

# Viewing Conditions and Chromatic Adaptation

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## Introduction:

In this report, we will summarize the contents of chapters 7 to 10 of [1]. First, we will talk about viewing conditions then chromatic adaptation, and finally, we will discuss various chromatic adaptation models.

### 1. Viewing condition:

We know that the conditions in which we are viewing the scene are very important in the response of our receptors. For instance, illumination is a very important issue. If you illuminate a white paper with a sun light or Florescent light, the receptors are receiving very different signals because the surface reflects the illumination light.

On the other hand, we know that a white piece of paper would perceive somehow the same under different illuminations. This means that our visual system adapts itself to the environment and tries to change our perception not to see the effect of illumination.

### 2. Chromatic adaptation:

We know about light and dark adaptation which is present in entering or exiting a theatre room in which our visual systems adapts itself to the environment. Both cone and rod sensors perform this adaptation and we know that rods are active in low illuminations.

Chromatic adaptation is very similar to light and dark adaptation. Actually, if you illuminate a scene with different light sources and take photos, you will have different colors in photos while you will perceive both the scenes more or less the same. Figure 1 shows an example.

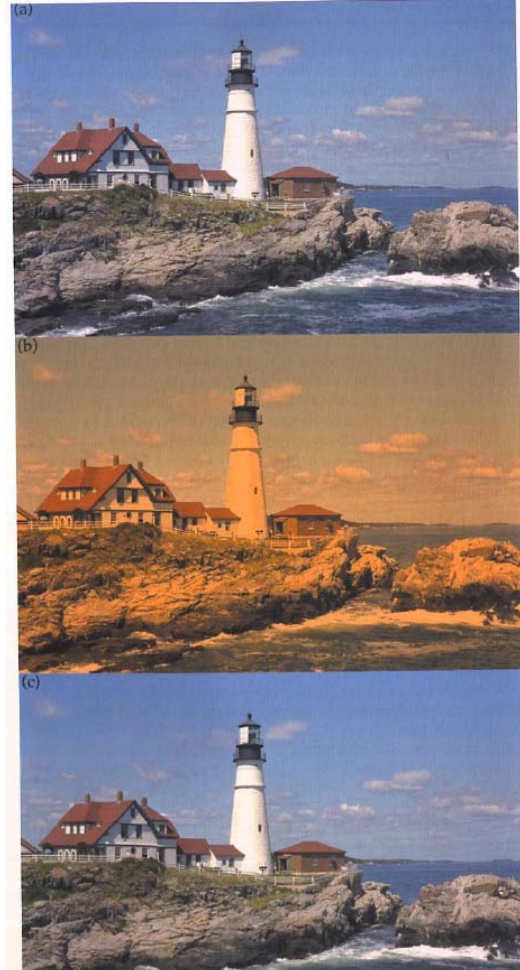


Figure 1 Chromatic adaptation: First: a scene illuminated by daylight. Second: the same scene illuminated by Tungsten light and seen by a visual system. Third: the perceived version of the second scene.

### 3. Chromatic adaptation models:

Now, we know that the human visual system performs chromatic adaptation. And if we want to have perceptually better systems, we have to model this phenomenon to be able to implement it in our artificial systems. First, we can go from XYZ tristimulus values to LMS cone responsivities easily using the following linear transformation.

$$\begin{bmatrix} L \\ M \\ S \end{bmatrix} = \begin{bmatrix} 0.400 & 0.708 & -0.081 \\ -0.226 & 1.165 & 0.046 \\ 0.000 & 0.000 & 0.918 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

### Von Kries model:

All chromatic adaptation models are a consequence of this model. It is a very simple model introduced by Von Kries in 1902. In this model, the adapted response of the cones is proportional to the actual response by a constant factor as follows:

$$L_a = k_L L$$

$$M_a = k_M M$$

$$S_a = k_S S$$

The coefficients could be found using the following relations by having the response for maximum stimulation:

$$k_L = 1/L_{\max} \quad \text{or} \quad k_L = 1/L_{\text{white}}$$

$$k_M = 1/M_{\max} \quad \text{or} \quad k_M = 1/M_{\text{white}}$$

$$k_S = 1/S_{\max} \quad \text{or} \quad k_S = 1/S_{\text{white}}$$

Hence, we would have the following matrix multiplication:

$$\begin{bmatrix} L_a \\ M_a \\ S_a \end{bmatrix} = \begin{bmatrix} 1/L_{\max} & 0.0 & 0.0 \\ 0.0 & 1/M_{\max} & 0.0 \\ 0.0 & 0.0 & 1/S_{\max} \end{bmatrix} \begin{bmatrix} L \\ M \\ S \end{bmatrix}$$

The main issue is that it is linear and the correlation between channels is not considered. As mentioned before, this modeling is helpful in designing systems to transform the colors from one adaptation condition to another. The procedure is as follows:

$$\begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \end{bmatrix} = \mathbf{M}^{-1} \begin{bmatrix} L_{\max 2} & 0.0 & 0.0 \\ 0.0 & M_{\max 2} & 0.0 \\ 0.0 & 0.0 & S_{\max 2} \end{bmatrix} \begin{bmatrix} 1/L_{\max 1} & 0.0 & 0.0 \\ 0.0 & 1/M_{\max 1} & 0.0 \\ 0.0 & 0.0 & 1/S_{\max 1} \end{bmatrix} \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix} = \mathbf{M} \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix}$$

where  $\mathbf{M}$  is the transform matrix from XYZ to LMS. It is interesting that this simple model works very well on the real data shown in Figure 2.

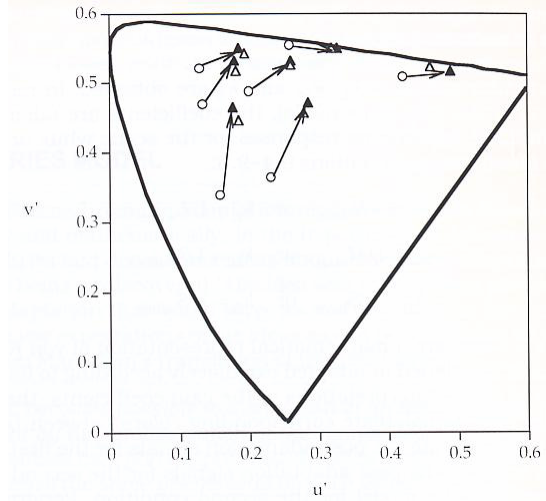


Figure 2: Von Kries model results. Dark triangles: output of the model

### Nayatani's model:

Nayatni and coworkers tried to change Von Kries model to enhance the performance by employing nonlinear functions. He has introduced the following model in which there are some constants that should be set using the experiments. Actually, they are trying to find a better analytic function which can estimate the adaptation procedure more accurately.

$$L_a = a_L \left( \frac{L + L_n}{L_0 + L_n} \right)^{\beta_L}$$

$$M_a = a_M \left( \frac{M + M_n}{M_0 + M_n} \right)^{\beta_M}$$

$$S_a = a_S \left( \frac{S + S_n}{S_0 + S_n} \right)^{\beta_S}$$

In this model, a noise term is also considered and the average response over the scene is considered to have better adaptation. The result of this model is also illustrated in Figure 3.

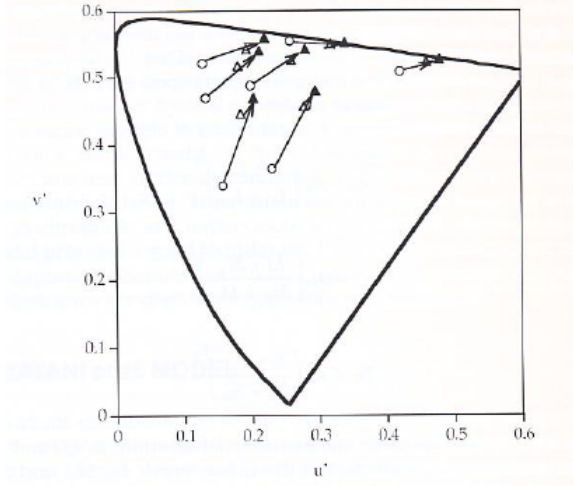


Figure 3: The result of Nayatani's model. Dark triangles: output of the model. By comparing Figure 1 and Figure 2, we can see that there is not a big difference between them. Hence unfortunately, by making the model more complicated, we do not earn significantly.

#### Guth's model:

This model is even more complicated than Nayatani's model and has better results. The model formulation is as follows:

$$L_a = L_r [1 - (L_{r0}/(\sigma + L_{r0}))]$$

$$L_r = 0.66L^{0.7} + 0.002$$

$$M_a = M_r [1 - (M_{r0}/(\sigma + M_{r0}))]$$

$$M_r = 1.0M^{0.7} + 0.003$$

$$S_a = S_r [1 - (S_{r0}/(\sigma + S_{r0}))]$$

$$S_r = 0.45S^{0.7} + 0.00135$$

Actually Guth's model is another interpretation of Von Kries model in which the coefficients are calculated nonlinearly using the following relation:

$$k_L = 1 - (L_{r0}/(\sigma + L_{r0}))$$

The result of this model is illustrated in Figure 4.

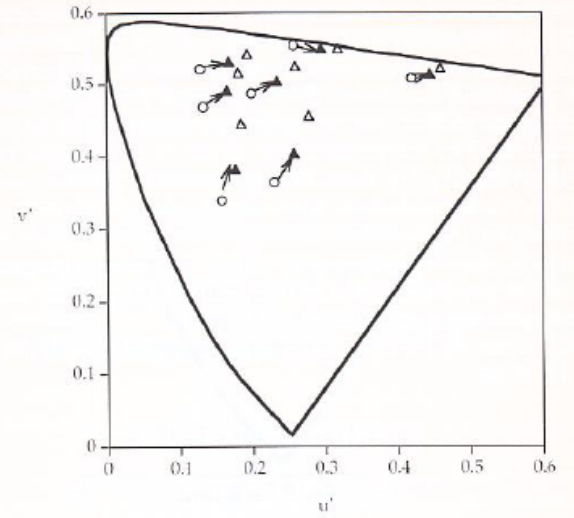


Figure 4: The result of Guth's model. Dark triangles: output of the model.

Again, we can see that the difference in comparison with Von Kries model is not significant.

