Announcements:

- Midterm 2 is Wed. Review sheet is on class webpage (in the list of lectures) and will be covered in discussion on Monday. Two sheets of notes are allowed, same rules as for the one sheet last time.
- Office hours today, Mon, Tues slightly revised from usual. See webpage.

Homework (due *Monday*):

Chapter 9: #50 (Each part counts for 1 point, so problem is worth 6 points.)

Sampling Distributions for Proportions: One proportion or difference in two

Chapter 9, Sections 4, 5, 9

Understanding Dissimilarity Among Samples

Key: Need to understand what kind of dissimilarity we should expect to see in various samples from the same population.

- Suppose knew most samples were likely to provide an answer that is within 10% of the population answer.
- Then would also know the population answer should be within 10% of whatever our specific sample gave.
- => Have a good guess about the *population value* based on just *one sample value*.

Statistics and Parameters

A **statistic** is a numerical value computed from a sample. Its value may differ for different samples. *e.g. sample mean* $\bar{x}$, *sample standard deviation s, and sample proportion* $\hat{p}$.

A **parameter** is a numerical value associated with a population. Considered fixed and unchanging. *e.g. population mean* μ, *population standard deviation* σ, *and population proportion p.*

Sampling Distributions

Each new sample taken =>
sample statistic will change.

The distribution of possible values of a statistic for repeated samples of the same size from a population is called the **sampling distribution** of the statistic.

Many statistics of interest have sampling distributions that are ***approximately normal*** distributions

9.4 Sampling Distribution for One Sample Proportion

- Suppose (unknown to us) **40% of a population carry the gene** for a disease, ($p = 0.40$).
- We will take a **random sample of 25** people from this population and count **X = number with gene**.
- Although we ***expect*** on average to find 10 people (40%) with the gene, we know the number will ***vary*** for different samples of $n = 25$.
- In this case, X is a **binomial random variable** with $n = 25$ and $p = 0.4$.

Many Possible Samples

Four possible random samples of 25 people:

Sample 1: X =12, proportion with gene =12/25 = 0.48 or 48%.

Sample 2: X = 9, proportion with gene = 9/25 = 0.36 or 36%.

Sample 3: X = 10, proportion with gene = 10/25 = 0.40 or 40%.

Sample 4: X = 7, proportion with gene = 7/25 = 0.28 or 28%.

Note:

- Each sample gave a different answer, which did not always match the population value of 40%.
- Although we cannot determine whether one sample statistic will accurately estimate the true population parameter, statisticians have determined probabilities for how far from the truth the sample values could be.

The Normal Curve Approximation Rule for Sample Proportions

Let p = population proportion of interest
or binomial probability of success.

Let $\hat{p}$ = sample proportion or proportion of successes.

If numerous random samples or repetitions of the same size n are taken, the distribution of possible values of $\hat{p}$ is **approximately** a **normal** curve distribution with

- **Mean** = p
- **Standard deviation** = s.d.($\hat{p}$) = $\sqrt{\dfrac{p(1-p)}{n}}$

