CS 262: Computational Complexity *Instructor: Sandy Irani*

Homework 2

Due: April 18, 2018

1. Show that if $\mathbf{L} = \mathbf{P}$, then $\mathbf{PSPACE} = \mathbf{EXP}$.

2. Define a *coding* κ to be a mapping from Σ to Σ. Note that κ need not be one-to-one. If $x = \sigma_1 \ldots \sigma_n$, where each $\sigma_i \in \Sigma$, then we define $\kappa(x) = \kappa(\sigma_1) \ldots \kappa(\sigma_n)$. If L is a language, then $\kappa(L)$ is defined to be $\{\kappa(x) | x \in L\}$.

   (a) Prove that NP is closed under codings. That is, show that if $L \in \text{NP}$ and κ is a coding defined on the alphabet of L, then $\kappa(L) \in \text{NP}$.

   (b) We expect that P is not closed under codings, but we can not prove this without establishing that $\text{P} \neq \text{NP}$. Instead, show that P is closed under codings if and only if $\text{P} = \text{NP}$.

3. Prove that if *every* unary language in **NP** is also in **P**, then **EXP**= **NEXP**. Recall that a language is unary if and only if it is a subset of 1^*.

   Note that the function $2^{(\log^k n)}$ is not polynomial if $k > 2$. Here is an easier version of this problem to start on: prove that if *every* unary language in **NP** is also in **P**, then any language that is is in $\mathbf{TIME}(2^{kn})$ for some k is also in $\mathbf{TIME}(2^{cn})$ for some c.