Note Title 6/3/2016

$n \geq 1$ $\quad \sum_{j=1}^{n} j \cdot 2^j = (n-1)\, 2^{n+1} + 2.$

Base: $n = 1$ $\quad \sum_{j=1}^{1} j \cdot 2^j = \underbrace{\cancel{(1-1)}}_{0} \cdot 2^{1+1} + 2$

$1 \cdot 2^1 = 2.$

Ind Step! Assume for $k \geq 1$ $\quad \boxed{\sum_{j=1}^{k} j \cdot 2^j = (k-1) \cdot 2^{k+1} + 2}$

prove $\quad \sum_{j=1}^{k+1} j \cdot 2^j = \underbrace{(k) \cdot 2^{k+2} + 2}.$

$$\underbrace{\sum_{j=1}^{k+1} j \cdot 2^j} = \underbrace{\sum_{j=1}^{k} j \cdot 2^j} + \underbrace{(k+1) \cdot 2^{k+1}}$$

$$= (k-1) \cdot 2^{k+1} + 2 + (k+1) \cdot 2^{k+1}$$ by the ind hyp

$$= \left[(k-1) + (k+1) \right] \cdot 2^{k+1} + 2.$$

$$= 2k \cdot 2^{k+1} + 2.$$

$$= k \cdot \underbrace{2^1 \cdot 2^{k+1}} + 2$$

$$= k \cdot 2^{k+2} + 2$$

HW3 #9.

Superpower (a, n) → returns $a^{(2^n)}$.

a real.
n int ≥ 0.

if (n = 0).
Return (a).

$a^{(2^0)} = a^1 = a$.

y = Superpower (a, n−1).
Return (y^2).

End.

$y = a^{(2^{n-1})}$

Want: $a^{(2^n)}$.

$= a^{(2 \cdot 2^{n-1})}$

x^y $\cancel{x^{2^4}}$

$y^2 = \left[a^{(2^{n-1})}\right]^2$

$a^{2 \cdot 2^{n-1}} = a^{2^n}$

HW5 12a

$N = 13 \cdot 17 = p \cdot q$
$\Phi = (p-1)(q-1) = 12 \cdot 16$.

d is mult inv of e mod Φ.

Lex order: 6-subsets from {1, 2, ..., 13}

{2, 6, 7, 11, 12, 13}   {2, 3, 4, 8, 9, 12}

{2, 6, 8, 9, 10, 11}.   {2, 3, 4, 8, 9, 13}

4

Perms of $(1,2,\ldots,6)$

$(2,3,1,7,6,5,4)$

$(2,3,4,7,6,5,1)$

$(2,3,4,1,5,6,7)$.

HW8 # 17c. 10 flips

prob of a run of ≥ 8 consec H.

$|S| = 2^{10}$.

A: H ——— H * *

B: * H ——— H *

C: * * H ——— H.

$\underbrace{\text{H ——— H}}_{8}$

$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$.

$4 \quad 4 \quad 4 \quad 2 \quad 1 \quad 2 \quad 1$

$3 \cdot 4 - 2 - 2 - \cancel{1} + \cancel{1} = 8.$

Prob 8 consec heads $\dfrac{8}{2^{10}}$.

Prob 8 cons flips same $= \dfrac{2 \cdot 8}{2^{10}} = \dfrac{2^4}{2^{10}} = \dfrac{1}{2^6}$.

HW8 #6.

50 cans.
8 varieties.

$$\binom{50+8-1}{8-1} = \binom{57}{7}$$

$\neg(\leq 20) = \geq 21$

$\#$ w/ $\underline{\leq 20}$ coke $=$ $\#$ no restrictions $-$ $\#$ ≥ 21 coke.

$$\binom{57}{7} - \binom{29+8-1}{8-1}$$

$\left(\leq 25 \ \# \text{ of any variety}\right) = \left(\# \text{ no restrictions}\right) - \left(\# \geq 26 \text{ of some variety}\right)$

$$\binom{50+8-1}{8-1} - 8\binom{24+8-1}{8-1}.$$

V_i = # selection w/ ≥ 26 from variety i.

$$|V_1 \cup V_2 \cup \cdots V_8| = 8 \cdot |V_1| = 8 \cdot \binom{24+8-1}{8-1}$$

$|V_1 \cap V_2| = 0$ only selection 50.

Sum Rule.

5 card hands: # cards w/ 3 Aces or 3 Kings.

hands w/ 2 Aces or 2 Kings.

Incl / Excl.

$$\sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k} = (x+y)^n.$$

$x \leftarrow 3$, $y \leftarrow -1$

$$(3+(-1))^n = (3-1)^n = 2^n.$$