ICS 6D	**Due: Wednesday, June 1, 2016, 3:00PM**

Homework 9

Instructor: Sandy Irani	Sections 11.1, 11.2

Leave your answer for the questions below as an arithmetic expression, including the $P(n,k)$ or $\binom{n}{k}$ notation. You do not have to compute a final numeric value.

1. What is the coefficient of x^5y^4 in $(-3x+4y)^9$?
2. What is the coefficient of x^4y^3 in $(5x-y)^7$?
3. What is the coefficient of x^3y^5 in $(3x-4y)^8$?
4. What is the coefficient of x^2y^7 in $(-2x+5y)^9$?
5. Use the Binomial Theorem to find the closed form for the following sums. Recall that a closed form is an expression without a summation.
   (a) $\sum_{k=0}^{n}\binom{n}{k}3^k(-1)^{n-k}$.
   (b) $\sum_{k=0}^{n}\binom{n}{k}2^k$.
6. The 6^{th} row of Pascal's triangle is $1, 6, 15, 20, 15, 6, 1$.
   (a) What is the 7^{th} row of Pascal's triangle?
   (b) Use your answer to the previous part to write the expanded form of $(x+y)^7$.
7. Write the following 5-tuples in increasing lexicographic order:

$$\begin{array}{ccc} (3,100,101,3,4) & (1,1,2,1,2) & (3,4,5,1,1) \\ (1,2,100,1,1) & (3,4,5,2,2) & (2,101,100,3,4) \\ (2,100,101,3,4) & (3,4,5,2,1) & (1,1,2,2,1) \end{array}$$

8. For each permutation of $\{1,2,3,4,5,6,7\}$, give the next largest permutation in lexocigraphic order or indicate that the permutation is the last one in lexicographic order:
   (a) $(1,2,3,4,5,6,7)$
   (b) $(7,6,5,4,3,2,1)$
   (c) $(1,7,6,4,2,3,5)$
   (d) $(3,7,6,5,4,2,1)$
   (e) $(5,4,7,6,3,2,1)$
9. Write the permutations of $\{1,2,3\}$ in lexicographic order.
10. For each 7-subset of $\{1,2,3,4,5,6,7,8,9,10,11,12,13,14\}$, give the next largest 7-subset in lexocigraphic order or indicate that the 7-subset is the last one in lexicographic order:
    (a) $\{1,2,3,4,5,6,7\}$
    (b) $\{2,4,5,9,11,12,13\}$

(c) $\{2, 4, 5, 11, 12, 13, 14\}$

(d) $\{2, 4, 5, 6, 11, 12, 14\}$

(e) $\{7, 8, 10, 11, 12, 13, 14\}$

(f) $\{8, 9, 10, 11, 12, 13, 14\}$

11. Write the 4-subsets of $\{1, 2, 3, 4, 5, 6\}$ in lexicographic order.

12. The rule to compare two n-tuples lexicographically can be generalized to tuples of different lengths. Let $(p_1, p_2, ..., p_m)$ and $(q_1, q_2, ..., q_n)$ be two tuples where $m < n$.

- If there is a j in the range 1 through m such that $p_j \neq q_j$, then let k be the smallest k such that $p_k \neq q_k$.
  - If $p_k > q_k$, then $(p_1, p_2, ..., p_m) > (q_1, q_2, ..., q_n)$
  - If $p_k < q_k$, then $(p_1, p_2, ..., p_m) < (q_1, q_2, ..., q_n)$
- If $p_j = q_j$ for every j in the range 1 through m, then $(p_1, p_2, ..., p_m) < (q_1, q_2, ..., q_n)$

The definition above corresponds exactly to how words are ordered in a dictionary. If one word is a prefix of another word, then the prefix comes first. For example, ”worth” comes before ”worthwhile” in the dictionary. Otherwise, the two words are ordered according to the first letter where the two words differ.

(a) Use the generalized definition of lexicographic ordering to order the following tuples according to lexicographic order:

$$(2, 3, 5, 6), (4, 3), (6), (2, 3, 5, 6, 2), (1, 2, 3, 4, 5, 6, 7), (6, 1),$$
$$(6, 1, 1), (4, 3, 2, 1), (4, 2, 1), (1, 1), (1, 2)$$

(b) Sets of different sizes can be ordered lexicographically by first sorting the elements in increasing order and then comparing the resulting sequences as if they were ordered tuples. For example, in comparing $\{2, 1, 3\}$ and $\{4, 1\}$, the elements of each set are sorted to get $\{1, 2, 3\}$ and $\{1, 4\}$. $\{1, 2, 3\}$ ¡ $\{1, 4\}$ because the first element is the same in both sets but $2 < 4$.

Write all the non-empty subsets of $\{1, 2, 3, 4\}$ with three or fewer elements in lexicographic order.