

## HW2 solutions - page 1

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1. Consider  $L \in \text{EXP}$ .

There is a TM that decides  $L$  in time  $2^{n^k}$ .  
Define  $\text{PAD}_L = \{x \# 2^{n^k} \mid x \in L \text{ and } n = |x|\}$

As seen in class  $\text{PAD}_L \in \text{P}$ .

If  $\text{P} = \text{L}$  then  $\text{PAD}_L \in \text{L}$ .

Let  $M$  be a TM that on input  $x \# 2^{n^k}$  uses  $\log(2^{n^k}) = |x|^k$  workspace to decide  $\text{PAD}_L$ .

Here is a TM  $M'$  that can decide  $L$  using only  $O(|x|^k)$  space (which implies that  $L \in \text{PSPACE}$ ).

- $M'$  • Work tapes are exactly the same as  $M$ .
- If the location of  $M$ 's input pointer is at location  $\leq |x|$  then the location of  $M'$ 's input pointer is the same.
- If  $M$ 's input pointer moves into the  $\#$ 's, keep a register that holds the location of the head. In this case the symbol being read on the input tape is  $\#$ .
- Start moving the pointer again if the contents of the register is  $\leq |x|$ .