Student ID Number:

Name:____________________________________

Final Exam

ICS 6D
Spring 2016
Monday, June 6, 2016
Instructor: Sandy Irani

Wait until instructed to turn over the cover page. Complete all of the following questions. There are a total of 100 points.

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Total	

1. *(4 points)* Below are two permutations of the set $\{1, 2, 3, 4, 5, 6, 7\}$. For each permutation, give the next permutation in lexicographic order.

   (a) $(2, 6, 4, 7, 5, 3, 1)$

   (b) $(4, 7, 6, 5, 3, 2, 1)$

2. *(2 points)* A search committee is formed to find a new software engineer. The committee will select five candidates to be interviewed. If 51 applicants apply for the job, how many ways are there to select a subset of five candidates to interview?

3. *(2 points)* A search committee is formed to find a new software engineer. They will forward to the company CEO a list of four candidates to interview. They will let the CEO know who is their first choice, their second choice, their third choice and their fourth choice. If there are 53 applicants for the job, then how many different outcomes are there from the search committee's selection process?

4. *(2 points)* How many ways are there to select 6 jelly beans from 52 different flavors of jelly beans? There is no restriction on the number of each flavor that can be chosen. Jelly beans of the same flavor are identical.

5. *(5 points)* The recursive algorithm below takes in two numbers, x, a real number, and n, a non-negative integer. The algorithm returns the product of x and n. Fill in the blanks to complete the recursive algorithm below. The only arithmetic operations your algorithm can use are addition or subtraction. There should be no loops in algorithm.

   Multiply(x, n)

   if (__________)   // Base Case

   Return(__________)

   y := Multiply(__________ , __________)   // Recursive Call

   Return(__________)   // Mathematical expression using any variables from x, y, n

   End

6. *(9 points)* The sequence $\{g_n\}$ is defined recursively as follows:

   - $g_0 = 51$
   - $g_1 = 348.$
   - $g_n = 5 \cdot g_{n-1} - 6 \cdot g_{n-2} + 20 \cdot 7^n$

   For this problem, we will consider a proof by induction of the following theorem:

   Theorem 1. *For the sequence $\{g_n\}$ (as defined above), and any $n \geq 0$,*

   $$g_n = 3^n + 2^n + 49 \cdot 7^n.$$

   (a) For which values of n must the theorem be proven in the base case?

   (b) The inductive step assumes a particular mathematical statement and uses the assumption to prove a different mathematical statement. State clearly below what would be assumed and what would be proven in the inductive step for a proof of the theorem. **You do not have to give the proof.**
   Assume:

   Prove:

7. *(6 points)* The seventh line of Pascal's triangle is: 1-7-21-35-35-21-7-1.

   (a) What is the 8^{th} line of Pascal's triangle?

   (b) What is the coefficient of x^6y^2 in $(x+y)^8$? (Your answer can be a mathematical expression with multiplication and exponentiation, but should not include "n choose k" notation).

   (c) What is the coefficient of x^3y^5 in $(3x-y)^8$? (Your answer can be a mathematical expression with multiplication and exponentiation, but should not include "n choose k" notation).

8. *(8 points)* Solve the following recurrence relation:

   - $h_n = 6 \cdot h_{n-1} - 9 \cdot h_{n-2}$
   - $h_0 = 2$
   - $h_1 = 18$

9. *(8 points)* Compute $gcd(248, 90)$ and write it in the form $248 \cdot s + 90 \cdot t$ for integers s and t.

10. *(12 points)* A company has 5 warehouses each in a different location. They must store 250 widgets in the warehouses and are deciding how to distribute the widgets between the warehouses. How many ways are there for them to distribute the widgets between the warehouses if:

    (a) The widgets are all the same and there is no restriction on the number of widgets that can be stored in a warehouse.

    (b) The widgets are all distinct and there is no restriction on the number of widgets that can be stored in a warehouse.

    (c) The widgets are the same and the same number of widgets are stored in each warehouse.

    (d) The widgets are all different and the same number of widgets are stored in each warehouse.

11. *(13 points)* A fair coin is flipped 10 times. Give the probability of the following events:

(a) Exactly 4 of the 10 coin flips come up heads.

(b) The first two flips both come up heads.

(c) The first two coin flips come up heads or eactly 4 of the 10 coin flips come up heads. (Use the inclusive or).

(d) At least one of the coin flips comes up heads.

(e) Somewhere in the sequence, there are at least two consecutive coin flips that are either both heads or both tails. (Two flips are "consecutive" if one comes right after the other.)

12. *(5 points)* Compute 4^{74} mod 13.

13. *(10 points)* 10 identical coupons are distributed to 20 customers shopping in a store. How many ways are there to distribute the coupons if:

   (a) Each customer gets at most one coupon.

   (b) There are no restrictions on the number of coupons a customer can get.

   (c) There are no restrictions on the number of coupons a customer can get, except that one particular costomer, Mr. Spendalot, gets at least three coupons.

   (d) There are no restrictions on the number of coupons a customer can get, except that one particular customer, Mr. Spendalot, gets at most three coupons.

14. *(4 points)* Below are two 6-subsets of the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$. For each 6-subsets, give the next 6-subsets in lexicographic order.

   (a) $\{1, 4, 9, 10, 11, 12\}$

   (b) $\{1, 4, 5, 8, 9, 11\}$

15. *(10 points)*

Let S be the set of all strings of length 10 over the alphabet $\{a, b, c\}$.

(a) How many strings from S have exactly 2 a's?

(b) How many strings from S have exactly 2 a's and exactly 3 b's?

(c) How many strings from S have exactly 2 a's or exactly 3 b's? (Use the inclusive or).

(d) How many strings from S have exactly 2 a's or exactly 2 b's or exactly 2 c's? (Use the inclusive or).