ICS 6B
Boolean Algebra & Logic

Lecture Notes for Summer Quarter, 2008

Michele Rousseau

Set 1 – Administrative Details, Ch. 1.1, 1.2

Today's Lecture

- Administrative details
  - Course Mechanics
  - Add/Drop
  - Grading
  - & etc..
- Chapter 1 (Sections 1.1 & 1.2)
  - Logic (1.1)
  - Propositional Equivalences (1.2)

Introductions

- **Instructor**
  - Michele Rousseau
  - Email: michele@ics.uci.edu
    - Please put ICS 6B in the Subject
  - Office Hours: by appointment
  - Office: DBH: 5204

Pre-requisites

- High School Mathematics through trigonometry
- Please let me know if you have not satisfied this requirements

Class Information

- **Website**
  - www.ics.uci.edu/~michele/Teaching/ICS6B-Sum08
  - Can access from my home page
    - www.ics.uci.edu/~michele

Course Materials

- **Required textbooks**
  - Rosen, Kenneth H.

    Discrete Mathematics and Its Applications, 6th edition, McGraw Hill, 2007.
    - This book is required, and it should be available at the UCI bookstore.
    - There is an online list of errata at: http://highered.mcgraw-hill.com/sites/dl/free/0072880082/299357/Rosen_errata.pdf
- **Additional Readings**
  - Will be announced on the website and in lecture

Course Mechanics

- **Lecture**
  - T Th 1p – 3:50p

How to be successful (1)

- **Attend class**
  - For summer classes missing one is a big deal
    - Material is core part of the exams
    - What is said in class supersedes all else
  - Official place for announcements
- **Do your Homework**
  - Really think about the problems

How to be successful (2)

- Ask Questions
- Read the Book
  - Review the lecture slides
- Visit course Web site on a regular basis
  - Assignments
  - Lecture Slides
- Use Office Hours

Grading

Assignments	**10%**
Quizzes	**40%**
Final	**50%**

- Will scale only if necessary

Assignments

- 2x a Week
- Package properly
  - Every assignment…
    - lists your Name & Student ID on every page
    - has a cover page with Class title, Name, student ID & assignment #
    - …is properly stapled
- Assignment grades are based on…
  - Correctness & Clarity
  - Sloppy, illegible, or unclear answers may be marked down even if they are correct
- Check the answers in the back
  - Let me know which problems you missed
- No Late Assignments

Exceptions for being late

- At the Instructor's discretion
  - Contact the instructor as soon as possible
  - Preferably before you are late
- Valid reasons
  - Serious illness, accident, family emergency, etc.
- Not-so-valid reasons
  - "Lost my pencil", "didn't know it was due today", "couldn't find parking", etc.

Quizzes

- Weekly (that's 1 a week)
- Quizzes will primarily be based on…
  - Lectures
  - Readings
  - Homework
- No Make-up Quizzes
- The Final will be comprehensive
- For all exams → Final answers must be in Pen for regrades

Grading

- **Disputes**
  - Let me know ASAP (by the next class)
  - Please don't play the "points-game"
    - I have limited time
    - Check your grading thoroughly and ASAP
    - Include a coversheet with your name, student ID, and a detailed description of the error
- **Re-grading**
  - Will only accept re-grades at the beginning of the class following the date they were returned
  - Must be accompanied with a clear explanation of what needs to be reconsidered and why
  - Entire assignment will be considered

Questions

When in doubt

- Ask Me!
  - Open door policy
  - Attend Office Hours
- Email me
  - If I think the whole class could benefit I'll forward it
  - let me know if you specifically don't want it forwarded
- Questions will generally be answered within 24 hours (except weekends)
- Ask your friends

Academic Dishonesty (ugh)

- Please don't Cheat
  - Know the academic dishonesty policies (for ICS & UCI)
  - ICS: http://www.ics.uci.edu/ugrad/policies/
  - UCI: http://www.editor.uci.edu/catalogue/appx/appx.2.htm
- If you do…
  - Final grade is an "F", irrespective of partial grades
  - Assignments, Quizzes, or Final
  - Letter in your UCI file
- Anything copied from a book or website needs to be quoted and the source provided