This approximate distribution is **sampling distribution of** $\hat{p}$**.**

Ex: Medicine cures 60%

Sample 200 people

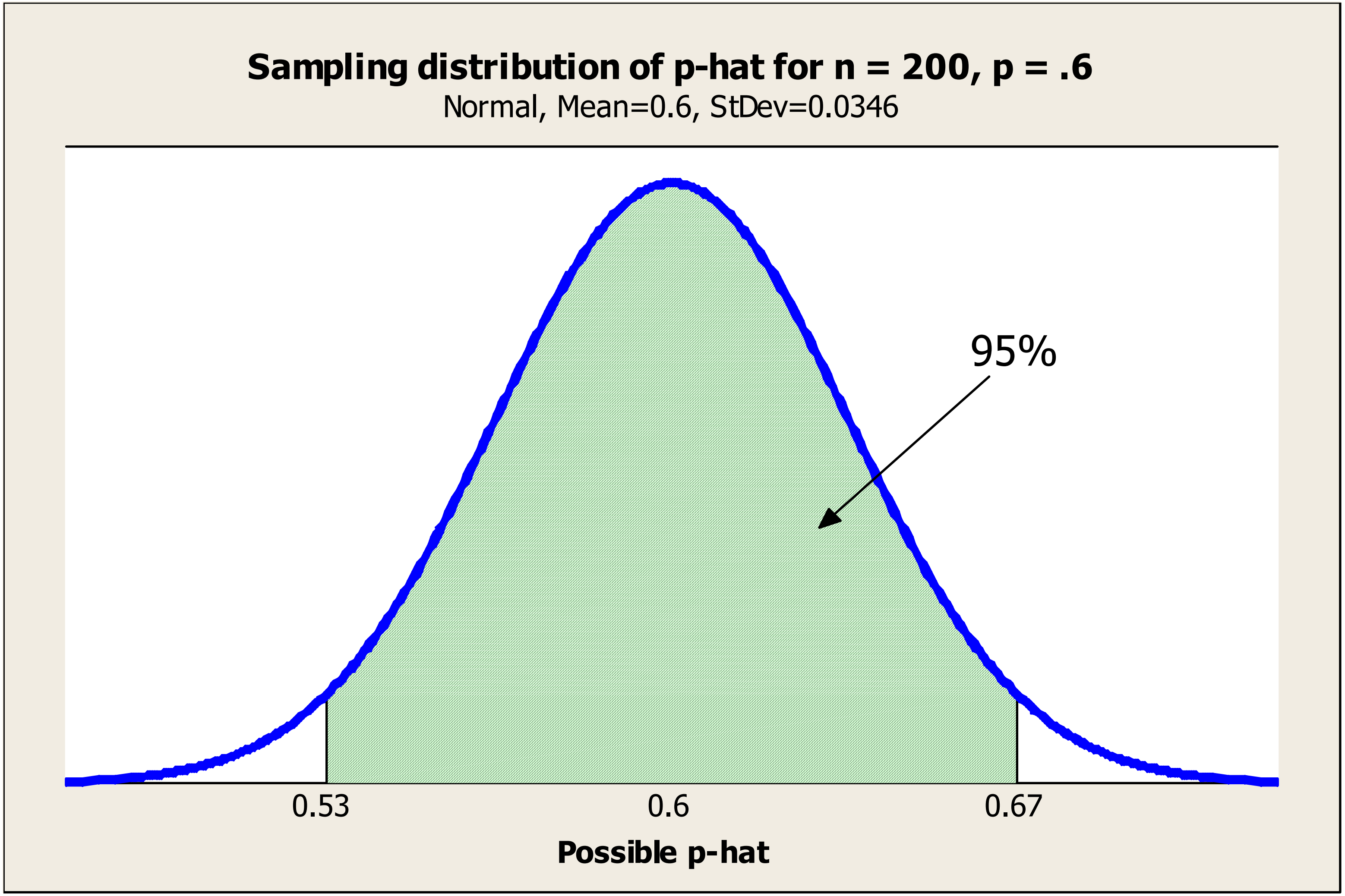
$\hat{p}$ = proportion of *sample* cured

Sampling distribution for $\hat{p}$ is:

- Approximately normal
- Mean = p = .60
- St. dev. = $\sqrt{\frac{(.4)(.6)}{200}} = .0346$

From Empirical Rule, expect 95% of samples to produce $\hat{p}$ to be in the interval mean ± 2s.d.

.60 ± 2(.0346) or .60 ± .07 or .53 to .67.


Sampling distribution of p-hat for n = 200, p = .6
Normal, Mean=0.6, StDev=0.0346
95%
0.53
0.6
0.67
Possible p-hat

The Normal Curve Approximation Rule for Sample Proportions

Normal Approximation Rule can be applied in ***two situations***:

Situation 1: A random sample is taken from a population.

Situation 2: A binomial experiment is repeated numerous times.

In each situation, ***three conditions*** must be met:

Condition 1: *The Physical Situation*
There is an actual population or repeatable situation.

