ICS 6D **Due: Wednesday, May 18, 2016, 3:00PM**

Homework 7

Instructor: Sandy Irani Sections 10.5-10.7

Leave your answer for the questions below as an arithmetic expression, including the $P(n,k)$ or $\binom{n}{k}$ notation. You do not have to compute a final numeric value.

1. Consider a function f that maps 5-permutations from a set $S = \{1, 2, ..., 20\}$ to 5-subsets from S. The function takes a 5-permutation and removes the ordering on the elements.
   (a) What is the value of f on input $(12, 1, 3, 15, 9)$?
   (b) Is $(12, 3, 12, 4, 19)$ a 5-permutation? Why or why not?
   (c) How many 5-permutations maps on to the subset $\{2, 5, 13, 14, 19\}$?
2. A teacher must select four members of the math club to participate in an upcoming competition.
   (a) How many ways are there for her to make her selection if the club has 12 members?
   (b) How many ways are there for her to make her selection if the club has 6 girls and 6 boys and she must select two girls and two boys.
3. How many strings are there of length 12 over the alphabet $\{a, b\}$ with exactly five a's?
4. How many strings are there of length 12 over the alphabet $\{a, b, c\}$ with exactly five a's?
5. There are 20 members of a basketball team.
   (a) The coach must select 12 players to travel to an away game. How many ways are there to select the players who will travel?
   (b) From the 12 players who will travel, the coach must select her starting line up. She will select a player for each of the five positions: center, right forward, left forward, right guard, left guard. How many ways are there for her to select the starting line-up?
   (c) From the 12 players who will travel, the coach must select her starting line up. She will select a player for each of the five positions: center, right forward, left forward, right guard, left guard. However, there are only three of the 12 players who can play center. Otherwise, there are no restrictions. How many ways are there for her to select the starting line-up?
6. There are 30 boys and 35 girls that try out for a chorus. The choir director will select 10 girls and 10 boys from the kids trying out. How many ways are there for the choir director to make his selection?
7. This question refers to a standard deck of playing cards. If you are unfamiliar with playing cards, there is an explanation in Section 12.1 of your text under the heading "Standard playing cards". A five-card hand is just a subset of 5 cards from a deck of 52 cards.
   (a) How many different five-card hands are there from a standard deck of 52 playing cards?
   (b) How many five-card hands have exactly two hearts?
   (c) How many five-card hands are made entirely of hearts and diamonds?
   (d) How many five-card hands have four cards of the same rank?

(e) A "full house" is a five-card hand that has two cards of the same rank and three cards of the same rank. For example, {queen of hearts, queen of spades, 8 of diamonds, 8 of spades, 8 of clubs}. How many five-card hands are a full house?

(f) How many five-card hands do not have any two cards of the same rank.

8. A teacher has five books to distribute to some of 20 kids in her class.

   - How many ways are there for her to distribute the books if they are all the same and no kid gets more than one?
   - How many ways are there fore her to distribute the books if they are different and no kid gets more than one? So, if Charlie gets "Green Eggs and Ham" and Amanda gets "The Cat in the Hat" that is a different way of distributing the books than if Amanda gets "Green Eggs and Ham" and Charlie gets "The Cat in the Hat".
   - How many ways are there to distribute the books if the books are all different and there is no restriction on the number of books that can be given to any kid.

9. How many ways are there to permute the letters in each of the following words:

   (a) NUMBER

   (b) DISCRETE

   (c) SUBSETS

10. Twelve employees of a company are being assigned to offices. There are four offices and each is large enough for three people. How many ways are there to assign employees to offices?

11. 20 different comic books will be distributed to five kids.

    (a) How many ways are there to distribute the comic books if there are no restrictions on how many go to each kid other than the fact that all 20 will be given out?

    (b) How many ways are there to distribute the comic books if they are divided evenly so that 4 go to each kid?

12. A family has four daughters. Their home has three bedrooms for the girls. Two of the bedrooms are only big enough for one girl. The other bedroom is will have two girls. How many ways are there to assign the girls to bedrooms?

13. A camp offers 4 different activities for an elective: archery, hiking, crafts and swimming. The capacity in each activity is limited so that at most 35 kids can do archery, 20 can do hiking, 25 can do crafts and 20 can do swimming. There are 100 kids in the camp. How many ways are there to assign the kids to the activities?

14. How many numbers greater than 3,000,000 can be formed from permutations of 1, 2, 2, 4, 6, 6, 6?

15. How many ways are there to deal hands from a standard playing deck to four players if

    (a) Each player gets exactly 13 cards.

    (b) Each player gets seven cards and the rest of the cards remain in the deck?