Help each other but don't share work

- To avoid being a cheater
  - Always do your work by yourself
    - *It is okay to…*
      - … ask your friends about how solve/approach a problem
      - … discuss an assignment
    - *It is not okay to…*
      - … ask for the answer/solution
      - … copy work
      - … have them do it for you!
      - …put your work on the Web
      - … borrow or lend work!
      - …post answers to assignments
    - When in doubt – ask me!
- Use good Judgment

Add/Drop/Change of Grade Policy

- Adding or Dropping the Class
  - Check with Summer Sessions
  - Check with the Department
  - If they are good with it – so am I
- Changing Grade to P/NP option
  - Check with Summer Sessions
  - Check with the Department
  - If they are good with it – so am I
- Please bring completed Add/Drop Cards
  - In Pen PLEASE ☺

Other Policies

- **Please use your ICS or UCI account**
  - This is for your privacy
  - Needs to be activated if you are a new student
- **Questions of general interest will be forwarded to the board**
  - if you don't want it forwarded for some reason please state that
- **If you need accommodations due to a disability, talk to me**

Miscellaneous

- **You get out of this class what you put into it**
  - Attend Class
  - Follow instructions
  - Do the homework
  - Read and study the textbook and slides
  - Help is available, do not be afraid to ask questions

Course Objective

- **To Teach You:**
  - Relations & their properties
  - Boolean algebra
  - Formal languages
  - Finite automata

Now to the fun part…

- **Chapter 1 (Sections 1.1 & 1.2) : Logic & Proofs**
  - Propositional Logic (1.1)
  - Propositional Equivalences (1.2)

Take a Break

- Stretch
- Get a drink / snack
- Use the restroom
- Relax…

When we return…

Chapter 1.1

Chapter 1: Section 1.1

Propositional Logic

What is Propositional Logic?

- *Logic* is the basis of all mathematical reasoning
- A *proposition* is a *declarative* statement that is either T (1) or F (0) → *Binary Logic*
- For example:
  - "Irvine is in California"
  - "California is on the East Coast of the USA"
  - "1+1=436"
- *Propositional Logic* is the area of logic that deals with propositions
- *Propositional Variables* – Typically *p,q,r,s...*
- *Truth Values* – denoted by T(1) or F(0)
- *Compound propositions* – combining propositions using logical operators

Section 1.1 – Propositional Logic

Which of the following are propositions?

- "It is sunny today" Yes → There is a clearly defined truth value
- 1+2=3 or 2+2=5 Yes → The 1st is true and the 2nd is false
- "Can I have a cookie?" No → This is a question.
- "Rose is very clean." Yes → No "free" variables.
- "Take out the Trash" No → Imperative statement.

Definition 1: Negation

Given a Proposition *p* the negation is "not *p*" or "it is not that case that *p*"

- Notated $\neg p$ *or* $\overline{p}$
  - For example:
    - p: "It is my turn"
    - $\neg p$: "It is not my turn" or
      "It is not that case that it is my turn"
    - p: "Easter is a national holiday in the USA"
    - $\neg p$: "Easter is not a national holiday in the USA"
    - p: "It rained on Monday"
    - $\neg p$: "it is not the case that it rained on Monday"

Truth Table for $\neg p$

- **Truth tables show the value of a proposition**

p	$\neg p$
T	F
F	T

All Possible Values of p

Result of applying the proposition $\neg p$

Constructing Truth Tables

- How many rows do you need for each propositional variable? (i.e. How many Ts & Fs?)
  - $2^{(\text{\# of variables})}$