#### Fairchild's model:

This model is even more complicated than the previous ones. It uses Von Kries model with different coefficients:

$$\begin{bmatrix} L_1 \\ M_1 \\ S_1 \end{bmatrix} = \mathbf{M} \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix}$$

$$\mathbf{M} = \begin{bmatrix} 0.4002 & 0.7076 & -0.0808 \\ -0.2263 & 1.1653 & 0.0457 \\ 0.0 & 0.0 & 0.9182 \end{bmatrix}$$

$$\begin{bmatrix} L'_1 \\ M'_1 \\ S'_1 \end{bmatrix} = \mathbf{A}_1 \begin{bmatrix} L_1 \\ M_1 \\ S_1 \end{bmatrix}$$

$$\mathbf{A}_1 = \begin{bmatrix} a_L & 0.0 & 0.0 \\ 0.0 & a_M & 0.0 \\ 0.0 & 0.0 & a_S \end{bmatrix}$$

$$\alpha_M = \frac{p_M}{M_n}$$

$$p_M = \frac{(1 + Y_n^v + m_E)}{(1 + Y_n^v + 1/m_E)}$$

$$m_E = \frac{3(M_n/M_E)}{L_n/L_E + M_n/M_E + S_n/S_E}$$

He also introduced the interchannel term to consider the correlation between channels.

$$\begin{bmatrix} L_a \\ M_a \\ S_a \end{bmatrix} = \mathbf{C}_1 \begin{bmatrix} L'_1 \\ M'_1 \\ S'_1 \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} 1.0 & c & c \\ c & 1.0 & c \\ c & c & 1.0 \end{bmatrix}$$

$$c = 0.219 - 0.0784 \log_{10}(Y_n)$$

Hence, in transforming from one environment to another we can use the following relation:

$$\begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \end{bmatrix} = \mathbf{M}^{-1} \mathbf{A}_2^{-1} \mathbf{C}_2^{-1} \mathbf{C}_1 \mathbf{A}_1 \mathbf{M} \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix}$$

But later they found out that it doesn't help and removed this part from their next model. The result is illustrated in Figure 5.

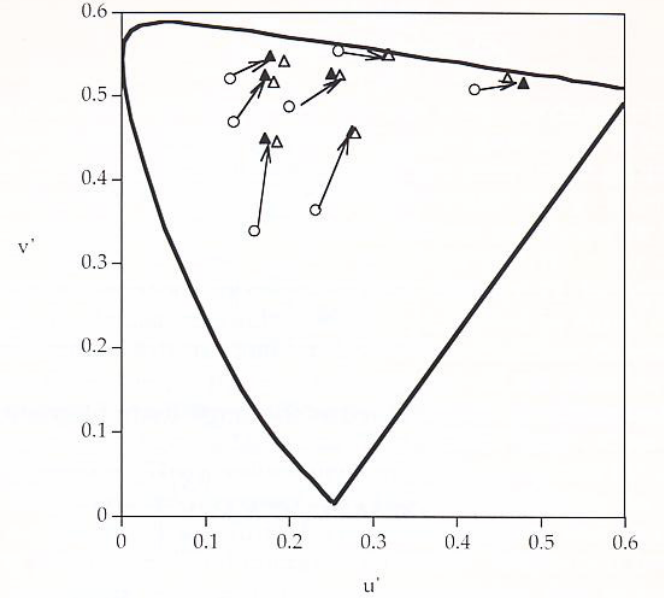


Figure 5: The result of Fairchild's model. Dark triangles: output of the model.

#### CAT02:

Finally, after comparing and working on all these models, researchers in developing CIE 2004 standard, decided to reconsider Von Kries model and use the following transformation which is very simple and close to Von Kries model.

$$\begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \end{bmatrix} = \mathbf{M}_{CAT02}^{-1} \begin{bmatrix} R_{skpt2} & 0.0 & 0.0 \\ 0.0 & G_{skpt2} & 0.0 \\ 0.0 & 0.0 & B_{skpt2} \end{bmatrix} \begin{bmatrix} 1/R_{skpt1} & 0.0 & 0.0 \\ 0.0 & 1/G_{skpt1} & 0.0 \\ 0.0 & 0.0 & 1/B_{skpt1} \end{bmatrix} \mathbf{M}_{CAT02} \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix}$$

#### Reference:

[1] Color Appearance Models, Mark D. Fairchild.

# Color Appearance Models

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## 1. INTRODUCTION

The world of color measurement is full of various descriptors of color such as tristimulus values, chromaticity coordinates, uniform chromaticity scales, uniform color spaces and 'just plain-old' color spaces. The problem is that it is difficult to keep all names and distinctions straight. What is it that sets a color appearance model apart from all of these other types of color specification? CIE Technical Committee 1-34 (TC1-34) agreed on the following definition: A color appearance model is any model that includes predictors of at least the relative color-appearance attributes of lightness, chroma, and hue. This definition enables some fairly simple uniform color spaces, such as the CIE 1976  $L^*a^*b^*$  color space (CIELAB) and the CIE 1976  $L^*u^*v^*$  color space (CIELUV), to be considered color appearance models.

All color appearance models for practical applications begin with the specification of the stimulus and viewing conditions in terms of CIE XYZ tristimulus values (along with certain absolute luminances for some models). The first process for these data is generally a linear transformation from XYZ tristimulus values to cone responses in order to more accurately model the physiological processes in the human visual system. The importance of beginning with CIE tristimulus values is a matter of practicality.

The accurate reproduction of color images in different media has a number of requirements.<sup>1</sup> One of the most notable is the need to specify and reproduce color appearance across a range of media and viewing conditions. This cannot be accomplished using traditional colorimetry, which is only capable of predicting color matches under identical viewing conditions for the original and reproduction. When viewing conditions such as the luminance level, white-point chromaticity, surround relative luminance, and cognitive interpretation of the medium vary, a color-appearance model is necessary to predict the appropriate image transformation to produce an image that closely resembles the color appearances of the original.

The RLAB color-appearance space was developed by Fairchild and Berns for cross-media color reproduction applications in which images are reproduced with differing white points, luminance levels, and/or surrounds.<sup>2</sup> Since its development, the RLAB space has been subjected to a series of psychophysical comparisons with other color-appearance models. This paper reviews the RLAB space. RLAB was derived to have color-appearance predictors similar to those of the CIELAB color space. RLAB includes predictors of lightness, redness-greenness, yellowness-blueness, chroma, and hue angle.

## Categories and Subject Descriptors

H.4 [Visual Perception]: Miscellaneous

## General Terms

Color Appearance Models

## Keywords

Color Appearance Models

## 2. CIELAB MODEL

### 2.1 Calculating CIELAB coordinates

To calculate CIELAB coordinates, one must begin with two sets of CIE XYZ tristimulus values, stimulus, XYZ, and those of reference white,  $X_n Y_n Z_n$ . These data are utilized in a modified form of the von Kries chromatic adaptation transform by normalizing the stimulus tristimulus values by those of the white (i.e.,  $X/X_n$ ,  $Y/Y_n$ , and  $Z/Z_n$ ). Note that the CIE tristimulus values are not transformed to cone responses first as would be necessary for a true von Kries adaptation model. Then, these signals are subject to a compressive non-linearity represented by a cube root in the equations. This nonlinearity is designed to model the compressive response typically found between physical energy measurements and perceptual responses. After that these signals are combined into three response dimensions related to the light-dark, red-green, and yellow-blue responses of the opponent theory of color vision. Finally, appropriate multiplicative constants are incorporated into the equations to provide the required uniform perceptual spacing and proper relationship between the three dimensions. The full CIELAB equations are given in the following equations.

$$L^* = 116f(Y/Y_n) - 16 \quad (1)$$

$$a^* = 500[f(X/X_n) - f(Y/Y_n)] \quad (2)$$

$$b^* = 200[f(Y/Y_n) - f(Z/Z_n)] \quad (3)$$

If input  $w$  is more than 0.008856,

$$f(w) = (w)^{\frac{1}{3}} \quad (4)$$

Otherwise,

$$f(w) = 7.787(w) + \frac{16}{116} \quad (5)$$

$L^*$  is perceived lightness approximately ranging from 0.0 for black to 100.0 for white.  $a^*$  represents red-green chroma perception and  $b^*$  represents yellow-blue chroma perception. They can be both negative and positive values. Both  $a^*$  and  $b^*$  have values of 0.0 for achromatic stimuli (i.e., white, gray, black). Their Maximum values are limited by physical properties of materials rather than the equations themselves.

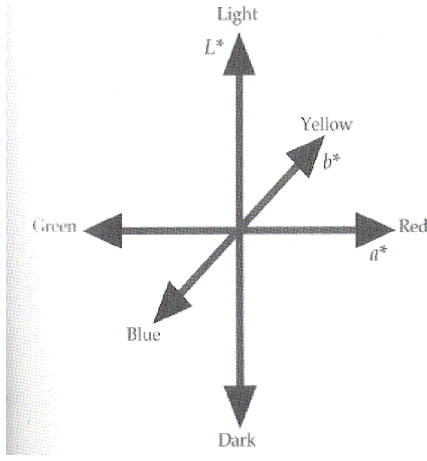


Figure 1: Cartesian representation of the CIELAB color space

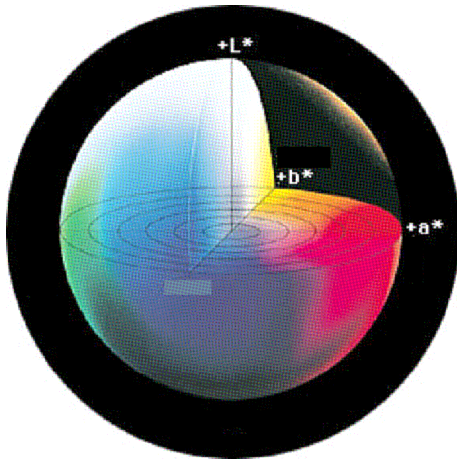


Figure 2: Visualization of the CIELAB color space

The CIELAB  $L^*$ ,  $a^*$ , and  $b^*$  dimensions are combined as Cartesian coordinates to form a three-dimensional color space.

This system provides predictors of chroma  $C_{ab}^*$  and hue  $h_{ab}$  (hue angle in degrees) as expressed in the following equations

$$C_{ab}^* = \sqrt{(a^{*2} + b^{*2})} \quad (6)$$

$$h_{ab} = \arctan(b^*/a^*) \quad (7)$$

$C^*$  has the same units as  $a^*$  and  $b^*$ . Achromatic stimuli have  $C^*$  values of 0.0 (i.e., no chroma). Hue angle,  $h_{ab}$ , is expressed in positive degrees starting from  $0^\circ$  at the positive  $a^*$  axis and progressing in a counter-clockwise direction. The following table provides worked examples of CIELAB calculations.