Condition 2: *Data Collection*
A random sample is obtained or situation repeated many times.

Condition 3: *The Size of the Sample or Number of Trials*
The size of the sample or number of repetitions is relatively large, np and $np(1-p)$ must be at least 5 and preferable at least 10.

How well does the approximation work?

It depends on *n* and *p*. Try this applet:

http://bcs.whfreeman.com/pbs/cat_050/pbs/CLT-Binomial.html

Examples for which Rule Applies

- ***Polls*:** to estimate proportion who favor a candidate; units = all voters.
- ***Television Ratings*:** to estimate proportion of households watching TV program; units = all households with TV.
- ***Consumer Preferences*:** to estimate proportion of consumers who prefer new recipe compared with old; units = all consumers.
- ***Testing ESP*:** to estimate probability a person can successfully guess which of 5 symbols on a hidden card; repeatable situation = a guess.

Example: Belief in evolution

Gallup Poll. Feb. 6-7, 2009. N=1,018 adults nationwide. Margin of error given as +/-3%.

"Now, thinking about another historical figure: Can you tell me with which scientific theory Charles Darwin is associated?" Options rotated

Correct response (Evolution, natural selection, etc.)	55%
Incorrect response	10%
Unsure/don't know	34%
No answer	1%

Example, continued

"In fact, Charles Darwin is noted for developing the theory of evolution. Do you, personally, believe in the theory of evolution, do you not believe in evolution, or don't you have an opinion either way?“

(Poll based on n = 1018 adults)

Believe in evolution	**39%**
Do not believe in evolution	**25%**
No opinion either way	**36%**
No answer	**1%**

Example, continued

Let p = *population proportion* who believe in evolution.

Our observed $\hat{p}$ = .39, from sample of 1018.

Based on samples of n = 1018, $\hat{p}$ comes from a distribution of possible values, which is approximately normal with mean μ = p and standard deviation $\sigma = \sqrt{\dfrac{p(1-p)}{1018}}$

Based on this, can we use $\hat{p}$ to estimate p?

Estimating the Population Proportion from a Single Sample Proportion

In practice, we don't know the true population proportion p, so we cannot compute the standard deviation of $\hat{p}$,

$$\text{s.d.}(\hat{p}) = \sqrt{\frac{p(1-p)}{n}}.$$

In practice, we only take one random sample, so we only have one sample proportion $\hat{p}$. Replacing p with $\hat{p}$ in the standard deviation expression gives us an estimate that is called the **standard error of** $\hat{p}$.

$$\text{s.e.}(\hat{p}) = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}.$$

If $\hat{p} = 0.39$ and $n = 1018$, then the standard error is 0.0153. So the *true proportion* who believe in evolution is ***almost surely*** between 0.39 – 3(0.0153) = $\boxed{0.344}$ and 0.39 + 3(0.0153) = $\boxed{0.436}$.

Independent Samples

Two samples are called **independent samples** when the measurements in one sample are not related to the measurements in the other sample.

- **Random samples** taken separately from two populations and same response variable is recorded.
- **One random sample** taken and a variable recorded, but units are **categorized** to form two populations.
- Participants **randomly assigned** to one of two treatment conditions, and same response variable is recorded.

Parameter 2: Difference in two population proportions, based on independent samples

Example research questions:

- How much difference is there between the proportions that would quit smoking if wearing a nicotine patch versus if wearing a placebo patch?
- How much difference is there in the proportion of UCI students and UC Davis students who are an only child?
- Were the proportions believing in evolution the same in 1994 and 2005?

Population parameter:

$\boldsymbol{p_1 - p_2}$ = difference between the two *population* proportions.

Sample estimate:

$\hat{p}_1 - \hat{p}_2$ = difference between the two *sample* proportions.