For 1?
$2^1 = 2$

p
T
F

For 2?
$2^2 = 4$

p	*q*
T	T
T	F
F	T
F	F

How Many T's to start in the 1st Column?
4 / 2 = 2

How about 4?
$2^4 = 16$

How Many T's to start in the 1st Column?
16 / 2 = 8

How Many T's to start in the 2nd Column?
8/ 2 = 4

How Many T's to start in the 3rd Column?
4 / 2 = 2

p	*q*	*r*	*s*
T	T	T	T
T	T	T	F
T	T	F	T
T	T	F	F
T	F	T	T
T	F	T	F
T	F	F	T
T	F	F	F
F	T	T	T
F	T	T	F
F	T	F	T
F	T	F	F
F	F	T	T
F	F	T	F
F	F	F	T
F	F	F	F

We can also use 0's & 1's

- How many rows do we need for 3 variables?
  - $2^3 = 8$

How Many 1's to start in the 1st Column?
8/ 2 = 4

How Many 1's to start in the 2nd Column?
4/ 2 = 2

p	*q*	*r*
1	1	1
1	1	0
1	0	1
1	0	0
0	1	1
0	1	0
0	0	1
0	0	0

Definition 2: Conjunction

Given two propositions *p* and *q*. The ***conjunction*** is true when both "*p* and *q*" are true.

- Notated: $p \wedge q$

p: "I am going out to dinner."

q: "I am going to the movies."

$p \wedge q$*:* "I am going out to dinner *and* I am going to the movies."

- First, fill in *p* & *q*
- Then fill in $p \wedge q$
  - What is the 1st Value for $p \wedge q$?
  - What is the 2nd Value?
  - What is the 3rd Value?
  - What is the 4th Value?

p	*q*	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Definition 3: Disjunction

AKA Inclusive Or. Given two propositions *p* and *q*. The ***disjunction*** is true when either "*p* or *q*" are true.

- Notated: $p \vee q$

p: "My neighbor's dog is barking."

q: "My cat is howling."

$p \vee q$*:* "My neighbor's dog is barking *or* my cat is howling."

Note: 1 of p or q or both need to be True – inclusive.

- Fill in *p* and *q*
- Fill in $p \vee q$
  - What is the 1st Value for $p \vee q$?
  - What is the 2nd Value?
  - What is the 3rd Value?
  - What is the 4th Value?

p	*q*	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Definition 4: Exclusive Or

Given two propositions *p* and *q*. The ***exclusive or*** is true when exactly one of *p* or *q* are true.

- Notated: $p \oplus q$

p: "I am going out to dinner."

q: "I am going to the movies."

$p \oplus q$*:* "*Either* I am going out to dinner *or* I am going to the movies."

How is this different from the previous or ***(∨)?***

- Fill in $p \oplus q$
  - What is the 1st Value for $p \oplus q$?
  - What is the 2nd Value?
  - What is the 3rd Value?
  - What is the 4th Value?

p	*q*	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

Inclusive Or and Exclusive Or

- Which of the following is Inclusive or Exclusive
  - "I will stay home or go to the party." Exclusive
  - "If I am late or I forget my ticket I'll miss the train" Inclusive
  - "To take software engineering I need to have taken a Java class or a C++ class. " Inclusive
  - "I will get an A or a B in this class" Exclusive

p	*q*	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

p	*q*	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

Definition 5: Implication

Let p & q be props. The conditional statement $p \longrightarrow q$ (p implies q) is only false when p is true and q is false, otherwise it is true. NOTE: if p is false then $p \longrightarrow q$ is true!

- Notated: $p \longrightarrow q$

  p: "I am going buy gasoline."

  q: "I will be broke."

  $p \longrightarrow q$: "*If* I am going to buy gasoline *then* I will be broke."

  Note: I can be broke whether or not I buy gas, but if I buy gas then I will definitely be broke.