Quantity	Case 1	Case 2	Case 3	Case 4
$X$	19.01	57.06	3.53	19.01
$Y$	20.00	43.06	6.56	20.00
$Z$	21.78	31.96	2.14	21.78
$X_n$	95.05	95.05	109.85	109.85
$Y_n$	100.00	100.00	100.00	100.00
$Z_n$	108.88	108.88	35.58	35.58
$L^*$	51.84	71.60	30.78	51.84
$a^*$	0.00	44.22	-42.69	-13.77
$b^*$	-0.01	18.11	2.30	-52.86
$C_{ab}^*$	0.01	47.79	42.75	54.62
$h_{ab}$	270.0	22.3	176.9	255.4

While the CIELAB space provides a simple of a color appearance model, there are some known limitations. The perceptual uniformity of the CIELAB space can be evaluated by examining plots of constant hue and chroma contours from the Munsell Book of color. Such a plot is illustrated in the following figure.

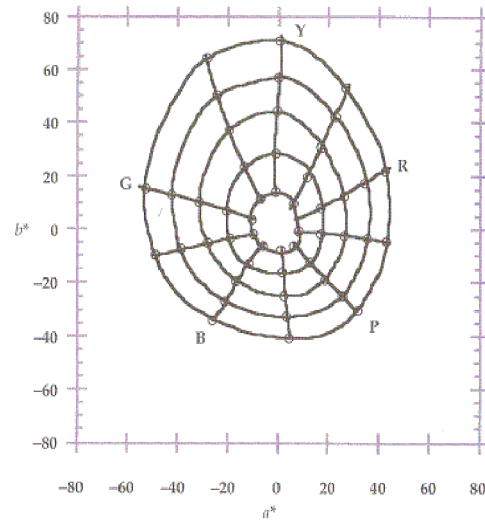


Figure 3: Contours of constant Munsell chroma and hue plotted in the CIELAB  $a^*b^*$  plane

Since the Munsell system is designed to be perceptually uniform in terms of hue and chroma, the plot should ideally be



a set of concentric circles representing the constant chroma contours with straight lines radiating from the center representing constant hue. As you can see in the figure, the CIELAB space does a respectable job of representing the Munsell system uniformly. However, further examinations using a CRT system (capable of achieving higher chroma than generally available in the Munsell Book of Color) have illustrated discrepancies between observed and predicted results. The following figure show constant perceived hue lines from Hung and Berns (1995). It is clear that these lines are curved in the CIELAB space, particularly for red and blue hues.

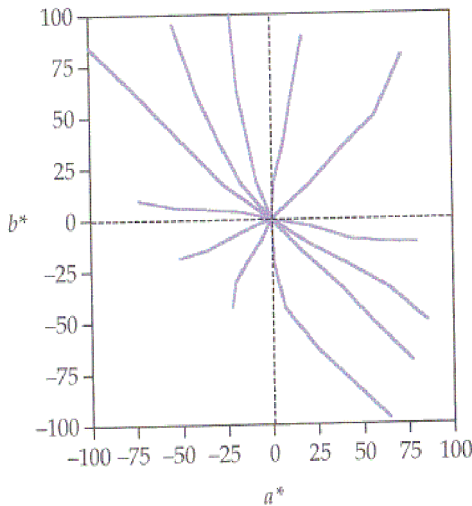


Figure 4: Contours of constant perceived hue from Hung and Berns (1995) plotted in the CIELAB  $a^*b^*$  plane

It is also worth noting that the perceptual unique hues (red, green, yellow and blue) do not align directly with the CIELAB  $a^*b^*$  axis. The unique hues under daylight illumination lie approximately at hue angles of  $24^\circ$  (red),  $90^\circ$  (yellow),  $162^\circ$  (green), and  $246^\circ$  (blue).

A similar examination of the CIELAB lightness scale can be made by plotting Munsell value as a function of  $L^*$  as shown in the following figure. Clearly, the  $L^*$  function predicts lightness, as defined by Munsell value, quite well.

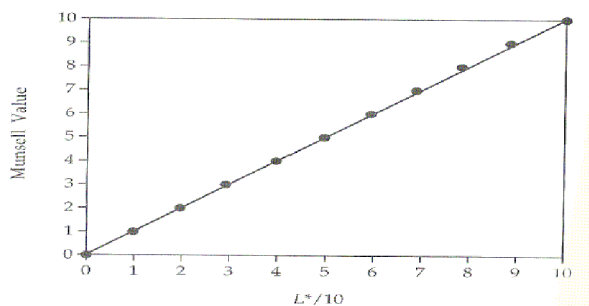


Figure 5: Munsell value plotted as a function of CIELAB  $L^*$

## 2.2 Summary of CIELAB

Given that CIELAB is a well-established, de facto international standard, color space that has been widely used for two decades and that it is capable of color appearance predictions, why are any other color appearance models necessary? First, the modified von Kries adaptation transformation incorporated into the CIELAB equations is clearly less accurate than transformations that more closely follow known visual physiology. Next, there are limitations in CIELAB's ability to predict hue that prompt further work on appearance models.

There are also several aspects of color appearance that CIELAB is incapable of predicting. CIELAB incorporates no luminance-level dependency. Thus it is completely incapable of predicting luminance-dependent effects such as the Hunt effect and the Stevens effect. CIELAB also incorporates no background or surround dependency. Therefore it cannot be used to predict simultaneous contrast or the Bartleson-Breneman results showing a change in image contrast with surround relative luminance. CIELAB also has no mechanism for modeling cognitive effects, such as discounting the illuminant, that can become important in cross-media color reproduction applications. Lastly, CIELAB does not provide correlates for the absolute appearance attributes of brightness and colorfulness. This long list of limitations seems to indicate that it should be possible to significantly improve upon CIELAB in the development of a color appearance model. The CIELAB space should be kept in mind as a simple model that can be used as a benchmark to measure whether more sophisticated models are indeed improvements.

## 3. HUNT MODEL

This model is the most extensive, complete, and complex color appearance model that has been developed. The Hunt color appearance model is not simple, but it is designed to predict a wide range of visual phenomena and, as Hunt himself has stated, the human visual system is not simple either. While there are applications in which simpler models are adequate, it is certainly of great value to have a complete model that can be adapted to a wider range of viewing conditions for more well defined or unusual circumstances. The Hunt model serves this purpose well and many of the other color appearance models can trace many of their features back to ideas that originally appeared in Hunt's model.

### 3.1 Input data

The Hunt model requires an extensive list of input data. All colorimetric coordinates are typically calculated using the CIE 1931 standard colorimetric observer ( $2^\circ$ ). The chromaticity coordinates ( $x, y$ ) of the illuminant and the adapting field are required. Typically, the adapting field is taken to be the integrated chromaticity of the scene, which is assumed to be identical to that of the illuminant (or source). Next, the chromaticities ( $x, y$ ) and luminance factors  $Y$  of the background, proximal field, reference white, and test sample are required. If separate data are not available for the proximal field, it is generally assumed to be identical to the background. Also, the reference white is often taken to have the same chromaticities as the illuminant with a luminance factor of 100 if specific data are not available.

All of these data are relative colorimetric values. Absolute luminance levels are required to predict several luminance-dependent appearance phenomena. Thus the absolute luminance levels, in  $\text{cd}/\text{m}^2$ , are required for the reference white and the adapting field. If the specific luminance of the adapting field is not available, it is taken to be 20 percent of the luminance of the reference white under the assumption that scenes integrate to a gray with a reflectance factor of 0.2. Additionally, scotopic luminance data are required in order to incorporate rod responses into the model (another unique feature of the Hunt model). Thus, the scotopic luminance of the adapting field in scotopic  $\text{cd}/\text{m}^2$  is required. Since scotopic data are rarely available, the scotopic luminance of the illuminant  $L_{AS}$  can be approximated from its photopic luminance  $L_A$  and correlated color temperature  $T$  using the following equation.

$$L_{AS} = 2.26L_A[(T/4000) - 0.4]^{1/3} \quad (8)$$

The scotopic luminance of the test stimulus relative to the scotopic luminance of the reference white is also required. Again, since such data are rarely available, an approximation is often used by substituting the photopic luminance of the sample relative to the reference white for the scotopic values.

Lastly, there are several input variables that are decided based on the viewing configuration. Two of these are the chromatic  $N_c$  and brightness  $N_b$  surround induction factors. Hunt (1995) suggests using values optimized for the particular viewing situation. Since this is often not possible, the nominal values listed in the following table are recommended.

Situation	$N_c$	$N_b$
Small areas in uniform backgrounds/surrounds	1.0	300
Normal scenes	1.0	75
Television and CRT displays in dim surrounds	1.0	25
Large transparencies on light boxes	0.7	25
Projected transparencies in dark surrounds	0.7	10

The last two input parameters are the chromatic  $N_{cb}$  and brightness  $N_{bb}$  background induction factors. Again, Hunt recommends optimized values. Assuming these are not available, the background induction factors are calculated from the luminances of the reference white  $Y_w$  and background  $Y_b$  using the following equations.

$$N_{cb} = 0.725(Y_w/Y_b)^{0.2} \quad (9)$$

$$N_{bb} = 0.725(Y_w/Y_b)^{0.2} \quad (10)$$

A final decision must be made regarding discounting-the-illuminant. Certain parameters in the model are assigned different values for situations in which discounting-the-illuminant occurs. Given all of the above data one can then continue with the calculations of the Hunt model parameters.

### 3.2 Adaptation model

As with many other models, the first step is a transformation from CIE tristimulus values to cone responses. In Hunt model, the cone responses are denoted  $\rho\gamma\beta$  rather than LMS. The transformation is given in the following equation. For the Hunt model, this transformation is normalized such that the equal-energy illuminant has equal  $\rho\gamma\beta$  values.

$$\begin{pmatrix} \rho \\ \gamma \\ \beta \end{pmatrix} = \begin{pmatrix} 0.38971 & 0.68898 & -0.07868 \\ -0.22981 & 1.18340 & 0.04641 \\ 0.0 & 0.0 & 1.0 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} \quad (11)$$

The transformation from XYZ to  $\rho\gamma\beta$  values must be completed for the reference white, background, proximal field, and test stimulus.