Sampling distribution for the difference in two proportions $\hat{p}_1 - \hat{p}_2$

- Approximately normal
- Mean is $p_1 - p_2$ = true difference in the *population* proportions
- Standard deviation of $\hat{p}_1 - \hat{p}_2$ is

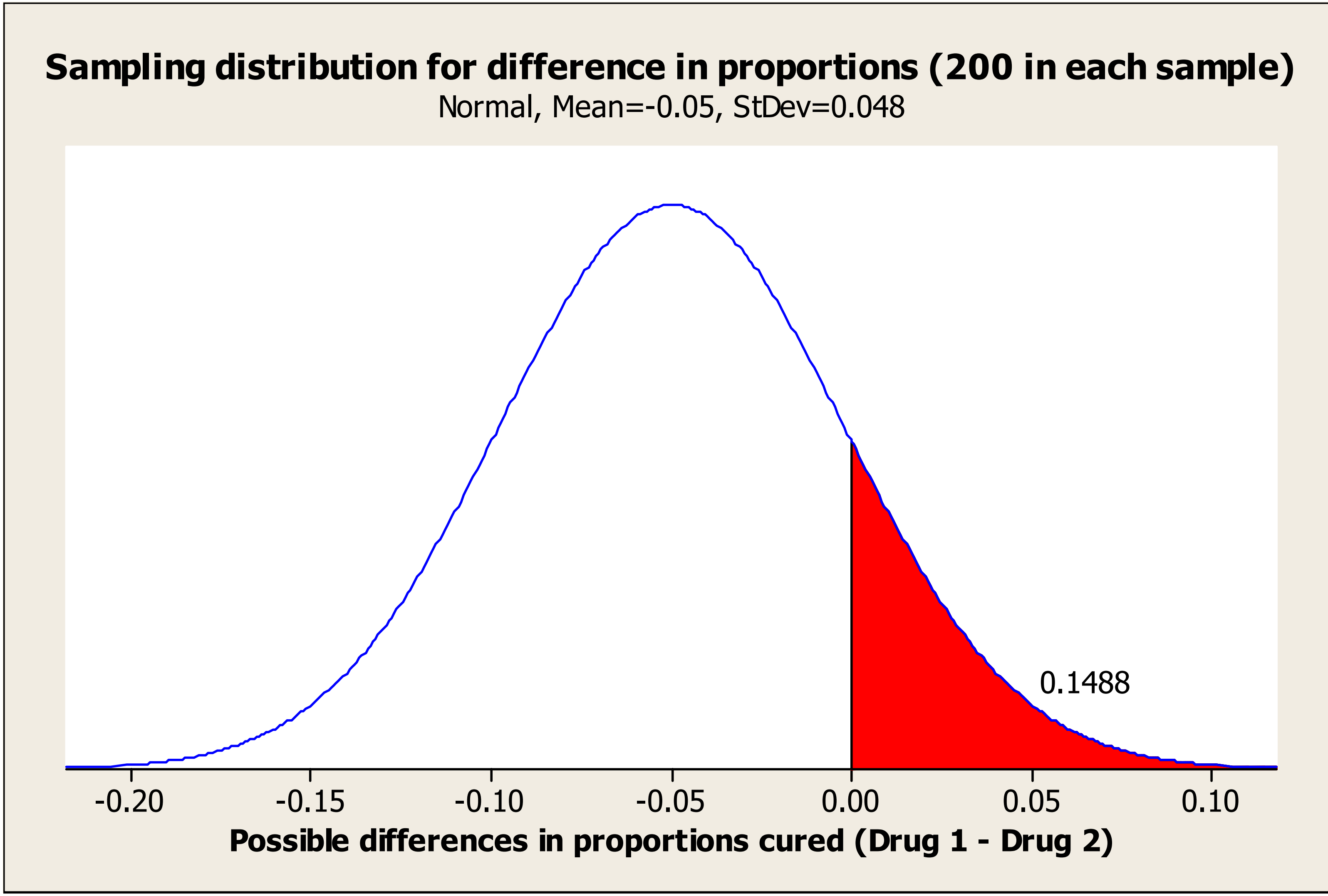
$$s.d.(\hat{p}_1 - \hat{p}_2) = \sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$$

Ex: 2 drugs, cure rates of 60% and 65%, what is probability that drug 1 will cure more in the *sample* than drug 2 if we sample 200 taking each drug? Want P($\hat{p}_1 - \hat{p}_2 > 0$)

Sampling distribution for $\hat{p}_1 - \hat{p}_2$ is:

- Approximately normal
- Mean = –.05
- s.d. $= \sqrt{\dfrac{.6(1-.6)}{200} + \dfrac{.65(1-.65)}{200}} = .048$

See picture on next slide.


