Visual Computing Midterm  
Winter 2018

Total Points: 80 points

Name:______________________________________________

Number:____________________________________________

Pledge: I neither received nor gave any help from or to anyone in this exam.

Signature:______________________________

Useful Tips
1. All questions are multiple choice questions --- please indicate your answers very clearly.  
   You can circle them or write out the exact choice.
2. **Questions may have more than one answer. You get full points only if you choose all  
   the correct answers.**
3. Use the blank pages as your worksheet. Put the question number when working out the  
   steps in the worksheet. Also, do your work clearly. This will help us give partial credit.
4. If you need more work sheets, feel free to ask for extra sheets.
5. Staple all your worksheets together with the paper at the end of the exam. If pages of  
   your exam are missing since you took them apart, we are not responsible for putting them  
   together.
6. **The number of minutes you should spend on each question is roughly equal to the  
   number of points assigned to the question.**
1) Assume that the maximum bandwidth of audio signal to be 75Hz. Let us consider amplitude modulation of such audio signals by multiple stations using high frequency carrier signals.
   a. Which of the following cannot be the frequency of a carrier signal.
      i. 125 Hz
      ii. 100 Hz
      iii. 200 Hz
      iv. 400 Hz
   b. What is the minimum gap that should exist between two carrier frequencies?
      Since the max bandwidth is 75Hz then there should be at least 2*75=150Hz gap between two carriers to avoid overlap of signals
      i. 75 Hz
      ii. 150 Hz
      iii. 200 Hz
      iv. 400 Hz
   c. Not maintaining this minimum gap results in
      i. Ghosting
      ii. Amplifying
      iii. Aliasing
   d. Amplitude modulation in frequency domain is equivalent to
      For amplitude modulation we multiply the signal with the carrier signal in spatial domain which equivalent of Convolving carrier frequency cosine wave with the signal
      i. Adding carrier frequency cosine wave to the signal
      ii. Multiplying carrier frequency cosine wave with the signal
      iii. Convolving carrier frequency cosine wave with the signal

2) Consider a double-torus manifold constructed of triangles. Its genus is 2.
   a. The euler characteristic of this double torus is
      \[ e = 2 - 2g = 2 - 4 = -2 \]
      i. -2
      ii. -1
      iii. 0
      iv. 1
      v. 2
   b. \( V \) (# of vertices) and \( F \) (# of faces) in this manifold is related by the following
      \[ e = F - E + V = -2 \]
      we can also write a equation between number of edges and number of faces. Since each face has 3 edge and each edge belongs to two face we have \( E = 3F/2 \)
      By replacing this equation in above equation we get \( V = (F-4)/2 \)
      i. \( V = (F+4)/2 \)
      ii. \( V = (F-4)/2 \)
      iii. \( V = 2F-2 \)
      iv. \( V = 2F+2 \)
   c. Now consider a torus. What is its genus?
Genus number shows the number of handles which is 1 for a torus

i. -2
ii. -1
iii. 0
iv. 1
v. 2

d. Is it possible to morph a double torus to the torus?
morphing a double torus to a torus without changing the connectivity of the vertices is no possible. We can not morph objects with different genus numbers.

i. Yes
ii. No

3) [4x2=8] Match the images from left column with the image of the magnitude of their DFT from right column.
For solving this question look at the orientation of the edges in each image. Then the DFT of the image should have same direction for the edges.
i) 0

ii) 1

iii) 7

iv) 8

v) 2

a) 

b) 

c) 

d) 

e)
4) \[2+3+3+2=10\] Consider a signal of 16 samples given by \[x(i) = 4 + \cos(\pi i/4) + 3\sin(3\pi i/8) + 2\cos(\pi i/2) + 5\sin(5\pi i/8) + 7\cos(3\pi i/4) + 9\sin(7\pi i/8) + \cos(\pi i)\].

For solving this question we can use the following equation

\[
x[i] = \sum_{k=0}^{N/2} \hat{x}_c[k] \cos \left(\frac{2\pi k i}{N}\right) + \sum_{k=0}^{N/2} \hat{x}_s[k] \sin \left(\frac{2\pi k i}{N}\right)
\]

\(xc\) and \(xs\) are the RX and IX and the DC component is the scalar value which is 4

a. The DC component of this signal is
   i. 1
   ii. 4
   iii. 8
   iv. 16

b. The array RX is given by
   i. [4, 1, 2, 7, 1]
   ii. [4, 1, 2, 7, 1, 0, 0, 0]
   iii. [4, 0, 1, 0, 2, 0, 7, 0, 1]
   iv. [0, 0, 0, 0, 4, 1, 2, 7, 1]

c. The array IX is given by
   i. [0, 3, 5, 9, 0]
   ii. [0, 3, 5, 9, 0, 0, 0, 0]
   iii. [0, 0, 3, 0, 5, 0, 9, 0, 0]
   iv. [0, 0, 0, 3, 0, 5, 0, 9, 0]

d. Consider the cosine and sine waves in \(x\) that make 4 cycles through the 16 samples. The amplitudes of these two waves are respectively
   i. 2, 5
   ii. 2, 0
   iii. 1, 9
   iv. 0,

