

Midterm Exam

Due: May 16, 2018

**Important note:** Please remember that you should not discuss this exam with anyone else or search the internet to look for solutions.

1. Show that if  $\mathbf{NP} \subseteq \mathbf{BPP}$  then  $\mathbf{RP} = \mathbf{NP}$ .
2. Show that if  $f(n) \geq n$  and  $g(n) \geq n$ , are proper functions, then  $\mathbf{TIME}(f(n)) = \mathbf{NTIME}(f(n))$  implies that  $\mathbf{TIME}(f(g(n))) = \mathbf{NTIME}(f(g(n)))$ .
3. Suppose that  $L_1, L_2 \in \mathbf{NP}$ . Is  $L_1 \cup L_2$  in  $\mathbf{NP}$ ? What about  $L_1 \cap L_2$ ? Prove your answers.
4. A complexity class  $\mathcal{C}$  is closed under complement if  $L \in \mathcal{C}$  implies that  $co-L \in \mathcal{C}$ .
  - (a) Are  $\mathbf{BPP}$  or  $\mathbf{ZPP}$  closed under complement? Justify your answer.
  - (b) If either  $\mathbf{RP}$  or  $\mathbf{co-RP}$  are closed under complement then which complexity classes (among the ones we have defined in class) are equivalent to  $\mathbf{RP}$ ?