The chromatic adaptation model embedded in Hunt's color appearance model is a significantly modified form of the von Kries hypothesis. The adapted cone signals  $\rho_a\gamma_a\beta_a$  are determined from the cone responses for the stimulus  $\rho\gamma\beta$  and those for the reference white  $\rho_w\gamma_w\beta_w$  by using the following equations.

$$\rho_a = B_\rho[f_n(F_L F_\rho \rho / \rho_w) + \rho_D] + 1 \quad (12)$$

$$\gamma_a = B_\gamma[f_n(F_L F_\gamma \gamma / \gamma_w) + \gamma_D] + 1 \quad (13)$$

$$\beta_a = B_\beta[f_n(F_L F_\beta \beta / \beta_w) + \beta_D] + 1 \quad (14)$$

There are many other parameters need to be defined in the equations. First, function  $f_n$  is a general hyperbolic function given in the following equation that is used to model the nonlinear behavior of various visual responses.

$$f_n[I] = 40[I^{0.73}/(I^{0.73} + 2)] \quad (15)$$

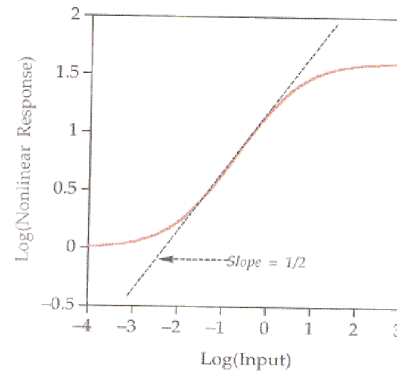


Figure 6: The nonlinear function  $f_n$  of the Hunt color appearance model

Figure 6 illustrates the form of Hunt's nonlinear function on log-log axes. In the central operating range, the function is



linear and therefore equivalent to a simple power function (in this case with an exponent of about 1/2). However, this function has the advantage that it models threshold behavior at low levels (the gradual increase in slope) and saturation behavior at high levels (the decrease in slope back to zero). Such a nonlinearity is required to model the visual system over the large range in luminance levels that the Hunt model addresses.

$F_L$  is a luminance-level adaptation factor incorporated into the adaptation model to predict the general behavior of light adaptation over a wide range of luminance level. It also reintroduces the absolute luminance level prior to the nonlinearity, allowing appearance phenomena such as the Stevens effect and Hunt effect to be predicted.  $F_L$  is calculated using the following equations.

$$f_L = 0.2k^4(5L_A) + 0.1(1 - k^4)^2(5L_A)^{1/3} \quad (16)$$

$$k = 1/(5L_A + 1) \quad (17)$$

$F_\rho$ ,  $F_\gamma$ , and  $F_\beta$  are chromatic-adaptation factors that are introduced to model the fact that chromatic adaptation is often incomplete. These factors are designed such that chromatic adaptation is always complete for the equal-energy illuminant (sometimes referred to as illuminant E). This means that the chromaticity of illuminant E always appears achromatic according to the model and thus  $F_\rho$ ,  $F_\gamma$ , and  $F_\beta$  are all equal to one. As  $F_\rho$ ,  $F_\gamma$ , and  $F_\beta$  depart from unity, chromatic adaptation is predicted to be less complete. The formulation of  $F_\rho$ ,  $F_\gamma$ , and  $F_\beta$  is given in the following equations.

$$f_\rho = (1 + L_A^{1/3} + h_\rho)/(1 + L_A^{1/3} + 1/h_\rho) \quad (18)$$

$$f_\gamma = (1 + L_A^{1/3} + h_\gamma)/(1 + L_A^{1/3} + 1/h_\gamma) \quad (19)$$

$$f_\beta = (1 + L_A^{1/3} + h_\beta)/(1 + L_A^{1/3} + 1/h_\beta) \quad (20)$$

$$h_\rho = 3\rho_w/(\rho_w + \gamma_w + \beta_w) \quad (21)$$

$$h_\gamma = 3\gamma_w/(\rho_w + \gamma_w + \beta_w) \quad (22)$$

$$h_\beta = 3\beta_w/(\rho_w + \gamma_w + \beta_w) \quad (23)$$

The parameters  $h_\rho$ ,  $h_\gamma$ , and  $h_\beta$  can be thought of as chromaticity coordinates scaled relative to illuminant E (since  $\rho/\gamma/\beta$  themselves are normalized to illuminant E). They take on values of 1.0 for illuminant E and depart further from 1.0 as the reference white becomes more saturated. These parameters, taken together with the luminance level dependency  $L_A$  produce values that depart from 1.0 by increasing amounts as the color of the reference white moves away from illuminant E (becoming more saturated) and the adapting luminance increases. If discounting-the-illuminant occurs, then chromatic adaptation is taken to be complete and  $F_\rho$ ,  $F_\gamma$ , and  $F_\beta$  are set equal to values of 1.0.

The parameters  $\rho_D$ ,  $\gamma_D$ , and  $\beta_D$  are included to allow prediction of the Helson-Judd effect. This is accomplished by

additive adjustments to the cone signals that are dependent upon the relationship between the luminance of the background  $Y_b$ , the reference white  $Y_w$ , and the test stimulus as given in the following equations.

$$\rho_D = f_n[(Y_b/Y_w)F_L F_\gamma] - f_n[(Y_b/Y_w)F_L F_\rho] \quad (24)$$

$$\gamma_D = 0.0 \quad (25)$$

$$\beta_D = f_n[(Y_b/Y_w)F_L F_\gamma] - f_n[(Y_b/Y_w)F_L F_\beta] \quad (26)$$

The Helson-Judd effect does not occur in most typical viewing situations. In such cases  $\rho_D$ ,  $\gamma_D$ , and  $\beta_D$  should be set equal to 0.0. In cases for which discounting-the-illuminant occurs, there is no Helson-Judd effect and  $\rho_D$ ,  $\gamma_D$ , and  $\beta_D$  are forced to 0.0 since  $F_\rho$ ,  $F_\gamma$ , and  $F_\beta$  are set equal to 1.0. There are some situations in which it is desirable to have  $F_\rho$ ,  $F_\gamma$ , and  $F_\beta$  take on their normal values while  $\rho_D$ ,  $\gamma_D$ , and  $\beta_D$  are set to 0.0. These include the viewing of images projected in a darkened surround or viewed on CRT displays.

The last factors in the chromatic-adaptation formulas are the cone bleach factors  $B_\rho$ ,  $B_\gamma$ , and  $B_\beta$ . These factors are only necessary to model visual responses over extremely large ranges in luminance level. They are formulated to model photopigment depletion (i.e., bleaching) that occurs at high luminance levels resulting in decreased photoreceptor output, as shown in the following equations.

$$B_\rho = 10^7/[10^7 + 5L_A(\rho_w/100)] \quad (27)$$

$$B_\gamma = 10^7/[10^7 + 5L_A(\gamma_w/100)] \quad (28)$$

$$B_\beta = 10^7/[10^7 + 5L_A(\beta_w/100)] \quad (29)$$

The cone bleaching factors are essentially 1.0 for most normal luminance levels. As the adapting luminance  $L_A$  reaches extremely high levels, the bleaching factors begin to decrease, resulting in decreased adapted cone output. In the limit, the bleaching factors will approach zero as the adapting luminance approaches infinity. This would result in no cone output when the receptors are fully bleached (sometimes referred to as a retinal burn). Such adapting levels are truly dangerous to the observer and would cause permanent damage. However, the influence of the cone bleaching factors does begin to take effect at high luminance levels that are below the threshold for retinal damage such as outdoors on a sunny day. In such situations, one can observe the decreased range of visual response due to 'too much light' and a typical response is to put on sunglasses. These high luminance levels are not found in typical image reproduction applications (except, perhaps, in some original scenes).

The adaptation formulas are completed with the addition of 1.0 designed to represent noise in the visual system.

If the proximal field and background differ from a gray, chromatic induction is modeled by adjusting the cone signals for

the reference white used in the adaptation equations. This suggests that the state of adaptation is being influenced by the local color of the proximal field and background in addition to the color of the reference white. This type of modeling is completely consistent with observed visual phenomena. Hunt (1991b) has suggested one algorithm for calculating adjusted reference white signals  $\rho'_w$ ,  $\gamma'_w$ , and  $\beta'_w$  from the cone responses for the background  $\rho_b$ ,  $\gamma_b$ , and  $\beta_b$ , and proximal field  $\rho_p$ ,  $\gamma_p$ , and  $\beta_p$ , given in the following equations.

$$\rho'_w = \frac{\rho_w[(1-p)\rho_p + (1+p)/\rho_p]^{1/2}}{[(1+p)\rho_p + (1-p)/\rho_p]^{1/2}} \quad (30)$$

$$\gamma'_w = \frac{\gamma_w[(1-p)\gamma_p + (1+p)/\gamma_p]^{1/2}}{[(1+p)\gamma_p + (1-p)/\gamma_p]^{1/2}} \quad (31)$$

$$\beta'_w = \frac{\beta_w[(1-p)\beta_p + (1+p)/\beta_p]^{1/2}}{[(1+p)\beta_p + (1-p)/\beta_p]^{1/2}} \quad (32)$$

$$p_\rho = (\rho_p/\rho_b) \quad (33)$$

$$p_\gamma = (\gamma_p/\gamma_b) \quad (34)$$

$$p_\beta = (\beta_p/\beta_b) \quad (35)$$

Values of  $p$  in the above equations are taken to be between 0 and -1 when simultaneous contrast occurs and between 0 and +1 when assimilation occurs. In most practical applications, the background and proximal field are assumed to be achromatic and adjustments such as those given in the above equations are not used.

Now that the adapted cone signals  $\rho_a$ ,  $\gamma_a$ , and  $\beta_a$  are available, it is possible to move onward to the opponent responses and color appearance correlates. The rod signals and their adaptation will be treated at the point they are incorporated in the achromatic response.

### 3.3 Opponent color dimation

Given the adapted cone signals  $\rho_a$ ,  $\gamma_a$ , and  $\beta_a$ , one can calculate opponent-type visual responses very simply in the following equations.

$$A_a = 2\rho_a + \gamma_a + (1/20)\beta_a - 3.05 + 1 \quad (36)$$

$$C_1 = \rho_a - \gamma_a \quad (37)$$

$$C_2 = \gamma_a - \beta_a \quad (38)$$

$$C_3 = \beta_a - \rho_a \quad (39)$$

The achromatic post-adaptation signal  $A_a$  is calculated by summing the cone responses with weights that represent their relative population in the retina. The subtraction of 3.05 and the addition of 1.0 represent removal of the earlier noise components followed by the addition of new noise. The three color difference signals  $C_1$ ,  $C_2$ , and  $C_3$ , represent

all of the possible chromatic opponent signals that could be produced in the retina. These may or may not have direct physiological correlates, but they are convenient formulations and used to construct more traditional opponent responses.