- Fill in $p \longrightarrow q$
  - What is the 1st Value for $p \longrightarrow q$?
  - What is the 2nd Value?
  - What is the 3rd Value?
  - What is the 4th Value?

p	q	$p \longrightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Definition 5: Implication (2)

If → Then means different things in different contexts

- In English, it implies cause and effect
- In programming, it means if this is true then execute some code
- In Math, it is based on truth values (not causality)

p	q	$p \longrightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Definition 5: Implication (3)

- Many ways to express $p \rightarrow q$

p is the premise, hypothesis, or antecedent and q is the conclusion (or consequence)

"If p, then q"

"if p, q"

"q if p"

"q when p"

"p implies q"

"p is sufficient for q"

"a sufficient condition for q is p"

"a necessary condition for p is q"

"q is necessary for p"

"p only if q"

"q whenever p"

"q unless $\neg p$"

"q follows from p"

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Definition 5: Implication (4)

- Rephrase the following to If → Then

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

If it rains, I'll go home. ("If p, q")
=> If it rains, then I'll go home.

I go walking whenever it rains. ("q whenever p")
=> If it rains then I go walking.

To go on the trip it is necessary that you get a passport.
("q is necessary for p") or ("a necessary condition for p is q")
⇔ Getting a passport is necessary for going on the trip.
=> If you go on the trip, then you must get a passport.

To pass the class it is sufficient that you get a high grade on the exam.
("p is sufficient for q") or ("a sufficient condition for q is p")
⇔ Getting a high grade on the exam is sufficient for passing the class.
If you get a high grade on the exam, then you will pass the class.

Converse, Inverse & Contrapositive

Related conditionals

- For $p \rightarrow q$
  - Converse $q \rightarrow p$
  - Inverse $\neg p \rightarrow \neg q$
  - Contrapositive $\neg q \rightarrow \neg p$

- Converse of $p \rightarrow q$
  - Truth table for $p \rightarrow q$
  - Now let's find the truth values for $q \rightarrow p$

p	q	$p \rightarrow q$	$q \rightarrow p$
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T	T

Inverse of $p \rightarrow q$

- **Inverse** $\neg p \rightarrow \neg q$

1. Get $\neg p$ & $\neg q$
2. We know $p \rightarrow q$
3. Then get $\neg p \rightarrow \neg q$

p	q	$\neg p$	$\neg q$	$p \rightarrow q$	$\neg p \rightarrow \neg q$
T	T	F	F	T	T
T	F	F	T	F	T
F	T	T	F	T	F
F	F	T	T	T	T

Contrapositive of $p \rightarrow q$

- **Contrapositive** $\neg q \rightarrow \neg p$
  1. We know $\neg p$, $\neg q$, & $p \rightarrow q$
  2. Now fill in $\neg q \rightarrow \neg p$

When the truth tables are the same -- we say they are EQUIVALENT

p	q	$\neg p$	$\neg q$	$p \rightarrow q$	$\neg q \rightarrow \neg p$
T	T	F	F	T	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

Converse, Inverse, & Contrapositive

- What are the *Converse*, *Inverse*, & *Contrapositive* of the following conditional statement?
- It rains whenever I wash my car.
  - <u>Converse</u>
  1. Assign variables to each component proposition
     (it might be easier to first convert it to If → then format.)
     "q whenever p" thus => If I wash my car, then it rains.
     p: I wash my car
     q: It rains
  2. State the conversion in symbols
     The converse of $p \rightarrow q$ is $q \rightarrow p$
  3. Convert the symbols back to words
     "If it rains, then I wash my car" or
     "I wash my car whenever it rains"

p	q	$p \rightarrow q$	$q \rightarrow p$
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T	T

Converse, Inverse, & Contrapositive

- **Inverse**

1. p: I wash my car.
   q: It rains.
2. The inverse of $p \rightarrow q$ is $\neg p \rightarrow \neg q$
   $\neg p$: It is not the case that I will wash my car. ⇔ I don't wash my car.
   $\neg q$: It is not the case that it will rain. ⇔ It won't rain.
3. "If I don't wash my car, then It won't rain" or
   "It won't rain whenever I don't wash my car"

p	q	$p \rightarrow q$	$\neg p$	$\neg q$	$\neg p \rightarrow \neg q$
T	T	T	F	F	T
T	F	F	F	T	T
F	T	T	T	F	F
F	F	T	T	T	T

