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2. There is a parsimonious reduction
Cycle-Cover $\propto$ SAT
s.t. the # of satisfying assignments to a Boolean formula ϕ = # cycle covers in G.

So we will show that permanent $\in P^{\#\text{Cycle}}$.

Let A be an $n \times n$ matrix & let $A_{max} = \max_{i,j} |A_{i,j}|$

permanent of $A \leq n! (A_{max})^n$

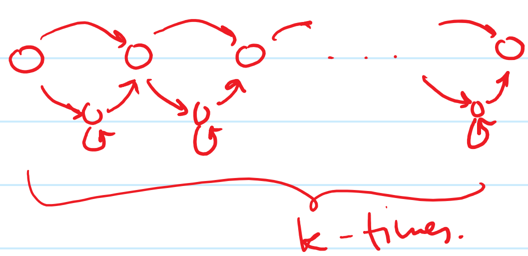
Select $t = \log(n!(A_{max})^n) + 2$
$= n^2 \log n \log(A_{max}) + 2$

polynomial in the # bits required to specify A.

$2 \cdot \text{permanent} < 2^t$

Create a graph G : vertices $\{1, .., n\}$ plus some auxiliary variables in gadgets.

k-gadget:



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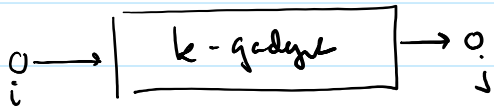
For $A_{ij} > 0$ for each k s.t. k^{th} bit of $A_{ij} = 1$
add a k-gadget from i to j:



$$\#\text{ paths from } i \text{ to } j \text{ using only gadgets} = A_{ij}$$

For $A_{ij} < 0$ for each k s.t. k^{th} bit of $A_{ij} = 1$
add a $(t-k)$-gadget from i to j

$$\#\text{ paths from } i \text{ to } j = \sum_{\substack{k \\ k^{th}\text{ bit of } A_{ij}=1}} \left(2^t - 2^k\right) = c \cdot 2^t + A_{ij}$$

$$\#\text{ Cycle covers in } C = \sum_{\pi} \prod_{i=1}^{n} \#\text{ paths from } i \text{ to } \pi(i)$$

$$= \sum_{\pi} \prod_{i=1}^{n} \left(A_{ij} + c \cdot 2^t\right)$$

$$= \left(\text{multiple of } 2^t\right) + \text{perm}(A)$$

if $\text{perm}(A) > 0$ then $(\#\text{ cycle covers in } G) \bmod 2^t < 2^{t-1}$
$\text{perm}(A) = (\#\text{ cycle covers in } G) \bmod 2^t$

if $\text{perm}(A) < 0$ then $(\#\text{ cycle covers in } G) \bmod 2^t > 2^{t-1}$
$\text{perm}(A) = (\#\text{ cycle covers in } G) \bmod 2^t - 2^t$.

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2. In class, we saw a reduction from 3-CNF formulas ϕ to weighted graphs G such that:

$$4^{3m}(\#\text{ satisfying assignments for }\phi) = W(G)$$

$m = \#$ clauses in ϕ

If $A(G)$ is an ϵ-approx for $W(G)$ then:

If ϕ is satisfiable, $W(G) \geq 4^{3m}$

$A(G) \geq \epsilon \cdot W(G) > 0$

If ϕ is not satisfiable, $W(G) = 0$

$A(G) \leq \frac{W(G)}{\epsilon} = 0$

To determine if ϕ is satisfiable, apply reduction to obtain G. Then run the approximation alg. for W on G. $A(G) > 0$ iff ϕ is satisfiable.

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3. Consider $L \in IP$ then there is an interactive proof system with a deterministic prover (P,V), a special case of a proof system w/ a probabilistic prover.

$x \in L$ $\quad \mathrm{Prob}_r[(P,V_r)(x) \text{ accepts}] \geq 2/3$

$x \notin L$ $\quad \forall P^* \; \mathrm{Prob}_r[(P^*,V_r)(x) \text{ accepts}] \leq 1/3$

if this true for every P^*, then it is also true for any distribution P_r^* over provers

$\forall P_r^* \; \mathrm{Prob}[(P_{r'}^*,V_r)(x) \text{ accepts}] \leq 1/3$ ✓

$\Rightarrow L \in IP'$

Now suppose $L \in IP'$.

$x \in L$ $\quad \mathrm{Prob}_{r,r'}[(P_r,V_{r'})(x) \text{ accepts}] \geq 2/3$

select the $r(x)$ that maximizes this probability

Define deterministic prover $\bar{P}$ that on input x, acts according to $P_{r(x)}$

$x \in L \Rightarrow \mathrm{Prob}_{r'}[(\bar{P},V_{r'})(x) \text{ accepts}] \geq 2/3$

$x \notin L$ $\quad$ for any distribution over provers P_r^*

$\mathrm{Prob}_{r,r'}[(P_r^*,V_{r'})(x) \text{ accepts}] \leq 1/3$

Since deterministic provers are a special case of probabilistic provers

$\forall P^* \; \mathrm{Prob}_{r'}[(P^*,V_{r'})(x) \text{ accepts}] \leq 1/3$

$\Rightarrow L \in IP'$