### 3.4 Hue

Hue angle in the Hunt color appearance model is calculated just as it is in other models once red-green and yellow-blue opponent dimensions are specified as appropriate combinations of the color difference signals. Hue angle  $h_s$  is calculated using the following equation.

$$h_s = \arctan\left[\frac{(1/2)(C_2 - C_3)/4.5}{C_1 - (C_2/11)}\right] \quad (40)$$

Given the hue angle  $h_s$ , a hue quadrature value  $H$  is calculated by interpolation between specified hue angles for the unique hues with adjustment of an eccentricity factor  $e_s$ . The interpolating function is given by the following equation.

$$H = H_1 + \frac{(100[(h_s - h_1)/e_1])}{[(h_s - h_1)/e_1 + (h_2 - h_s)/e_2]} \quad (41)$$

$H_1$  is defined as 0, 100, 200, or 300 based on whether red, yellow, green, or blue, respectively, is the unique hue with the hue angle nearest to and less than that of the test sample. The values of  $h_1$  and  $e_1$  are taken from the following table as the values for the unique hue having the nearest lower value of  $h_s$  while  $h_2$  and  $e_2$  are taken as the values of the unique hue with the nearest higher value of  $h_s$ .

Hue	$h_s$	$e_s$
Red	20.14	0.8
Yellow	90.00	0.7
Green	164.25	1.0
Blue	237.53	1.2

Finally, an eccentricity factor  $e_s$  must be calculated for the test stimulus to be used in further calculations of appearance correlates. This is accomplished through linear interpolation using the hue angle  $h_s$  of the test stimulus and the data in the table.

### 3.5 Saturation

As a step toward calculating a correlate of saturation, yellowness-blueness and redness-greenness responses must be calculated from the color difference signals according to the following equations.

$$M_{YB} = 100[(1/2)(C_2 - C_3)/4.5][e_s(10/13)N_cN_{cb}F_t] \quad (42)$$

$$M_{RG} = 100[C_1 - (C_2/11)][e_s(10/13)N_cN_{cb}] \quad (43)$$

The constant values in the equations are simply scaling factors.  $N_c$  and  $N_{cb}$  are the chromatic surround and background induction factors determined at the outset.  $F_t$  is a low-luminance tritanopia factor calculated using the following equation.

$$F_t = L_A / (L_A + 0.1) \quad (44)$$

Low-luminance tritanopia is a phenomenon whereby observers with normal color vision become more and more tritanopic (yellow-blue deficient) as luminance decreases since the luminance threshold for short-wavelength-sensitive cones is higher than that for the other two cone types. As can be seen in the above equation,  $F_t$  is essentially 1.0 for all typical luminance levels. It approaches zero as the adapting luminance  $L_A$  approaches zero, forcing the yellowness-blueness response to decrease at low luminance levels. This factor is also of little importance in most practical situations, but necessary to model appearance over an extremely wide range of luminance levels. For most applications, it is better to avoid this situation by viewing samples at sufficiently high luminance levels.

Given the yellowness-blueness and redness-greenness responses defined above, an overall chromatic response  $M$  is calculated as their quadrature sum as shown in the following equation.

$$M = (M_{YB}^2 + M_{RG}^2)^{1/2} \quad (45)$$

Finally, saturation  $s$  is calculated from  $M$  and the adapted cone signals using the following equation.

$$s = 50M / (\rho_a + \gamma_a + \beta_a) \quad (46)$$

This calculation follows the definition of saturation (colorfulness of stimulus relative to its own brightness) if one takes the overall chromatic response  $M$  to approximate colorfulness, and the sum of the adapted cone signals in the denominator of the equation to approximate the stimulus brightness.

### 3.6 Brightness

Further development of the achromatic signals is required to derive correlates of brightness and lightness. Recall that the Hunt color appearance model is designed to function over the full range of luminance levels. In order to do that, it must also incorporate the response of rod photoreceptors, which are active at low luminance levels. The rod response is incorporated into the achromatic signal (which in turn impacts the predictors of chroma and colorfulness). The rod response after adaptation  $A_S$  is given by the following equation.

$$A_S = 3.05B_S[f_n(F_{LS}S/S_W)] + 0.3 \quad (47)$$

The subscript  $S$  is derived from the word scotopic, which is used to describe vision at the low luminance levels for

which the rod response dominates. The value 3.05 is simply a scaling factor and the noise level of 0.3 (rather than 1.0) is chosen since the rods are more sensitive than cones.

The formulation of the adapted rod signal is analogous to the formulation of the adapted cone signals. At its heart is a von Kries-type scaling of the scotopic response for the stimulus  $S$ , by that for the reference white  $S_W$ .  $F_{LS}$  is a scotopic luminance level adaptation factor given by the following equations that is similar to the cone luminance adaptation factor.

$$F_{LS} = 3800j^2(5L_{AS}/2.26) + 0.2(1 - j^2)^4(5L_{AS}/2.26)^{1/6} \quad (48)$$

$$j = 0.00001 / [(5L_{AS}/2.26) + 0.00001] \quad (49)$$

Finally, a rod bleaching factor  $B_S$  is added to reduce the rod contribution to the overall color appearance as luminance level increases and the rods become less active. This factor is calculated using the following equation.

$$B_S = 0.5 / (1 + 0.3[(5L_{AS}/2.26)(S/S_W)]^{0.3}) + 0.5 / (1 + 5[5L_{AS}/2.26]) \quad (50)$$

Given the achromatic cone signal  $A_a$ , the adapted scotopic signal  $A_S$ , and the brightness background induction factor determined at the outset, an overall achromatic signal  $A$  is calculated as follows:

$$A = N_{bb}[A_a - 1 + A_S - 0.3 + (1^2 + 0.3^2)^{1/2}] \quad (51)$$

All the constant values in the equation represent removal of the earlier noise terms and then their reintroduction through quadrature summation.

The achromatic signal  $A$  is then combined with the overall chromatic signal  $M$  to calculate a correlate of brightness  $Q$  using the following equation.

$$Q = (7[A + (M/100)])^{0/6} N_1 - N_2 \quad (52)$$

The correlate of brightness  $Q$  depends on both the achromatic  $A$  and chromatic  $M$  responses in order to appropriately model the Helmholtz-Kohlrausch effect. The equation also includes two terms  $N_1$  and  $N_2$  that account for the effects of surround on perceived brightness. These terms are calculated from the achromatic signal for the reference white  $A_W$  and the brightness surround induction factor  $N_b$  (also determined at the outset), through the following equations.

$$N_1 = (7A_W)^{0.5} / (5.33N_b^{0.13}) \quad (53)$$

$$N_2 = 7A_W N_b^{0.362} / 200 \quad (54)$$

Note that since the achromatic signal for the reference white  $A_W$  is required, it is necessary to carry through all of the model calculations described above for the reference white in addition to the test stimulus. The brightness of the reference white  $Q_W$ , must also be calculated for use in later equations.

Another form of brightness, referred to as whiteness-blackness  $Q_{WB}$ , can be calculated in the Hunt model. This is a bipolar value similar to the  $Q$  value in the Nayatani et al. model that illustrates that black objects look darker and white objects look brighter as the adapting luminance level increases (another way to state the Stevens effect).  $Q_{WB}$  is calculated according to the following equation using the brightness of the background (which also must be calculated through the model).

$$Q_{WB} = 20(Q^{0.7} - Q_b^{0.7}) \quad (55)$$

### 3.7 Lightness

Given the brightness of the test stimulus  $Q$  and the brightness of the reference white  $Q_W$  the Hunt color appearance model correlate of lightness  $J$  is calculated as shown below.

$$J = 100(Q/Q_W)^z \quad (56)$$

This formulation for lightness follows the CIE definition that lightness is the brightness of the test stimulus relative to the brightness of a white. This ratio is raised to a power  $z$  that models the influence of the background relative luminance on perceived lightness according to the following equation.

$$z = 1 + (Y_b/Y_W)^{1/2} \quad (57)$$

The exponent  $z$  increases as the background becomes lighter, indicating that dark test stimuli will appear relatively more dark on a light background than they would on a dark background. This follows the commonly observed phenomenon of simultaneous lightness contrast.

### 3.8 Chroma

The Hunt color appearance model correlate of chroma  $C_{94}$  is determined from saturation  $s$  and the relative brightness (approximately lightness). It follows the general definition that indicate chroma can be represented as saturation multiplied by lightness. The precise formulation is given below.

$$C_{94} = 2.44s^{0.69}(Q/Q_W)^{Y_b/Y_W}(1.64 - 0.29^{Y_b/Y_W}) \quad (58)$$

This equation illustrates that chroma depends on the relative brightness of the stimulus  $Q/Q_W$  and on the relative luminance of the background  $Y_b/Y_W$ . The formulation for chroma given by the equation was derived empirically based upon the results of a series of appearance scaling experiments (Hunt 1994, Hunt and Luo 1994).

### 3.9 Colorfulness

Given chroma, colorfulness can be determined by factoring in the brightness (or at least the luminance level). This is accomplished in the Hunt color appearance model by multiplying chroma  $C_{94}$  by the luminance level adaptation factor  $F_L$  raised to a power of 0.15 as shown below.

$$M_{94} = F_L^{0.15} C_{94} \quad (59)$$

Thus  $M_{94}$  is the colorfulness correlate for the Hunt color appearance model. It was also derived empirically through analysis of visual scaling results.

### 3.10 Summary of the Hunt model

It seems that the Hunt color appearance model seems to be able to do everything that anyone could ever want a color appearance model to do. Why not use it as the single standard color appearance model for all applications? The main reason could be the very fact that it is so complete. In its completeness also lies its complexity. However, the complexity of the Hunt model also allow it to be very flexible. The Hunt model is generally capable of making accurate predictions for a wide range of visual experiments. This is because the model is flexible enough to be adjusted to the required situations. Clearly, this flexibility and general accuracy are great features of the Hunt model. However, often it is not possible to know just how to apply the Hunt model (i.e., to decide on the appropriate parameter values) until after the visual data have been obtained. In other cases, the parameters actually need to be optimized, not just chosen, for the particular viewing situation. This is not a problem if the resources are available to derive the optimized parameters. However, when such resources are not available and the Hunt model must be used 'as is' with the recommended parameters, the model can perform extremely poorly. This is because the nominal parameters used for a given viewing condition are being used to make specific predictions of phenomena that may or may not be important in that situation. Thus, if it is not possible to optimize (or optimally choose) the implementation of the Hunt model, its precision might result in predictions that are worse than much simpler models for some applications.