Sampling distribution for difference in proportions (200 in each sample)
Normal, Mean=-0.05, StDev=0.048
0.1488
-0.20
-0.15
-0.10
-0.05
0.00
0.05
0.10
Possible differences in proportions cured (Drug 1 - Drug 2)

General format for all sampling distributions in Chapter 9

The sampling distribution of the sample estimate (the sample statistic) is:

- Approximately normal
- Mean = population parameter
- Standard deviation is called the *standard deviation of* _______, where the blank is filled in with the name of the statistic (p-hat, x-bar, etc.)
- The estimated standard deviation is called the *standard error of* _______.

Standard Error of the Difference Between Two Sample Proportions

$$s.e.(\hat{p}_1 - \hat{p}_2) = \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

Are more UCI than UCD children an only child?

$n_1 = 358$ (UCI, 2 classes combined) $n_2 = 173$ (UCD)

UCI: 40 of the 358 students were an only child = $\hat{p}_1 = .112$

UCD: 14 of the 173 students were an only child = $\hat{p}_2 = .081$

So, $\hat{p}_1 - \hat{p}_2 = .112 - .081 = .031$

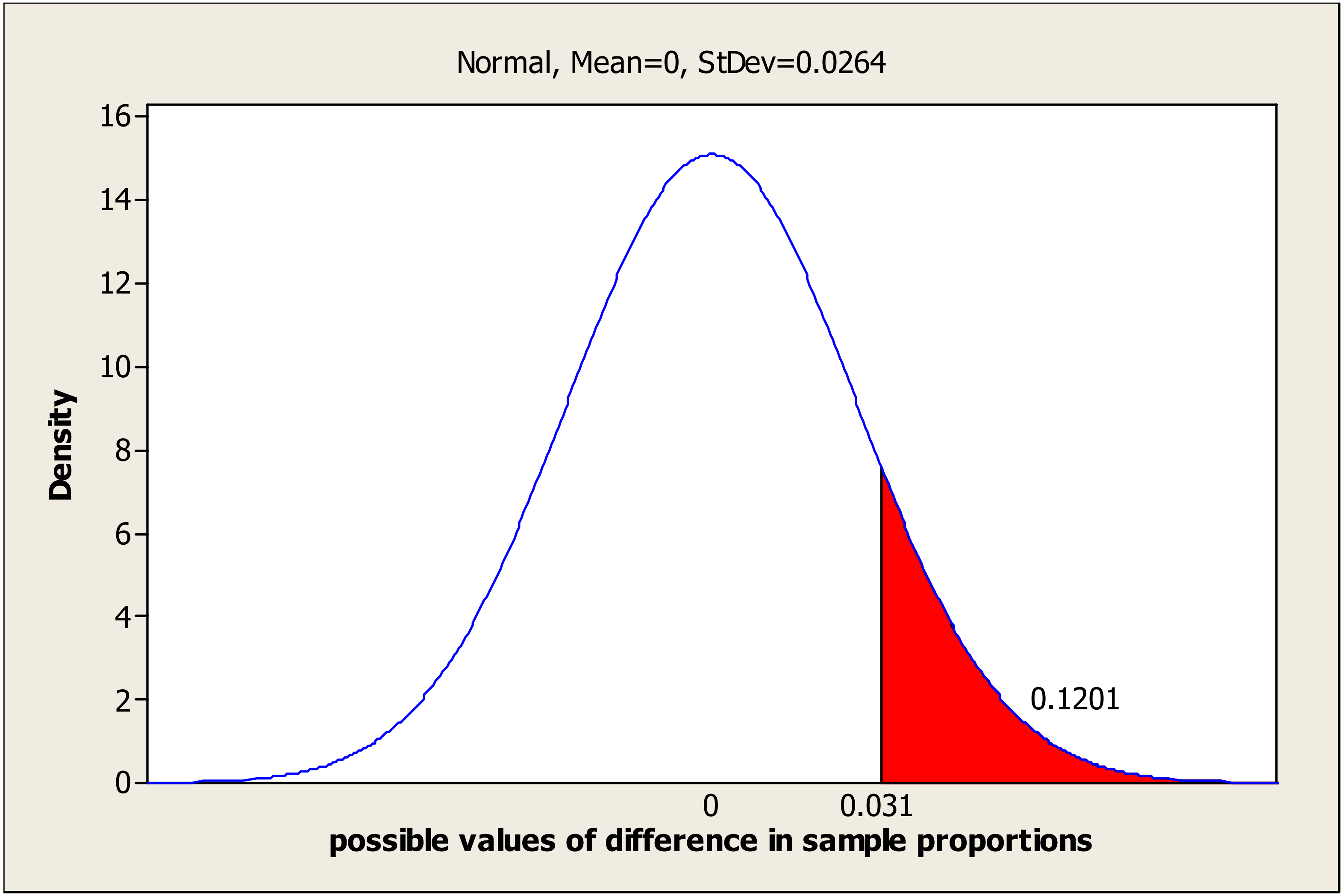
and $s.e.(\hat{p}_1 - \hat{p}_2) = \sqrt{\frac{.11(1-.11)}{358} + \frac{.08(1-.08)}{173}} = .0264$

