Homework 3

Due: April 25, 2018

1. A directed graph $G = (V, E)$ is *strongly connected* if for every pair of verties (x, y), there is a directed path from x to y and a directed path from y to x. Define STRONGLY-CONN to be the language consisting of all graphs that are strongly connected. Either show that this problem is in **L** or show a complexity consequence that results if this problem is in **L**.

2. A *strong* nondeterministic Turing Machine has, in addition to q_{acc} and q_{rej} states, a special state $q_?$. The state $q_?$, like q_{acc} and q_{rej}, is a terminating state in that there are no transitions out of state $q_?$. A strong NTM *accepts* its input if all computation paths lead to q_{acc} or $q_?$ states. A strong NTM *rejects* its input if all computation paths lead to q_{rej} or $q_?$ states. Also, on each input, there is at least one computation path that leads to q_{acc} or q_{rej}. In order for a language L to be decided by a strong NTM, the NTM must accept or reject every input x according to whether $x \in L$. Note that the fact that a strong NTM decides a language implies that there can be no input that has compuation paths that lead to both q_{acc} and q_{rej}.

   Show that the class of languages decided by a strong nondeterministic Turing Machine in polynomial time is exactly **NP** $\cap$ **co-NP**.

3. An alternative definition of the class **NL** makes use of a Turing Machine with a special read-once tape. The head on a read-once tape starts at the left-most end of the non-blank symbols written on the tape and can only move to the right or stay in the same place (i.e. it can never move left). The alternative definition says that a language L is in **NL** if there is a deterministic Turing Machine R (called a *verifier*) with a special read-once tape and a polynomial p such that for every $x \in \Sigma^*$,

   $$x \in L \Leftrightarrow \exists u \in \Sigma^{p(|x|)} \text{ such that } R(x, u) = 1,$$

   where $R(x, u)$ is the output of R when x is placed on the input tape and u is placed on its special read-once tape and R uses $O(\log n)$ space on its work tape for every input x.

   Prove that this definition of **NL** is equivalent to the definition using non-deterministic Turing Machines discussed in class.