Other negative aspects that counteract the positive features of the Hunt color appearance model are that it cannot be easily inverted and that it is computationally expensive, difficult to implement, and requires significant user knowledge to use consistently. The Hunt model also uses functions with additive offsets to predict contrast changes due to variation in surround relative luminance. These functions can result in predicted corresponding colors with negative tristimulus values for some changes in surround.

## 4. RLAB MODEL

The RLAB color appearance model arose from studies of Chromatic Adaptation and practical implications in cross media image reproductions. CIELAB can be used as an approximate color appearance model, it has its own significant limitations. These include an absence of a chromatic adaptation transform, no luminance-level dependency and no surround dependency. While CIELAB has many advantages,

these are some of the practical limitations which are encountered for cross media image reproduction applications. CIELAB is also quite accurate in its own respect, also being very simple to use. RLAB tries to keep all these positive aspects in mind, and trying to overcome its limitations.

The RLAB color-appearance space was developed by Fairchild and Berns for cross-media color reproduction applications in which images are reproduced with differing white points, luminance levels, and/or surrounds.<sup>2</sup> Since its development, the RLAB space has been subjected to a series of psychophysical comparisons with other color-appearance models.

The chromatic-adaptation transform utilized in RLAB has several unique features. The first is the capability to predict incomplete levels of chromatic adaptation that allow highly chromatic "white-points" to retain some of their chromatic appearance. In addition, the incomplete-chromaticadaptation feature can be turned on or off depending on whether cognitive "discounting-the-illuminant" mechanisms are active. These mechanisms are active when viewing hard-copy images in an illuminated environment and inactive when viewing soft-copy images. A final unique feature of RLAB is a matrix in the transformation that models interaction between the cone types allowing the prediction of luminance-dependent appearance effects such as the Hunt effect (increase in perceived colorfulness with luminance).

Another aspect of the RLAB model is that the power-function nonlinearities in the CIELAB equations (cube root) are allowed to vary depending on the image-surround conditions.<sup>5</sup> This models the change in image contrast caused by changes in the relative luminance of the image surround. For example, the dark surround in which projected slides are typically viewed causes the perceived contrast to be lower than if the same image luminances were presented in an average surround as is typical of a printed image

RLAB was derived to have color-appearance predictors similar to those of the CIELAB color space.<sup>3</sup> RLAB includes predictors of lightness, LR, redness-greenness, aR, yellowness-blueness, bR, chroma, CR, and hue angle, hR. These appearance predictors are calculated using equations virtually identical to the CIELAB equations after the stimulus tristimulus values are transformed to the corresponding tristimulus values for the reference viewing condition (D65, 318 cd/m<sup>2</sup>, hard copy). The transformation is accomplished using a modified von Kries-type chromatic adaptation transformation previously formulated by Fairchild.<sup>4</sup> The end result is that the RLAB color space is identical to (and takes advantage of the excellent performance of) the CIELAB color space for the reference viewing conditions and average surround relative luminance. However, for other viewing conditions, the more accurate chromatic-adaptation transform replaces the normalization of tristimulus values inherent in the CIELAB equations.

For average daylight, CIELAB gives adequate perceptual uniformity in average daylight illumination. However, due to CIELAB's "wrong von kries" chromatic adaptation transform, the good perceptual spacing of CIELAB quickly degrades as one moves away from average daylight illumina-

tion. RLAB essentially takes the CIELAB model and tries to improve its performance in average daylight. This was done by defining a reference set of viewing conditions for which the CIELAB space is used, and then using a more accurate chromatic adaptation transform to determine the corresponding colors between the test viewing conditions and the reference viewing conditions. Thus, test tristimulus values are first transformed into corresponding colors under the reference viewing condition and then a modified CIELAB space is used to describe the appearance correlates.

Also, the compressive nonlinearity of the CIELAB space is adapted to become a function of the surround relative luminance. This allows the prediction of decrease in perceived image contrast as the surround becomes darker. The improved chromatic adaptation transform and the surround dependence enhance the CIELAB model in the two areas that are very critical to image reproduction.

## 5. FAIRCHILD'S CHROMATIC ADAPTATION MODEL

Fairchild 1990 aimed at measuring the degree of chromatic adaptation to various forms of adapting stimuli. This led to the modification of the von Kries hypothesis that included the ability to predict the degree of adaptation based on the adapting stimulus itself. This model is designed to be a relatively simple model to account for the Hunt model.

The first step is a transformation from CIE tristimulus values XYZ to fundamental tristimulus values LMS for the first viewing

$$\begin{pmatrix} L_1 \\ M_1 \\ S_1 \end{pmatrix} = M \begin{pmatrix} X_1 \\ Y_1 \\ Z_1 \end{pmatrix} \quad (60)$$

where,

$$M = \begin{pmatrix} 0.4002 & 0.7076 & -0.0808 \\ -0.2263 & 1.1653 & 0.0457 \\ 0.0 & 0.0 & 0.9182 \end{pmatrix} \quad (61)$$

The next step is to modify the von Kries chromatic adaptation transform that takes incomplete chromatic adaptation into account as shown below:

$$\begin{pmatrix} L'_1 \\ M'_1 \\ S'_1 \end{pmatrix} = A_1 \begin{pmatrix} L_1 \\ M_1 \\ S_1 \end{pmatrix} \quad (62)$$

where,

$$A_1 = \begin{pmatrix} a_L & 0 & 0 \\ 0 & a_M & 0.0 \\ 0.0 & 0.0 & a_S \end{pmatrix} \quad (63)$$

$$a_M = \frac{p_M}{M_n} \quad (64)$$

$$p_M = \frac{1 + Y_n^v + m_E}{1 + Y_n^v + \frac{1}{m_E}} \quad (65)$$

$$m_E = \frac{3 \frac{M_n}{M_e}}{\frac{L_n}{L_e} + \frac{S_n}{S_e} + \frac{M_n}{M_e}} \quad (66)$$

The p and a terms for the short (S) and long wavelength (L) sensitive cones are derived in a similar fashion. The Y term is the luminance of the adapting stimulus. E subscripts refer to the equal energy illuminant. The a terms are the modified von Kries coefficients. They depart from the 1.0 as adaptation becomes incomplete. The p depend on the adapting luminance and color.

The final step is the computation of post adaptation signals which is done as follows:

$$\begin{pmatrix} L_a \\ M_a \\ S_a \end{pmatrix} = C_1 \begin{pmatrix} L'_1 \\ M'_1 \\ S'_1 \end{pmatrix} \quad (67)$$

where,

$$C_1 = \begin{pmatrix} 1 & c & c \\ c & 1 & c \\ c & c & 1 \end{pmatrix} \quad (68)$$

$$c = 0.219 - 0.0784 \log_{10}(Y_n) \quad (69)$$

## 6. CHROMATIC ADAPTATION MODEL

Chromatic adaptation is the ability of the human visual system to discount the colour of a light source and to approximately preserve the appearance of an object. For example, a white piece of paper appears to be white when viewed under sky light and tungsten light. However, the measured tristimulus values (the product of surface reflectance, spectral power distribution of the light source, and cone sensitivities, integrated over the visible spectrum) are quite different for the two viewing conditions: sky light is “bluer”, it contains more short wavelength energy than tungsten light.

Digital imaging systems, such as digital cameras and scanners, do not have the ability to adapt to the light source. Scanners usually use fluorescent light sources. For digital cameras, the light source varies with the scene, and sometimes within a scene. Therefore, to achieve the same appearance of the original or original scene under different display conditions (such as a computer monitor or a light booth), the captured image tristimulus values have to be transformed to take into account the light source of the display viewing conditions. Such transformations are called chromatic adaptation transforms.

This section describes the chromatic adaptation model built into RLAB, based on the model of incomplete chromatic adaptation described by Fairchild (described in previous section).

First,

$$\begin{pmatrix} L \\ M \\ S \end{pmatrix} = M \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} \quad (70)$$

where,

$$M = \begin{pmatrix} 0.3897 & 0.6890 & -0.0787 \\ -0.2298 & 1.1834 & 0.0464 \\ 0.0 & 0.0 & 1.000 \end{pmatrix} \quad (71)$$

The transformation to cone responses is the same as that used in the previous model. Matrix M is normalized such that tristimulus values for the equal energy illuminant ( $X = Y = Z = 100$ ) produces equal cone responses ( $L = M = S = 100$ ).

The next step is the computation of the A matrix that is used to model the chromatic adaptation transform.

$$A = \begin{pmatrix} a_L & 0 & 0 \\ 0 & a_M & 0 \\ 0 & 0 & a_S \end{pmatrix} \quad (72)$$

The A matrix represents the von Kries adaptation coefficients that are applied to the cone responses for the test stimulus. The von Kries - like coefficients are computed as :

$$a_L = \frac{p_L + D(1.0 - p_L)}{L_n} \quad (73)$$

$$a_M = \frac{p_M + D(1.0 - p_M)}{M_n} \quad (74)$$

$$a_S = \frac{p_S + D(1.0 - p_S)}{S_n} \quad (75)$$

The p terms describe the proportion of the von Kries adaptation that is occurring. These terms are equivalent to the one used in the Hunt Model.

$$p_L = \frac{1.0 + Y_n^{\frac{1}{3}} + l_E}{1.0 + Y_n^{\frac{1}{3}} + \frac{1}{l_E}} \quad (76)$$

$$p_M = \frac{1.0 + Y_n^{\frac{1}{3}} + m_E}{1.0 + Y_n^{\frac{1}{3}} + \frac{1}{m_E}} \quad (77)$$

$$p_S = \frac{1.0 + Y_n^{\frac{1}{3}} + s_E}{1.0 + Y_n^{\frac{1}{3}} + \frac{1}{s_E}} \quad (78)$$



$$l_E = \frac{3.0L_n}{L_n + M_n + S_n} \quad (79)$$

$$m_E = \frac{3.0M_n}{L_n + M_n + S_n} \quad (80)$$

$$s_E = \frac{3.0S_n}{L_n + M_n + S_n} \quad (81)$$

The Y term is the absolute adapting luminance. The cone response L,M,N (with the n subscript) refers to values for the adapting stimulus derived from the relative tristimulus values. The D factor allows various proportions of cognitive discounting-the-illuminant. D should be 1.0 for hard copy images, 0.0 for soft copy images. The value of D can be used to account for the various levels of chromatic adaptations found in the infinite variety of practical viewing conditions. The exact choice of intermediate values will depend upon the specific conditions. When no visual data is available, a value of 0.5 is chosen and refined with experience.