5) \[2+2+2+1+2=9\] Consider the Laplacian filter.

a. It provides the following at any pixel in an image.
   Laplacian filter finds the second derivative of the image which shows the curvature of the image
   i. Gradient in \(x\) direction
   ii. Gradient in \(y\) direction
   iii. Curvature
   iv. Strength and direction of edges

b. Consider an image on which the Laplacian filter is applied for edge detection. An edge in the image corresponds to the following in the filtered image.
   The edge is the point of transition from positive curvature to negative curvature which is the zero crossing of the second derivative.
   i. Zero
ii. Zero Crossings
iii. Maxima
iv. Minima

c. If the image is noisy, what kind of filter should be applied to the image before applying the Laplacian filter. 
By removing the high frequencies we can reduce the noise so we need to use a low pass filter
i. A low pass filter
ii. A high pass filter
iii. A gradient filter

d. The degrades which of the following property of a good edge detector
Low pass filter will blur the image and as a result the location of the detected edges can be changed
i. Detection
ii. Localization
iii. Single Response

e. By changing the size of this preprocessing filter, we can detect edges of
i. Different Lengths
ii. Different Resolutions
iii. Different Contrasts

6) \[3+3+4=9\] We want to use Hough Transform to identify the presence of different types of conics (e.g. parabolas, circles, ellipses) in an edge image.

a. Consider an ellipse given by equation \[\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1\] The Hough space for identifying circle is
The unknown variables are a and b. So the hough transform has two variables.
i. one dimensional
ii. two dimensional
iii. three dimensional

b. The ellipse in space corresponds to the following in the Hough space
If we assume a and b are the variables of the above equation and x and y are the coefficients then the equation belongs to parabolas
i. parabolas
ii. Sphere
iii. lines

c. The general equation of the entity corresponding to the parabola in the Hough space is given by
This question was wrong
i. \((x-a)^2+(y-b)^2-z^2=0\)
ii. \(ax+by+z=c\)
iii. \(ax^2+by^2+cz^2=0\)
iv. \(x^2+y^2+z^2 = c\)
7) \[1+2+2=5\] Consider the Canny Edge detector.
   a. It is a
      i. Gradient based edge detector
      ii. Curvature based edge detector
   b. The non-maximal suppression achieves which of the following in Canny edge detector
      non-maximal suppression assures to choose one point for each edge
      i. Detection
      ii. Localization
      iii. Single Response
      iv. Classification of edges based on their strengths
   c. The process of double thresholding helps in
      Using double thresholding we can categorize each point as non-edge, weak edge and strong edge
      i. Detection
      ii. Localization
      iii. Single Response
      iv. Classification of edges based on their strengths

8) \[2+3+3+2=10\] Consider a 2D rectangle ABCD where A=(0,0), B=(3,0), C=(3, 1) and D=(0,1). We want to apply a 2D transformation to this rectangle which makes it a paralleloipipped ABEF where E = (6,1) and F= (3,1).
   a. What kind of transformation is this?
      As you can see after transformation the value of x is increase proportional to value of y so we have \( x = x + 3y \). So this is a shear
      i. Scale
      ii. Rotate
      iii. Shear
      iv. Translate
   b. The 3x3 matrix \( M \) achieving this transformation is given by
      i. \[ [1 \quad 3 \quad 0 ; \quad 0 \quad 1 \quad 0 ; \quad 0 \quad 0 \quad 1] \]
      ii. \[ [1 \quad 0 \quad 0 ; \quad 3 \quad 1 \quad 0 ; \quad 0 \quad 0 \quad 1] \]
      iii. \[ [1 \quad 0 \quad 3 ; \quad 0 \quad 1 \quad 0 ; \quad 0 \quad 0 \quad 1] \]
      iv. \[ [1 \quad 0 \quad 0 ; \quad 0 \quad 1 \quad 0 ; \quad 3 \quad 0 \quad 1] \]
   c. What additional transformation \( N \) we would need to apply to ABEF to get the paralleloipipped A’B’E’F’ where A’ = (0, 0), B’ = (6,0), E’= (12, 3), and F’ = (6,3).
      i. Rotation by 45
      ii. Scale by (2, 3)
      iii. Translate by (3, 2)
   d. What is the final concatenated matrix in terms of \( M \) and \( N \) that will transform ABCD to A’B’E’F’?
      M is the first transformation so it should be on the right. Then when we multiply \( NM \) with coordinate of each point M will multiply first
      i. \( MN \)
      ii. \( NM \)
      iii. \( M^{-1}N \)
9) Consider the following matrix \[ \begin{bmatrix} 3 & 2 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \] Note: \( \frac{1}{\sqrt{2}} = 0.707 \).

