different movies will be placed on servers in the network with at most one movie per server. How many ways are there to select where the movies will be placed? Note that it matters which movie is placed on which server.

8. Below is a permutation of the set $\{1, 2, 3, 4, 5, 6\}$. Give the permutation that comes next in lexicogrpahical order:

$$(3, 2, 6, 5, 4, 1)$$

9. How many strings of length 9 over the alphabet $\{a, b, c, d, e\}$ have exactly two a's and exactly three b's?

10. A homeowner employs eight kids to do chores every weekend. At the end of the year, she has 100 one-dollar bills that she will distribute to the kids as a year-end bonus. If there is no restriction on the number of dollars that she can give to each kid, how many ways are there for her to distribute the money? You can assume that all the dollar bills are indistinguishable.

11. Below is a 6-subset (a subset with 6 elements) chosen from the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$. Give the 6-subset that comes next in lexicogrpahical order:

$$\{1, 2, 4, 5, 8, 9\}$$

12. A tour guide is planning an itinerary for a group visiting a large city. The group will visit exactly seven different museums on the tour. The tour lasts seven days and the group can visit at most one museum each day. How many different ways are there to plan the itinerary?

13. A pair of dice (one red and one blue) are thrown. Each die has six sides. The dice are fair, so each outcome is equally likely. What is the probability that the number that comes up on the blue die is one more than the number on the red die?

14. Each month, Ms. Jenkins selects one student to be the ”student of the month” from her class of 20 students. There are nine months in the school year and a student can not be selected more than once in the same year. How many ways are there for Ms. Jenkins to make her selections for the year? Note that it matters which student is chosen in which month. For example, Alicia chosen in January and Frederico chosen in February is different than Frederico chosen in January and Alicia chosen in February.

15. Anika buys three binders for school. The binders at the store come in 10 different colors but are otherwise identical. Anika will select three binders that are all different colors. How many ways are there for her to make her selection?

16. The Binomial Theorem says that for any positive integer n and any real numbers x and y,

$$\sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k} = (x+y)^n.$$

Use the Binomial Theorem to determine the value for $\sum_{k=0}^{n} (-1)^k \binom{n}{k}$. Your answer should not be a sum of terms.

17. How many binary strings of length 10 have exactly five 1's or begin with a 0?

18. How many strings of length 10 over the alphabet $\{a, b, c\}$ have exactly two a's or have exactly two b's or have exactly two c's?

19. A coin is tossed 12 times. The coin is fair so each of the 2^{12} outcomes is equally likely. What is the probability that the number of flips that come up heads is equal to the number of flips that come up tails?

20. 20 choclate truffles are given to 5 people. The truffles are all different flavors (e.g., chocolate-raspberry, chocolate-caramel, etc.). Each person is given the same number of truffles. How many ways are there to distribute the truffles? Note that who gets which flavor is important.

21. The 7^{th} row of Pascal's triangle is:

$$1 \quad 7 \quad 21 \quad 35 \quad 35 \quad 21 \quad 7 \quad 1$$

Give the 8^{th} row of Pascal's triangle.

22. Three different colored dice (red, blue, and green) are thrown. Each die has six sides. The dice are fair so each outcome is equally likely. What is the probability that at least one of the die comes up 6?

23. A basketball team has 12 players. The coach must select her starting line-up. She will select a player for each of the five positions: center, right forward, left forward, right guard, left guard. However, there are only three of the 12 players who can play center. Otherwise, there are no restrictions on who can play which position. How many ways are there for her to select the starting line-up?

24. A basketball team has 12 players. The coach must select her starting line-up. She will select a player for each of the five positions: center, right forward, left forward, right guard, left guard. How many ways are there for her to select the starting line-up?

25. A wedding party consisting of eight people line up for a photo. How many ways are there to line up the people in the wedding party so that the bride is **not** in the leftmost position?

26. 15 different tasks are assigned to 8 different people. There are no restrictions on the number of tasks that can be given to any one person. How many ways are there to assign the tasks? Note that who gets which task is important.

27. Cornelia goes into a candy store and selects 12 pieces of taffy. The candy store offers 85 varieties of taffy. How many ways are there for Cornelia to select her 12 pieces of taffy?

28. How many solutions are there to the equation below? Each variable $x_1, \ldots, x_5$ must be a **positive** integer.

$$x_1 + x_2 + x_3 + x_4 + x_5 = 27$$

29. A girls scout troop consisting of 13 people line up single file to hike on a narrow path. The 13 people in the troop consist of 11 scouts and 2 leaders. How many ways are there for the troop to line up so that one of the leaders is at the beginning and the other is at the end of the line?

30. Suzy is selecting 20 donuts to bring to her club meeting. The donut store sells seven varieties of donuts. Donuts of the same variety are all the same. There is a large supply of each variety of dounts, except for jelly donuts. There are only five jelly donuts available. How many ways are there for Suzy to select the donuts?

Part II

31. *(2 points)* Give the base 9 representation of the decimal number 101.

32. *(2 points)* The greatest common divisor of 133 and 162 is 1, and

$$1 = 33 \cdot 162 - 65 \cdot 133.$$

What is the multiplicative inverse of 133 mod 162?

33. *(2 points)* Give the binary representation of $(A6D1)_{16}$.

34. *(3 points)* Fill in the blanks for the recursive algorithm below so that it returns $(n!)$. The input n can be any non-negative integer. The value returned in the last line should be a mathematical expression that uses variables y and/or n and only uses multiplication operations.

```
Factorial(n)

    If ( n = 0 ) Return( ______________)        // Base case

    y := Factorial(______________) // Recursive Call

    Return( ________________________________)
End
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35. *(3 points)* Let $S(n)$ be a statement parameterized by a positive integer n. Consider a proof that uses strong induction to prove that for all $n \geq 3$, $S(n)$ is true. The base case proves that $S(3)$, $S(4)$, $S(5)$ and $S(6)$ are all true. Fill in the blanks below to get a correct statement of what is assumed and proven in the inductive step.

For $k \geq$____________, suppose that S(j) is true for any $j =$____________to k. We will show that

____________is true.

36. *(3 points)* Compute 40^{19} mod 7.

37. *(2 points)* The equation below is a characteristic equation for a recurrence relation that describes a sequence $\{f_n\}$. Give the *general solution* to the recurrence equation. That is, express f_n as a linear combination of terms. Note that you will not know the actual coefficients in the linear combination since you are not given the initial values for the sequence, so you can use α_1, α_2, etc.

$$(x-5)(x-5)(x+6)=0.$$

38. *(3 points)* A set S is a subset of $\{a,b\}^*$ and is defined recursively as follows:

Basis: $\lambda \in S$ and $a \in S$.

Recursive rules: If string x is in S, then

- $xaa \in S$
- $xb \in S$

Exclusion statement: A string x is in S only if x is given in the basis or x can be constructed by repeatedly applying the recursive rule starting with a string given in the basis.

List all the strings of length 3 in S:

This area is for scratch work.