Announcements:

•Discussion today is review for midterm, no credit. You may attend more than one discussion section.

•Bring 2 sheets of notes and calculator to midterm. We will provide Scantron form.

Homework: (Due Wed)

Chapter 10: #5, 22, 42

Chapter 10

Estimating Proportions with Confidence

Confidence interval example from Fri lecture

Gallup poll of n = 1018 adults found 39% believe in evolution. So $\hat{p} = .39$

A **95% confidence interval** or **interval estimate** for the proportion (or percent) of *all* adults who believe in evolution is **.36** to **.42** (or **36%** to **42%**).

> **Confidence interval:** an interval of estimates that is *likely* to capture the population value.

Goal today: Learn to calculate and interpret confidence intervals for p and for $p_1 - p_2$ *and* learn general format.

Remember population versus sample:

- **Population proportion:** the fraction of the ***population*** that has a certain trait/characteristic or the probability of success in a binomial experiment – denoted by p. The value of the ***parameter*** p is not known.
- **Sample proportion:** the fraction of the ***sample*** that has a certain trait/characteristic – denoted by $\hat{p}$. The ***statistic*** $\hat{p}$ is an estimate of p.

> The **Fundamental Rule for Using Data for Inference:** Available data can be used to make inferences about a much larger group *if the data can be considered to be representative with regard to the question(s) of interest.*

Some Definitions:

- **Point estimate:** A *single number* used to estimate a population parameter. For our five situations:

  point estimate = **sample statistic** = **sample estimate**

  = $\hat{p}$ for one proportion

  = $\hat{p}_1 - \hat{p}_2$ for difference in two proportions
- **Interval estimate:** An *interval* of values used to estimate a **population parameter**. Also called a **confidence interval**. For our five situations, always:

> Sample estimate ± multiplier × standard error

Details for proportions:

> Sample estimate ± multiplier × standard error

Parameter	Sample estimate	Standard error
p	$\hat{p}$	$s.e.(\hat{p}) = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$
$p_1 - p_2$	$\hat{p}_1 - \hat{p}_2$	See p. 424 for formula

Multiplier and Confidence Level

- The **multiplier** is determined by the desired confidence level.
- The **confidence level** is the probability that the procedure used to determine the interval *will* provide an interval that includes the population parameter. Most common is .95.
- If we consider *all possible* randomly selected samples of same size from a population, the *confidence level* is the fraction or percent of those samples for which the confidence interval includes the population parameter.

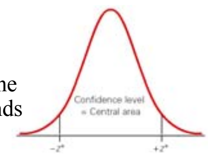
  See picture on board.
- Often express the confidence level as a percent. Common levels are 90%, 95%, 98%, and 99%.

More about the Multiplier

Confidence Level	Multiplier	Confidence Interval
90	1.645 or 1.65	$\hat{p} \pm 1.65$ standard errors
95	1.96, often rounded to 2	$\hat{p} \pm 2$ standard errors
98	2.33	$\hat{p} \pm 2.33$ standard errors
99	2.58	$\hat{p} \pm 2.58$ standard errors

Note: Increase confidence level => larger multiplier.

Multiplier, denoted as z^*, is the standardized score such that the area between $-z^*$ and z^* under the standard normal curve corresponds to the desired confidence level.



Formula for C.I. for proportion

Sample estimate ± multiplier × standard error

For one proportion: A confidence interval for a population proportion p, based on a sample of size n from that population, with sample proportion $\hat{p}$ is:

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

Example of different confidence levels

Poll on belief in evolution:

$n = 1018$

Sample proportion = .39

Standard error = $\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{.39(1-.39)}{1018}} = .0153$

90% confidence interval

$.39 \pm 1.65(.0153)$ or $.39 \pm .025$ or .365 to .415

95% confidence interval:

$.39 \pm 2(.0153)$ or $.39 \pm .03$ or .36 to .42

99% confidence interval

$.39 \pm 2.58(.0153)$ or $.39 \pm .04$ or .35 to .43

Interpretation of the confidence interval and confidence level:

- We are ***90% confident*** that the proportion of *all* adults in the US who believe in evolution is between **.365 and .415**.
- We are ***95% confident*** that the proportion of *all* adults in the US who believe in evolution is between **.36 and .42.**
- We are ***99% confident*** that the proportion of *all* adults in the US who believe in evolution is between **.35 and .43**.

Interpreting the confidence level of 99%:

The interval .35 to .43 ***may or may not*** capture the true proportion of adult Americans who believe in evolution

But, ***in the long run*** this ***procedure*** will produce intervals that capture the unknown population values about 99% of the time. So, we are 99% confident that it worked this time.