Suppose population proportions are the same, so true difference $p_1 - p_2 = 0$

Then the sampling distribution of $\hat{p}_1 - \hat{p}_2$ is:

- Approximately normal
- Mean = population parameter = 0
- The estimated standard deviation is .0264
- *Observed* difference of .031 is z = 1.174 standard errors above the mean of 0.
- See picture on next slide; area above .031 = .1201

Sampling distribution of $\hat{p}_1 - \hat{p}_2$



Standardized Statistics for sampling distributions

Recall the general form for standardizing a random variable x when it has a normal distribution:

$$z = \frac{x - \mu}{\sigma}$$

For all 5 parameters we will consider, we can find where our *observed* sample statistic falls if we hypothesize a specific number for the population parameter:

$$z = \frac{sample\ statistic - population\ \ parameter}{s.d.(sample\ statistic)}$$

Example: *Do college students watch less TV?*

In general, there isn't much correlation between age and hrs/TV per day. In 2008 General Social Survey (very large n), 73% watched ≥2 hours per day. So assume population proportion is .73.

In a sample of 175 college students (at Penn State), 105 said they watched 2 or more hours per day.
Is it likely that the population proportion for students is also .73?

$$\hat{p} = \frac{105}{175} = .6 \qquad s.d.(\hat{p}) = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.73(1-0.73)}{175}} = 0.034$$

$$z = \frac{.6 - .73}{.034} = -3.82$$

This z-score is too small! Area below it is .00007. Students are different from general population.

Case Study 9.1 *Do Americans Really Vote When They Say They Do?*

Election of 1994:

- *Time Magazine Poll*: $n = 800$ adults (two days after election), **56% reported that they had voted**.
- Info from Committee for the Study of the American Electorate: **only 39% of American adults had voted**.

If $p = 0.39$ then sample proportions for samples of size $n = 800$ should vary approximately normally with …

$$\text{mean} = p = 0.39 \text{ and s.d.}(\hat{p}) = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.39(1-0.39)}{800}} = 0.017$$

Case Study 9.1 *Do Americans Really Vote When They Say They Do?*

If respondents were telling the truth, the sample percent should be no higher than 39% + 3(1.7%) = 44.1%, nowhere near the reported percentage of 56%.

If 39% of the population voted, the **standardized score** for the reported value of 56% is …

$$z = \frac{0.56 - 0.39}{0.017} = 10.0$$

It is virtually ***impossible*** to obtain a standardized score of 10.