After the A matrix is calculated, the tristimulus values for a stimulus color are converted to a corresponding tristimulus values under the reference viewing conditions as follows

$$\begin{pmatrix} X_{ref} \\ Y_{ref} \\ Z_{ref} \end{pmatrix} = RAM \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} \quad (82)$$

where,

$$R = \begin{pmatrix} 1.9569 & -1.1882 & 0.2313 \\ 0.3612 & 0.6388 & 0.0 \\ 0.0 & 0.0 & 1.000 \end{pmatrix} \quad (83)$$

The R matrix is the inverse of the M and A matrices for the reference viewing conditions (which is then normalized).

## 7. OPPONENT COLOR DIMENSIONS

The second step in the determination of a color appearance model is the application of opponent color theory to it. In short, opponent color theory states that the human visual system interprets information about color by processing signals from cones and rods in an antagonistic manner. The three types of cones have some overlap in the wavelengths of light to which they respond, so it is more efficient for the visual system to record differences between the responses of cones, rather than each type of cone's individual response. The opponent color theory suggests that there are three opponent channels: red versus green, blue versus yellow, and black versus white (the latter type is achromatic and detects light-dark variation, or luminance).[1] Responses to one color of an opponent channel are antagonistic to those to the other color. For RLAB, opponent color responses are determined as follows

$$L^R = 100(Y_{ref})^\sigma \quad (84)$$

$$a^R = 430[(X_{ref})^\sigma - (Y_{ref})^\sigma] \quad (85)$$

$$b^R = 170[(Y_{ref})^\sigma - (Z_{ref})^\sigma] \quad (86)$$

The  $L^R$  term refers to the achromatic response, whereas the  $a^R$  and  $b^R$  terms are the red-green and yellow-blue responses respectively. For reference viewing conditions, the RLAB model would be approximately equal to the CIELAB coordinates. For different (low) tristimulus values, CIELAB has different functions. This is replaced in RLAB with simple power functions. This results in different exponents and scaling factors. The exponents in vary depending on the luminance of the surround. For an average surround,  $\sigma = 1/2.3$ , a dim surround  $\sigma = 1/2.9$  and for a dark surround  $\sigma = 1/3.5$ . It is best to use intermediate values to model less severe changes in surround relative luminance.

## 8. RLAB PARAMETERS

Following are the formulas of the various color appearance parameters.

### 8.1 Lightness

And lightness is defined as the brightness relative to the brightness of White. The Lightness is given by  $L^R$ . It is computed by the formula given in the previous section.

### 8.2 Hue

Hue Angle is computed like in the CIELAB model by the following equation

$$h^R = \arctan(b^R/a^R) \quad (87)$$

Hue composition can be determined in RLAB using a procedure similar to that of the Hunt and Nayatani Models. This is useful when testing a color appearance model against magnitude estimation data and when it is desired to reproduce a named hue. Hue composition  $H^R$  can be calculated via linear interpolation of the values in the table 1.

$h^R$	R	B	G	Y	$H^R$
24	100	0	0	0	R
90	0	0	0	100	Y
162	0	0	100	0	G
180	0	21.4	78.6	0	B79G
246	0	100	0	0	B
276	17.4	82.6	0	0	R83B
0	82.6	17.4	0	0	R17B
24	100	0	0	0	R

### 8.3 Chroma

Chroma is defined as colorfulness to the brightness of White. The RLAB chroma is calculated the same way as the CIELAB chroma, as shown below:

$$C^R = \sqrt{(a^R)^2 + (b^R)^2} \quad (88)$$

## 8.4 Saturation

In some applications, such as the image color manipulation required for gamut mapping, it might be desirable to change colors along lines of constant saturation rather than constant chroma. Saturation is defined as colorfulness relative to brightness. Chroma is defined as colorfulness to the brightness of White. And lightness is defined as the brightness relative to the brightness of White. Saturation is thus computed as:

$$s^R = C^R / L^R \quad (89)$$

## 9. INVERSE MODEL

Since the RLAB model was designed with image reproduction applications in mind, computational efficiency and simple inversion were considered of significant importance. Thus, the RLAB model is very easy to invert and requires a minimum of calculations. Here is the procedure for implementing the RLAB model:

Step 1 : Obtain the colorimetric data for the test and adapting stimuli and the absolute luminance of the adapting stimulus. Decide on the discounting-the-illuminant factor and the exponent (based on the surround relative illuminance).

Step 2 : Calculate the chromatic adaptation matrix A.

Step 3 : Calculate the reference stimulus values.

Step 4 : Calculate the RLAB parameters  $L^R$ ,  $a^R$  and  $b^R$ .

Step 5 : Use  $a^R$  and  $b^R$  to compute  $C^R$  and  $h^R$ .

Step 6 : Use  $h^R$  to determine  $H^R$ .

Step 7 : Use  $s^R$  to determine  $C^R$  and  $L^R$ .

## 10. PHENOMENA PREDICTED

The RLAB model provides correlates for relative color appearance attributes only (lightness, chroma, saturation and hue). It cannot be used to predict brightness and colorfulness. RLAB includes a chromatic adaptation transform with a parameter for discounting-the-illuminant and predicts incomplete chromatic adaptation to certain stimuli. For eg, RLAB correctly predicts a CRT display with a D50 white point will retain a yellowish appearance. If it is necessary to predict absolute appearance properties, then we need to use the more complex Hunt or Nayatani models.

## 11. CONCLUSIONS - RLAB

The RLAB model is simple, straightforward, easy to use and is as accurate or more than other existing models. Since its a relatively simple model, it is not exhaustive in its prediction of color appearance phenomena. It does not include correlates of brightness and colourfulness. It cannot be applied over a wide range of luminance levels. It does not predict some color appearance phenomena like the Hunt effect, Stevens effect and the Helson Judd Effect. But the motivation behind RLAB is to use it as many practical applications as possible (which deal with cross media image reproductions). Thus, the above drawbacks can be ignored.

## 12. NAYATANI MODEL

This and the following sections describes the Nayatani Color Appearance model. The researchers who developed this come from the field of illumination engineering. This application provides significantly different challenges from those encountered in that of cross media image reproduction, and hence different from the RLAB model described above.

Despite the different pedigrees of the various models, it is worthwhile to stretch them to applications for which they are not designed. The best possible result is that they will work well (which implies a general model) and the worst outcome is that something more is learned about the important differences between various applications. This learning can provide some theoretical background, which can be priceless source of information.

The model attempts to predict a wide range of color appearance phenomenon like the Stevens effect, the Hunt effect and the Helson-Judd effect in addition to the effects of chromatic adaptation. It is designed to predict the color appearance of simple patches on uniform mid-to-light-gray backgrounds. The model includes output values designed to correlate with all important color appearance attributes like brightness, lightness, colorfulness, chroma and hue.

## 13. INPUT DATA

The input data for the model includes the colorimetric and photometric specification of the stimulus, the adapting illuminant and the luminance factor of the background. Specifically the following data is required:

1. The luminance factor of the achromatic background is expressed as a percentage  $Y_o$ .
2. The color of the illumination  $x_o, y_o$  is expressed in terms of its chromaticity coordinates for the CIE 1931 standard colorimetric observer.
3. The test stimulus is specified in terms of its chromaticity co-ordinates  $x, y$  and its luminance factor  $Y$ .
4. The absolute luminance of the stimulus and adapting field is defined by the illuminance of the viewing field  $E_o$  expressed in lux.
5. The normalizing illuminance  $E_{or}$  which is expressed in lux and usually in the range of 1000-3000 lux.
6. The noise term  $n$  is used in nonlinear chromatic adaptation model. Its value is usually taken to be 1.

Given that the background is a lambertian diffuser, the first step in the Nayatani Model is the calculation of adapting luminance and the normalizing luminance.

$$L_o = \frac{Y_o E_o}{100\pi} \quad (90)$$

$$L_{or} = \frac{Y_o E_{or}}{100\pi} \quad (91)$$

We compute the following intermediary values which form the basis of all the further computations:

$$\xi = (0.48105x_o + 0.78841y_o - 0.08081)/y_o \quad (92)$$

$$\eta = (-0.272x_o + 1.11962y_o + 0.04570)/y_o \quad (93)$$

$$\zeta = 0.91822(1 - x_o - y_o)/y_o \quad (94)$$

## 14. THE ADAPTION MODEL

First, the cone responses for the adapting field must be calculated in terms of the absolute luminance level. This relies on the chromaticity transform described in the above equations, the luminance level  $E_o$  and the luminance factor of the adapting background  $Y_o$  as follows:

$$\begin{pmatrix} R_o \\ G_o \\ B_o \end{pmatrix} = \frac{Y_o E_o}{100\pi} \begin{pmatrix} \xi \\ \eta \\ \zeta \end{pmatrix} \quad (95)$$

Given the adapting-level cone responses from above, the exponents of the nonlinear model of chromatic adaptation can be computed as:

$$\beta_1(R_o) = \frac{6.469 + 6.362(R_o)^{0.4495}}{6.649 + (R_o)^{0.4495}} \quad (96)$$

$$\beta_1(G_o) = \frac{6.469 + 6.362(G_o)^{0.4495}}{6.649 + (G_o)^{0.4495}} \quad (97)$$

$$\beta_2(B_o) = \frac{8.414 + 8.091(G_o)^{0.5128}}{8.414 + (G_o)^{0.5128}} \times 0.7844 \quad (98)$$

These exponents are used in the chromatic adaption model to obtain the cone signals after adaptation. Here are the equations for the adaptation model.

$$L_a = a_L \left( \frac{L + L_n}{L_o + L_n} \right)_L^\beta \quad (99)$$

$$M_a = a_M \left( \frac{M + M_n}{M_o + M_n} \right)_M^\beta \quad (100)$$

$$S_a = a_S \left( \frac{S + S_n}{S_o + S_n} \right)_S^\beta \quad (101)$$

In the above equations,  $L_a, M_a$  and  $S_a$  are the cone signals after adaptation.  $L, M, S$  are the cone excitations.  $L_n, M_n$  and  $S_n$

are the noise terms.  $L_o, M_o$  and  $S_o$  are the cone excitations for the adapting field. And  $a_L, a_M$  and  $a_S$  are coefficients determined by the principle that exact color constancy holds for a nonselective sample of the same luminance factor as the adapting background.

