

Homework 6

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Sections 10.1-10.4

Leave your answer for the questions below as an arithmetic expression, including the $P(n, k)$ notation. You do not have to compute a final numeric value.

1. Consider the following definitions for sets of characters:

- Symbols = $\{*, \&, \$, \#\}$
- Digits = $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$
- Letters = $\{a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z\}$

How many passwords that satisfy each set of constraints:

- (a) Strings of length 6. Characters can be symbols, digits, or letters, with no restrictions.
 - (b) Strings of length 7, 8, or 9. Characters can be symbols, digits, or letters, with no restrictions.
 - (c) Strings of length 7, 8, or 9. Characters can be symbols, digits, or letters. The first symbol can not be a letter.
 - (d) Strings of length 6. Characters can be symbols, digits, or letters, with no repeated characters.
 - (e) Strings of length 6. Characters can be symbols, digits, or letters, with no repeated characters. The first symbol can not be a symbol.
2. Let $B = \{0, 1\}$. B^n is the set of binary strings with n bits. Define E_n to be the set of n -bit binary strings that have an even number of 1's. Note that zero is an even number, so for example, $0000 \in E_4$.
- (a) Define a bijection between B^9 and E_{10} . Explain why the function is a bijection.
 - (b) What is $|E_{10}|$?

3. Ten kids line up for recess. The names of the kids are:

$$\{Abe, Ben, Cam, Don, Eli, Fran, Gene, Hal, Ike, Jan\}$$

Let S be the set of all possible ways to line up the kids. For example, one ordering might be:

$$(Fran, Gene, Hal, Jan, Abe, Don, Cam, Eli, Ike, Ben)$$

The names are listed in order from left to right, so Fran is at the front of the line and Ben is at the end of the line.

Let T be the set of all possible ways to line up the kids in which Gene is ahead of Don in the line. Note that Gene does not have to be immediately ahead of Don. For example, the ordering shown above is an element in T .

Now define a function f whose domain is S and whose target is T . Let x be an element of S , so x is one possible way to order the kids. If Gene is ahead of Don in the ordering x , then $f(x) = x$. If Don is ahead of Gene in x , then $f(x)$ is the ordering that is the same as x , except that Don and Gene have swapped places.

(a) What is the output of f on the following input?

$$(Fran, Gene, Hal, Jan, Abe, Don, Cam, Eli, Ike, Ben)$$

(b) What is the output of f on the following input?

(Eli, Ike, Don, Hal, Jan, Abe, Ben, Fran, Gene, Cam)

(c) Is the function f a k -to-1 correspondence for some positive integer k ? If so, for what value of k ? Justify your answer.

(d) There are 3268800 ways to line up the 10 kids with no restrictions on who comes before whom. That is, $|S| = 3268800$. Use this fact and the answer to the previous question to determine $|T|$.

4. How many strings are there over the set $\{a, b, c\}$ that have length 10 in which no two consecutive characters are the same? For example, the string "abc bcbabc b" would count and the strings "abbbcbabc b" and "aacbcbabc b" would not count.

5. At a certain university in the U.S., all phone numbers are 7-digits long and start with either 824 or 825.

(a) How many different phone numbers are possible?

(b) How many different phone numbers are there in which the last four digits are all different?

6. Consider a family with a mother and a father and five children who line up for a photo.

(a) How many ways are there for the family to line up for the photo?

(b) How many ways are there for the family to line up if the mother and father are next to each other?

(c) Suppose that there are two daughters and three sons in the family. How many ways are there for the family to line up if the two daughters are on either side of their mother?

7. Count the number of different functions with the given domain, target and additional properties.

(a) $f : \{0, 1\}^7 \rightarrow \{0, 1\}^7$.

Hint: to get some intuition for this problem, try counting the number of functions whose domain and target are $\{0, 1\}^2$. In order to specify a function, you need to determine the output of the function for every singly possible input value from the domain. Write down a sample specification of a function $f : \{0, 1\}^2 \rightarrow \{0, 1\}^2$. How many different choices had to be made? For each choice, how many possibilities were there to select from? Follow this same reasoning for all the parts to this question.

(b) $f : \{0, 1\}^7 \rightarrow \{0, 1\}^7$. The function f is one-to-one.

(c) $f : \{0, 1\}^5 \rightarrow \{0, 1\}^7$.

(d) $f : \{0, 1\}^5 \rightarrow \{0, 1\}^7$. The function f is one-to-one.