Notes about interval width

- Higher confidence <=> wider interval
- Larger n (sample size) <=> more narrow interval, because n is in the *denominator* of the standard error.
- So, if you want a more narrow interval you can either *reduce* your confidence, or *increase* your sample size.

Reconciling with Chapter 3 formula for 95% confidence interval

Sample estimate ± Margin of error
where (conservative) margin of error was $\frac{1}{\sqrt{n}}$

Now, "margin of error" is $2\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

These are the *same* when $\hat{p} = .5$. The new margin of error is *smaller* for *any* other value of $\hat{p}$ So we say the old version is *conservative.* It will give a *wider* interval.

Comparing three versions (Details on board)

For the evolution example, $n = 1018$, $\hat{p} = .39$

- *Conservative* margin of error = .0313 ≈ .03
- *Approximate* margin of error using z* = 2
  2 × .0153 = .0306 ≈ .03
- *Exact* margin of error using z* = 1.96
  1.96 × .0153 = .029988 ≈ .03

All very close to .03, and it really doesn't make much difference which one we use!

New example: compare methods

Marist Poll in Oct 2009 asked "How often do you text while driving?" $n = 1026$

Nine percent answered "Often" or "sometimes" so and $\hat{p} = .09$ $\quad s.e.(\hat{p}) = \sqrt{\frac{.09(.91)}{1026}} = .009$

- *Conservative* margin of error = .0312
- *Approximate* margin of error = 2 × .009 = .018.

This time, they are quite different!

The conservative one is too conservative, it's double the approximate one!

Comparing margin of error

- Conservative margin of error will be okay for sample proportions near .5.
- For sample proportions far from .5, closer to 0 or 1, don't use the conservative margin of error. Resulting interval is wider than needed.
- Note that using a multiplier of 2 is called the *approximate* margin of error; the *exact* one uses multiplier of 1.96. It will rarely matter if we use 2 instead of 1.96.

Factors that Determine Margin of Error

1. ***The sample size, n.***
   When sample size *increases*, margin of error *decreases*.
2. ***The sample proportion,*** $\hat{p}$.
   If the proportion is close to either 1 or 0 most individuals have the same trait or opinion, so there is little natural variability and the margin of error is smaller than if the proportion is near 0.5.
3. ***The "multiplier"* 2 or 1.96.**
   Connected to the "95%" aspect of the margin of error. Usually the term "margin of error" is used only when the confidence level is 95%.

General Description of the Approximate 95% CI for a Proportion

Approximate 95% CI for the population proportion:

$\hat{p} \pm 2$ standard errors

The standard error is $s.e.(\hat{p}) = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

Interpretation*:* For about 95% of all randomly selected samples from the population, the confidence interval computed in this manner captures the population proportion.

Necessary Conditions*:* $n\hat{p}$ and $n(1-\hat{p})$ are both greater than 10, and the sample is randomly selected.

Finding the formula for a 95% CI for a Proportion – use Empirical Rule:

For 95% of all samples, $\hat{p}$ is within 2 st.dev. of p

Sampling distribution of $\hat{p}$ tells us *for 95% of all samples*:

−2 standard deviations < $\hat{p} - p$ < 2 standard deviations

Don't know true standard deviation, so use standard error.
For approximately 95% of all samples,

−2 standard errors < $\hat{p} - p$ < 2 standard errors

which implies for approximately 95% of all samples,

$\hat{p}$ – 2 standard errors < p < $\hat{p}$ + 2 standard errors

Same holds for *any* confidence level; replace 2 with z*

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

where:

- $\hat{p}$ is the sample proportion
- z^* denotes the multiplier.
- $\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ is the standard error of $\hat{p}$.

Example 10.3 *Intelligent Life Elsewhere?*

Poll: Random sample of 935 Americans
Do you think there is intelligent life on other planets?

Results: 60% of the sample said "yes", $\hat{p}$ = .60

$$s.e.(\hat{p}) = \sqrt{\frac{.6(1-.6)}{935}} = .016$$

90% Confidence Interval: .60 ± 1.65(.016), or .60 ± .026
.574 to .626 or 57.4% to 62.6%

98% Confidence Interval: .60 ± 2.33(.016), or .60 ± .037
.563 to .637 or 56.3% to 63.7%

Note: entire interval is above 50% => high confidence that a majority believe there is intelligent life.

Confidence intervals and "plausible" values

- Remember that a confidence interval is an *interval estimate* for a population parameter.
- Therefore, any value that is covered by the confidence interval is a *plausible value* for the parameter.
- Values *not* covered by the interval are still possible, but not very likely (depending on the confidence level).

