

$$n \geq \underline{1} \quad \sum_{j=1}^n j \cdot 2^j = (n-1) 2^{n+1} + 2.$$

Base: $n=1$

$$\sum_{j=1}^1 j \cdot 2^j = \underbrace{(1-1)}_0 \cdot 2^{1+1} + 2$$

$$1 \cdot 2^1 = 2.$$

Ind Step: Assume for $k \geq 1$

$$\sum_{j=1}^k j \cdot 2^j = (k-1) \cdot 2^{k+1} + 2$$

prove

$$\sum_{j=1}^{k+1} j \cdot 2^j = (k) \cdot 2^{k+2} + 2.$$

$$\sum_{j=1}^{k+1} \underbrace{j \cdot 2^j}_{\text{ind step}} = \sum_{j=1}^k j \cdot 2^j + \underbrace{(k+1) \cdot 2^{k+1}}$$

$$= \underbrace{(k-1) \cdot 2^{k+1} + 2}_{\text{ind step}} + \underbrace{(k+1) \cdot 2^{k+1}}_{\text{ind step}}$$

$$= [(k-1) + (k+1)] \cdot 2^{k+1} + 2.$$

$$= 2k \cdot 2^{k+1} + 2.$$

$$= k \cdot \underbrace{2^1 \cdot 2^{k+1}}_{2^{k+2}} + 2$$

$$= k \cdot 2^{k+2} + 2$$

by the
ind step

HWS #9. \rightarrow returns $a^{(2^n)}$.

Superpower (a, \underline{n})

a real.
 n int ≥ 0 .

if ($n=0$).
Return (a).

$$a^{(2^0)} = a^1 = a.$$

$y =$ Superpower ($a, \underline{n-1}$).
Return (y^2).

$$y = a^{(2^{n-1})}$$

Want: $a^{(2^n)}$.

End.

$$= a^{(2 \cdot 2^{n-1})}$$

$$y^2 = \left[a^{(2^{n-1})} \right]^2$$

$$a^{2 \cdot 2^{n-1}} = a^{2^n}$$

HWS 12a

$$N = 13 \cdot 17 = p \cdot q$$

$$\phi = (p-1)(q-1) = 12 \cdot 16$$

d is mult inv of e mod ϕ .

lex order: 6-Subsets from $\{1, 2, \dots, 13\}$

$\{2, 6, 7, 11, 12, 13\}$

$\{2, 3, 4, 8, 9, 12\}$

$\{2, 6, 8, 9, 10, 11\}$

$\{2, 3, 4, 8, 9, 13\}$

Perms of $(1, 2, \dots, 6)$

$(2, 3, \textcircled{1}, 7, 6, 5, \textcircled{4})$

$(2, 3, 4, 7, 6, 5, 1)$

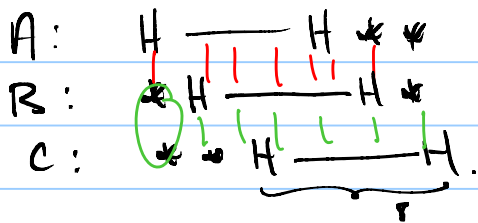
$(2, 3, 4, 1, 5, 6, 7)$

HW # 17c.

10 flips

prob of a run of ≥ 8 consec H.

$$|S| = 2^{10}$$



$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$

$$3 \cdot 4 - 2 - 2 - 1 + 1 = 8$$

Prob 8 consec heads $\frac{8}{2^{10}}$

$$\text{Prob 8 consec flips same} = \frac{2 \cdot 8}{2^{10}} = \frac{2^4}{2^{10}} = \frac{1}{2^6}$$

HWS #6.

50 cases.
8 varieties.

$$\binom{50+8-1}{8-1} = \binom{57}{7}$$

$$\neg(\leq 20) = \geq 21$$

$$\begin{aligned} \# \text{ w/ } \leq 20 \text{ coke} &= \# \text{ no restrictions} - \# \geq 21 \text{ coke.} \\ &= \binom{57}{7} - \binom{29+8-1}{8-1} \end{aligned}$$

$$\begin{aligned} \# (\leq 25 \text{ of any variety.}) &= \# \text{ no restrictions} - \# \geq 26 \text{ of some variety} \\ &= \binom{50+8-1}{8-1} - 8 \binom{24+8-1}{8-1} \end{aligned}$$

$V_i = \#$ selections w/ ≥ 26 for variety i .

$$|V_1 \cup V_2 \cup \dots \cup V_8| = 8 \cdot |V_1| = 8 \cdot \binom{24+8-1}{8-1}$$

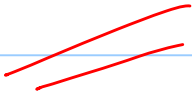
$$|V_1 \cap V_2| = \emptyset \quad \text{only selection 50.}$$

5 card hands: # hands w/ 3 Aces or 3 Kings.

Sum Rule.

hands w/ 2 Aces or 2 Kings.

↪ Incl / Excl.



$$\sum_{k=0}^n \binom{n}{k} x^k y^{n-k} = (x+y)^n.$$

↑ ↑
3 -1

$$(3+(-1))^n = (3-1)^n = 2^n.$$