

Homework 4

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Sections 9.1-9.4

1. Compute the value of the following expressions:

- (a) $344 \bmod 5$.
- (b) $344 \operatorname{div} 5$.
- (c) $-344 \bmod 5$.
- (d) $-344 \operatorname{div} 5$.
- (e) $38^7 \bmod 3$.
- (f) $(72 \cdot (-65) + 211) \bmod 7$.
- (g) $(77 \cdot (-65) + 147) \bmod 7$.
- (h) $(44^{12}) \bmod 6$.
- (i) $(17^{12}) \bmod 3$.

2. Give the multiplication table for \mathbb{Z}_6 .

3. Some numbers and their prime factorizations are given below.

$$\begin{aligned}140 &= 2^2 \cdot 5 \cdot 7 \\1078 &= 2 \cdot 7^2 \cdot 11 \\175 &= 5^2 \cdot 7 \\25480 &= 2^3 \cdot 5 \cdot 7^2 \cdot 13\end{aligned}$$

Express each of the quantities below as a product of primes:

- (a) $\gcd(1078, 140)$
- (b) $\gcd(1078, 25480)$
- (c) $\operatorname{lcm}(1078, 140)$
- (d) $\operatorname{lcm}(175, 25480)$
- (e) $\operatorname{lcm}(140, 25480)$
- (f) $175 \cdot 25480$
- (g) $25480/140$

4. Suppose that two positive integers x and y are expressed using a common set of primes as follows:

$$\begin{aligned}x &= p_1^{\alpha_1} \cdot p_2^{\alpha_2} \cdot p_3^{\alpha_3} \cdots p_r^{\alpha_r} \\y &= p_1^{\beta_1} \cdot p_2^{\beta_2} \cdot p_3^{\beta_3} \cdots p_r^{\beta_r}\end{aligned}$$

- (a) What is the prime factorization for $x \cdot y$?
- (b) If $y|x$, give the prime factorization for x/y . (It's OK to have some p^0 's in your prime factorization).

5. Let a , b and c be positive integers. Consider the statement:

If $a|b$ or $a|c$, then $a|bc$.

- (a) Show that the statement is true.
 - (b) Show that the converse of the statement is not true by giving a counter-example. (The converse of the statement says that if $a|bc$, then $a|b$ or $a|c$.)
 - (c) Is the converse of the statement true if a is a prime number?
6. Group the following numbers according to equivalence mod 13. That is, put two numbers in the same group if they are equivalent mod 13.

$$\{-63, -54, -41, 11, 13, 76, 80, 130, 132, 137\}$$

7. Use the prime number theorem to give an approximation for the number of prime numbers in the range 2 through 10,000,000.