Example of plausible values

- 98% Confidence interval for proportion who believe intelligent life exists elsewhere is:

  .563 to .637 or 56.3% to 63.7%

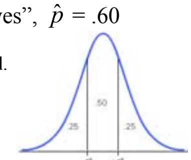
- Therefore, 56% is a *plausible value* for the population percent, but 50% is not very likely to be the population percent.
- Entire interval is above 50% => high confidence that a *majority* believe there is intelligent life.

New multiplier: let's do a confidence level of 50%

Poll: Random sample of 935 Americans
"Do you think there is intelligent life on other planets?"

Results: 60% of the sample said "yes", $\hat{p}$ = .60

We want a **50% confidence interval**.
If the area between $-z^*$ and z^* is .50,
then the area to the left of z^* is .75.
From Table A.1 we have $z^* \approx .67$.
(See next page for Table A.1)



50% Confidence Interval: .60 ± .67(.016), or .60 ± .011
.589 to .611 or 58.9% to 61.1%

Note: Lower confidence *level* results in a narrower *interval*.

Remember conditions for using the formula:

1. Sample is **randomly selected** from the population. ***Note***: Available data can be used to make inferences about a much larger group *if the data can be considered to be representative with regard to the question(s) of interest.*
2. Normal curve approximation to the distribution of possible sample proportions assumes a **"large" sample size**. Both $n\hat{p}$ and $n(1-\hat{p})$ should be at least 10 (although some say these need only to be at least 5).

In Summary*: Confidence Interval for a Population Proportion *p

General CI for *p*: $\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

Approximate 95% CI for *p*: $\hat{p} \pm 2\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

Conservative 95% CI for *p*: $\hat{p} \pm \frac{1}{\sqrt{n}}$

Section 10.4: Comparing two population proportions

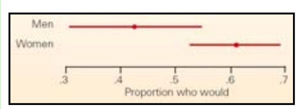
- Independent samples of size n_1 and n_2
- Use the two *sample* proportions as data.
- Could compute separate confidence intervals for the two population proportions and see if they overlap.
- Better to find a confidence interval for the *difference* in the two population proportions,

Case Study 10.3 *Comparing proportions*

Would you date someone with a great personality even though you did ***not*** find them attractive?

Women: .611 (61.1%) of 131 answered "yes." 95% confidence interval is .527 to .694.

Men: .426 (42.6%) of 61 answered "yes." 95% confidence interval is .302 to .55.



Conclusions:

- ***Higher proportion*** of ***women*** would say yes. CIs slightly overlap
- Women CI ***narrower*** than men CI due to larger sample size

Compare the two proportions by finding a CI for the difference

C.I. for the difference in two population proportions:

Sample estimate ± multiplier × standard error

$$(\hat{p}_1 - \hat{p}_2) \pm z^* \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

Case Study 10.3 *Comparing proportions*

Would you date someone with a great personality even though you did ***not*** find them attractive?

Women: .611 of 131 answered "yes." 95% confidence interval is .527 to .694.

Men: .426 of 61 answered "yes." 95% confidence interval is .302 to .55.

Confidence interval for the difference in *population proportions* of women and men who would say yes.

$$(.611 - .426) \pm z^* \sqrt{\frac{.611(1-.611)}{131} + \frac{.426(1-.426)}{61}}$$

95% confidence interval

- A 95% confidence interval for the difference is .035 to .334 or 3.5% to 33.4%.
- We are 95% confident that the *population* proportions of men and women who would date someone they didn't find attractive *differ* by between .035 and .334, with a lower proportion for men than for women.
- We can conclude that the two *population* proportions *differ* because 0 is not in the interval.

Section 10.5: Using confidence intervals to guide decisions

- A value *not* in a confidence interval can be rejected as a likely value for the population parameter.
- When a confidence interval for $p_1 - p_2$ does not cover 0 it is reasonable to conclude that the two population values differ.
- When confidence intervals for p_1 and p_2 do not overlap it is reasonable to conclude they differ, but if they do overlap, no conclusion can be made. In that case, find a confidence interval for the difference.

From the Midterm 2 review sheet for Chapter 10 - you should know these now

1. Understand how to interpret the *confidence level*
2. Understand how to interpret a *confidence interval*
3. Understand how the sampling distribution for $\hat{p}$ leads to the confidence interval formula (pg. 417-418)
4. Know how to compute a confidence interval for one proportion, including conditions needed.
5. Know how to compute a confidence interval for the difference in two proportions, including conditions needed.
6. Understand how to find the multiplier for desired confidence level.
7. Understand how margin of error from Chapter 3 relates to the 95% confidence interval formula in Chapter 10
8. Know the general format for a confidence interval for the 5 situations defined in Chapter 9 (see summary on page 483).