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1. 3SAT $\propto$ EXACT 3-SAT

Input: ϕ instance of 3-SAT
Output: ϕ' instance of EXACT 3-SAT
ϕ is satisfiable if & only if ϕ' is satisfiable

Enumerate the clauses of ϕ: $C_1 C_2 \ldots C_m$.
For $i := 1$ to N
if C_i has 3 literals, add C_i to ϕ'.
if C_i has 2 literals, add a new variable y_i
Add clauses $C_i \vee y_i$ and $C_i \vee \neg y_i$ to ϕ'.
if C_i has 1 literal, add new variables y_i & z_i
Add clauses
$C_i \vee y_i \vee z_i$, $C_i \vee y_i \vee \neg z_i$, $C_i \vee \neg y_i \vee z_i$, $C_i \vee \neg y_i \vee \neg z_i$
to ϕ'.

If there is an assignment to $x_1 \cdots x_n$ s.t. $\phi(x_1, \ldots, x_n)$ is satisfied then all the clauses in ϕ' are also satisfied. The y's & z's can be set arbitrarily and the resulting assignment is a satisfying assignment for ϕ'.

Suppose there is a satisfying assignment for ϕ'.
If every clause has an x-literal that is true, the assignment of truth values to the x-variables also satisfies ϕ.
Suppose there is a clause in which every x-literal is false
For example $C_i \vee y_i \vee z_i$ C_i is not satisfied but one of y_i and/or z_i is true.

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In one of the clauses

$c_i \vee y_i \vee z_i$, $c_i \vee y_i \vee \neg z_i$, $c_i \vee \neg y_i \vee z_i$, $c_i \vee \neg y_i \vee \neg z_i$

the y-literal & the z-literal must both be false. Since c_i is also not satisfied the whole clause is not satisfied which contradicts the assertion that the original assignment satisfies ϕ'.
Therefore if an assignment to the x's y's & z's satisfies ϕ' the assignment to the x's satisfies ϕ. //

2. Any non-increasing function will do, but I meant to ask for an increasing non-proper function.

We know the following language is undecidable.

$$L = \{ i \mid \langle i \; i \rangle \text{ is an accepting computation} \}$$

string i interpreted as an encoding of a TM M_i

string i interpreted as an input to M_i

Define $f(n) = 2n$ if the binary encoding of $n \in L$.

$f(n) = 2n+1$ if the binary encoding of $n \notin L$.

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If $f(n)$ can be computed in finite time by a TM M then we can use M to decide L:

Simulate M on input n. If M outputs an even number accept. If M outputs an odd number then reject.

3. Suppose $L \propto L'$ and $L' \in P$.
by poly time reduction R.

On input x compute $R(x)$ & put results on an extra tape. Use poly time procedure for L' to decide if $R(x) \in L'$. Output the result. $\Rightarrow$ Poly time total.

Since $x \in L$ iff $R(x) \in L'$ the answer is correct.
$\Rightarrow$ P is closed under reductions.

Suppose $L \propto L'$ and $L' \in$ PSPACE.
Same as above, except the procedure to decide L' is PSPACE. The resulting procedure for L is also PSPACE.
$\Rightarrow$ PSPACE is closed under reductions.

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TIME(n^2) is not closed under reductions.

Consider $L \in$ TIME(n^2)
$L' \in P$ and $L' \notin$ TIME(n^2).
We know that there is such an L' due to the Time complexity theorem.

We will show a poly-time reduction from L' to L which is sufficient to establish that NTIME(n^2) is not closed under reductions:

Select $x_0 \notin L$
$x_1 \in L$.

R: On input x use the poly-time procedure to determine if $x \in L'$.
If $x \in L'$, output x_1
If $x \notin L'$, output x_0

$x \in L$ iff $R(x) \in L'$.

4. Suppose $\mathcal{L} = P$ and $P = PSPACE$

then $\mathcal{L} = PSPACE$.

However we know that $\mathcal{L} \neq PSPACE$ due to the Space Hierarchy theorem.