The following matrix is created by multiplication of a scale matrix and a shear matrix.

\[ \begin{bmatrix} 3 & 0 & 2 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \]

a. This matrix achieves the following in the global coordinate system
   i. x-shear of 2 following by \( S_x \) of 3
   ii. \( S_x \) of 3 following by x-shear of 2
   iii. z-shear of 2 following by \( S_x \) of 3
   iv. \( S_x \) of 3 following by z-shear of 2

b. Consider a translation \( T \) of the local coordinate system following these transformations. The resultant transformation will be (\( T \) shows translation, \( S \) shows Scaling and \( H \) shows shear)

   Because \( T \) is in local coordinate system it should be in the right side.
   i. \( H_x S_x T \)
   ii. \( T H_x S_x \)
   iii. \( T S_x H_z \)
   iv. \( S_x H_z T \)

10) Assume two images related by a homography transformation.

a. How many correspondences do you need in the least to recover the homography? Homography matrix is a 3by3 matrix and if we assume the last element is always 1 then it has 8 unknowns having each correspondences we can write two equations for x and y coordinate so we need to have 4 point to have 8 equation to find 8 unknowns
   i. Three
   ii. Four
   iii. Five
   iv. Six

b. Which of the following scenarios result in a homography.

   If we can transform one image to another image using a sequence of projective transformation of planes then there is a homography between these images.
   i. The two cameras sharing the same center of projection with different orientation.
   ii. The two cameras having the same orientation but are translated with respect to each other.
   iii. The two cameras are capturing a 2D planar scene.

11) Consider a camera with focal length of 15mm and a square sensor of 2cmx2cm with a resolution of 5000x5000. The principal axis does not pass through the center of the image plane but is slightly shifted by 0.3mm in both horizontal and vertical direction. The camera is translated by 2mm in X, Y and Z direction each. Further, it is rotated by 45 degrees around the Z axis.

Wrong numbers for a c and d

a. The pixel pitch (size in one direction) of the camera is
The size of sensor is 2cm and sensor has 5000 pixels then the size of each pixel is 
\[ \frac{2 \text{cm}}{5000} = 4 \mu m \]

1. 2\(\mu\)m
2. 2.5\(\mu\)m
3. 1\(\mu\)m
4. 1.5\(\mu\)m

b. The pixel shape is a
1. Square
2. Rectangle
3. Rombus
4. Parallelepiped

c. The focal length of the camera is pixels is given by

Now we know the size of pixel is 4 \(\mu\)m and focal length is 15mm so the focal length of camera is 15mm/4 \(\mu\)m

1. 15000 pix
2. 25000 pix
3. 10000 pix

d. The intrinsic parameter matrix of the camera is given by
The intrinsic matrix of the camera is
\[
\begin{bmatrix}
f & 0 & 0 \\
0 & f & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

1. \[
\begin{bmatrix}
15000 & 0 & 0 \\
0 & 15000 & 0 \\
300 & 300 & 1
\end{bmatrix}
\]
2. \[
\begin{bmatrix}
15000 & 300 & 0 \\
0 & 15000 & 300 \\
0 & 0 & 1
\end{bmatrix}
\]
3. \[
\begin{bmatrix}
25000 & 500 & 0 \\
0 & 25000 & 0 \\
500 & 500 & 1
\end{bmatrix}
\]
4. \[
\begin{bmatrix}
25000 & 0 & 500 \\
0 & 25000 & 500 \\
0 & 0 & 1
\end{bmatrix}
\]
5. \[
\begin{bmatrix}
10000 & 300 & 0 \\
0 & 10000 & 0 \\
300 & 300 & 1
\end{bmatrix}
\]
6. \[
\begin{bmatrix}
10000 & 0 & 300 \\
0 & 10000 & 300 \\
0 & 0 & 1
\end{bmatrix}
\]

e. The extrinsic parameter matrix is given by
The rotation matrix around z is
\[
\begin{bmatrix}
\cos(\text{angle}) & -\sin(\text{angle}) & 0 & 0 \\
\sin(\text{angle}) & \cos(\text{angle}) & 0 & 0 \\
0 & 0 & 1 & 0
\end{bmatrix}
\]
and translation matrix is [2;2;2]
So the extrinsic matrix of the camera is [R|RT]

1. \[
\begin{bmatrix}
\sqrt{2}/2 & \sqrt{2}/2 & 0 & 0 \\
-\sqrt{2}/2 & \sqrt{2}/2 & 2/2 & 0 \\
0 & 0 & 1 & 2
\end{bmatrix}
\]
2. \[
\begin{bmatrix}
\sqrt{2}/2 & \sqrt{2}/2 & 0 & 0 \\
-\sqrt{2}/2 & \sqrt{2}/2 & 2/2 & 0 \\
0 & 0 & 1 & 2\sqrt{2}
\end{bmatrix}
\]
3. \[
\begin{bmatrix}
\sqrt{2}/2 & -\sqrt{2}/2 & 0 & 2 \\
\sqrt{2}/2 & \sqrt{2}/2 & 0 & 2 \\
0 & 0 & 1 & 2
\end{bmatrix}
\]
4. \[
\begin{bmatrix}
\sqrt{2}/2 & -\sqrt{2}/2 & 0 & 0 \\
\sqrt{2}/2 & \sqrt{2}/2 & 2/2 & 0 \\
0 & 0 & 1 & 2
\end{bmatrix}
\]