Coming back to the adaptation model, we need another exponential factor that depends on the normalizing luminance must also be calculated using the same functional form as the exponents for the middle and long wavelength sensitive cones as shown below

$$\beta_1(L_{or}) = \frac{6.469 + 6.362(L_{or})^{0.4495}}{6.649 + (L_{or})^{0.4495}} \quad (102)$$

The cone responses for the test stimulus are calculated from their tristimulus values using a more traditional linear transformation given below

$$\begin{pmatrix} R \\ G \\ B \end{pmatrix} = \begin{pmatrix} 0.4002 & 0.7076 & -0.8081 \\ -0.2263 & 1.16532 & 0.04570 \\ 0.0 & 0.0 & 0.9182 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} \quad (103)$$

Finally, two scaling coefficients  $e(R)$  and  $e(G)$  are introduced.

## 15. OPPONENT COLOR DIMENSIONS

The cone responses are transformed directly into intermediate values representing classical opponent dimensions of visual response: an achromatic and two chromatic channels. These equations, used to model these opponent processes, also incorporate the nonlinear chromatic adaptation model.

First, the achromatic response  $Q$  is calculated,

$$Q = \frac{41.69}{\beta_1(L_{or})} \left[ \frac{2}{3} \beta_1(R_o) e(R) \log \frac{R+n}{20\xi+n} + \frac{1}{3} \beta_1(G_o) e(G) \log \frac{G+n}{20\eta+n} \right] \quad (104)$$

The general structure of the above equation is such that achromatic response is calculated as a weighted sum of the outputs of the long and middle wavelengths cone responses with relative weights  $2/3$  and  $1/3$ , which correspond to their relative population in the human retina. They are first normalized after the addition of noise  $n$ , by the cone response for the adapting stimulus represented by  $\xi$  and  $\eta$  according to a von-kries type transformation. A logarithm transform is taken to model the compressive nonlinearity that is known to occur in the human visual system. Because of which, the exponents become multiplicative terms. The scaling factor 41.69 and the luminance dependent exponential adjustment  $\beta_1(L_{or})$ , to complete the equation.

The chromatic channel responses  $t$  (red-green) and  $p$  (yellow-blue) are calculated in a similar manner using the following:

$$t = \beta_1(R_o) \log \frac{R+n}{20\xi+n} - \frac{12}{11} \beta_1(G_o) \log \frac{G+n}{20\eta+n} + \frac{1}{11} \beta_2(B_o) \log \frac{B+n}{20\zeta+n} \quad B_{rw} = \frac{41.69}{\beta_1(L_{or})} \left[ \frac{2}{3} \beta_1(R_o) e(1.758) \log \frac{100\xi+n}{20\xi+n} \right. \quad (109)$$

$$\left. + \frac{1}{3} \beta_1(G_o) e(1.758) \log \frac{100\eta+n}{20\eta+n} \right] + \quad (110)$$

$$p = \frac{1}{9} \beta_1(R_o) \log \frac{R+n}{20\xi+n} + \frac{1}{9} \beta_1(G_o) \log \frac{G+n}{20\eta+n} - \frac{2}{9} \beta_2(B_o) \log \frac{B+n}{20\zeta+n} \quad \frac{50}{\beta_1(L_{or})} \left[ \frac{2}{3} \beta_1(R_o) + \frac{1}{3} \beta_1(G_o) \right] \quad (111)$$

(106)

The t response is a weighted combination of the post-adaptation signals from each of the cone types. The combination is a difference between the long and middle wavelength sensitive cones with a small input from the short wavelength that adds with the long wavelength response. This results in a red minus green response that also includes some reddish input from the short wavelength end of the spectrum that is often used to explain the violet appearance of those wavelengths. The p response is calculated in a similar response by adding the long and middle wavelength-sensitive cone outputs to produce a yellow response and then subtracting the short-wavelength cone output to produce the opposing blue response.

The t and p notation is derived from the terms tritanopic response. A tritanope has only the red-green response t and a protanope has only the yellow-blue response. The Q, t and p responses are used in further equations to calculate the correlates of brightness, lightness, saturation, colorfulness and hue.

One aspect of the hue correlate, the hue angle can be computed directly from the t and p parameters.

$$\theta = \arctan\left(\frac{p}{t}\right) \quad (107)$$

Hue angle is calculated as a positive angle from 0° to 360°. The hue angle is required to compute some other color appearance correlates since a hue dependent adjustment factor is required in some cases.

## 16. NAYATANI MODEL PARAMETERS

Following are the formulas of the various color appearance parameters.

### 16.1 Brightness

The Brightness Br is calculated as follows

$$B_r = Q + \frac{50}{\beta_1(L_{or})} \left[ \frac{2}{3} \beta_1(R_o) + \frac{1}{3} \beta_1(G_o) \right] \quad (108)$$

Q is the achromatic response, which is adjusted by the adaptation exponents in order to include the dependency upon the absolute luminance that is required for brightness, as opposed to lightness.

The brightness of an idea white can be determined by substituting R=100ξ and G=100η. This quantity is referred to as  $B_{rw}$ .

### 16.2 Lightness

The achromatic lightness of a test sample by simply adding 50 to the achromatic response Q as shown below.

$$L_P^* = Q + 50 \quad (112)$$

A second lightness correlate known as normalized achromatic lightness, is the brightness of the test sample to the brightness of white.

$$L_N^* = 100 \left( \frac{B_r}{B_{rw}} \right) \quad (113)$$

The differences between the two lightness correlates  $L_P^*$  and  $L_N^*$  are generally negligible.

### 16.3 Hue

Hue Angle is calculated as shown previously. More descriptive hue correlates are the hue quadratures H and hue composition  $H_C$ .

Hue Quadrature H is a 400 step hue scale on which unique hues take values of 0(Red), 100(Yellow), 200(Green) and 300(Blue). The hue quadrature is computed via linear interpolation using the hue angle of the test sample. Hue angles for the four unique hues are 20.14° (red), 90.00° (yellow), 164.25° (green) and 231.00° (blue).

The Hue Composition  $H_C$  describes hue in percentages of two of the unique hues from which the test hue is composed. For eg, an orange color might be expressed as 50Y50R, indicating that the hue is perceived to be halfway between unique red and unique yellow. Hue composition is computed by simply converting the hue quadrature into percent composition between the unique hues falling on either side of the test color.

### 16.4 Saturation

Saturation is derived directly and then the measurement of colorfulness and chroma are derived from it. Saturation is expressed in terms of a red-green component,  $S_{RG}$  derived from the t response and a yellow blue component  $S_{YB}$  derived from the p response.

$$S_{RG} = \frac{488.93}{\beta_1(L_{or})} E_s(\theta) t \quad (114)$$

$$S_{YB} = \frac{488.93}{\beta_1(L_{or})} E_s(\theta) p \quad (115)$$

$E_s(\theta)$  is a chromatic strength function which is a function of the hue angle.

$$E_s(\theta) = 0.9394 - 0.2478\sin(\theta) - 0.0743\sin(2\theta) \quad (116)$$

$$+ 0.0666\sin(3\theta) - 0.0186\sin(4\theta) - 0.0055\cos(\theta) \quad (117)$$

$$- 0.0521\cos(2\theta) - 0.0573\cos(3\theta) - 0.0061\cos(4\theta) \quad (118)$$

## 16.5 Chroma

Given the correlates for saturation, chroma correlates can be easily determined. The correlates for the red-green, yellow-blue and overall chroma of the test sample are given below

$$C_{RG} = \left(\frac{L_P^*}{50}\right)^0 .7S_{RG} \quad (119)$$

$$C_{YB} = \left(\frac{L_P^*}{50}\right)^0 .7S_{YB} \quad (120)$$

$$C = \left(\frac{L_P^*}{50}\right)^0 .7S \quad (121)$$

## 16.6 Colorfulness

The predictors of colorfulness in the Nayatani model can also be directly derived like Chroma, from the CIE definitions, which say that colorfulness is simply the chroma of a sample multiplied by the brightness of an ideal white

$$M_{RG} = C_{RG} \frac{B_{RW}}{100} \quad (122)$$

$$M_{YB} = C_{YB} \frac{B_{RW}}{100} \quad (123)$$

$$M = C \frac{B_{RW}}{100} \quad (124)$$

The normalizing value of 100 is the brightness of an ideal white under illuminant D65 at the normalizing illuminance. It provides a convenient place to tie down the scale.

## 17. THE INVERSE MODEL

In many applications, it is necessary to use a color appearance model in both forward and reverse directions. Thus, it is important or atleast highly convenient, that the equations can be analytically inverted. Fortunately, the nayatani model can be analytically inverted. In applying the model, it is useful to consider its implementation as a simple step by step process. Thus, the steps to implement and (in reverse order to invert it) are follows:

Step 1 : Obtain physical data.

Step 2 : Calculate Q, t and p.

Step 3 : Calculate  $\theta$ ,  $E_s(\theta)$ , H and  $H_c$ .

Step 4 : Calculate  $B_r$ ,  $B_{rw}$ ,  $L_P^*$ ,  $L_N^*$  and S.

Step 5 : Calculate C.

Step 6 : Calculate M.

## 18. PHENOMENON PREDICTED

The nayatani et al model accounts for changes in color appearance due to chromatic adaptation and luminance level (Steven and Hunt effect). It also predicts the Helson-Judd effect. The model can be used for different background luminance factors. It cannot be used to predict the effects of changes in background color or surround relative illuminance. The model also does not incorporate mechanisms mechanisms for predicting incomplete chromatic adaptation.

## 19. CONCLUSIONS - NAYATANI COLOR APPEARANCE MODEL

The Nayatani model is a complete model in terms of output correlates. It is pretty straightforward and analytically invertible. However, it cannot account for changes in background, surround or cognitive effects. Surround and cognitive factors are critical in image reproduction applications. It also does not predict adaptation level, which is also important in cross media applications. It has been derived and tested for small patches, which might limit its usefulness in many complex lighting/viewing conditions. Thus, we can conclude that the model cannot provide a complete answer for a color appearance model. But the nayatani model has been in used in practical situations for the past few years.

## 20. REFERENCES

Color Appearance Models, Mark D. Fairchild