Converse, Inverse, & Contrapositive

- **Contrapositive**

1. p: I wash my car.
   q: It rains.
2. The contrapositive of $p \rightarrow q$ is $\neg q \rightarrow \neg p$
   $\neg p$: It is not the case that I will wash my car. ⇔ I don't wash my car.
   $\neg q$: It is not the case that it will rain. ⇔ It won't rain.
3. "If it doesn't rain, then I don't wash my car " or
   "I don't wash my car whenever it doesn't rain"

p	q	$p \rightarrow q$	$\neg p$	$\neg q$	$\neg q \rightarrow \neg p$
T	T	T	F	F	T
T	F	F	F	T	F
F	T	T	T	F	T
F	F	T	T	T	T

Bi-Conditional

Let p & q be props. AKA *bi-implications.*
The biconditional statement is the proposition "*p if and only If q*"
$p \leftrightarrow q$ is true when p & q have the same truth value, and is false otherwise.

- Notation: $\leftrightarrow$
- p if and only if q (iff)
  - "*p* is necessary and sufficient for *q*"
  - "if *p*, then *q*, and conversely
- Truth Table for $p \leftrightarrow q$

"You must take ICS 52 if you pass this class."
"I will wash my car if and only if it rains"
"I wash my car exactly when it rains"

We don't really talk this way. It is usually implied

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Note: "*exactly*" takes the place of "if and only if"

Thus far…

- …We have learned the building blocks

Conjunction, Exclusive Or, Biconditional, Negation, Disjunction, Implication

p	q	$\neg p$	$\neg q$	$p \wedge q$	$p \vee q$	$p \oplus q$	$p \rightarrow q$	$p \leftrightarrow q$
T	T	F	F	T	T	F	T	T
T	F	F	T	F	T	T	F	F
F	T	T	F	F	T	T	T	F
F	F	T	T	F	F	F	T	T

- **Now we can combine them**

Precedence of logical operators

Before we move on you should note:

Operator	Precedence
$\neg$	1
$\wedge$	2
$\vee$	3
$\rightarrow$	4
$\leftrightarrow$	5

Note: It is best to use good ol' fashioned parentheses ( ) to avoid confusion

Compound Propositions

- **Construct the truth table for**
  - $(\neg p \vee q) \leftrightarrow (p \oplus \neg q)$

p	q	$\neg p$	$\neg q$	$\neg p \vee q$	$p \oplus \neg q$	$(\neg p \vee q) \leftrightarrow (p \oplus \neg q)$
T	T	F	F	T	T	T
T	F	F	T	F	F	T
F	T	T	F	T	F	F
F	F	T	T	T	T	T

We have to evaluate $p \oplus \neg q$

Finally, we have to evaluate $(\neg p \vee q) \leftrightarrow (p \oplus \neg q)$

Applying it to Computer Science

- Software Specifications are often written in natural language
  - Problem: Natural Language is ambiguous
  - Translating to "math" decreases ambiguity
- Translate the following into a logical expression.

"The online user is sent a notification of a link error if the network link is down."

1. Look for Key Words

2. Rephrase (if necessary)

"If the network link is down,
then the online user is sent a notification of a link error."

3. Define the propositions

l: The network link is down
n: online user is sent a notification of a link error.

4. Construct your statement

$l \longrightarrow n$

Note: There are many other applications in CS
Read the book

Take a Break

- Stretch
- Get a drink / snack
- Use the restroom
- Relax…

When we return…

Chapter 1.2

Chapter 1: Section 1.2

Propositional Equivalences

Definitions

- *Tautology:* When a compound proposition is always true (eg. $p \lor \neg p$)
- *Contradiction:* When a compound proposition is always false (eg. $p \land \neg p$)
- *Contingency:* When a compound proposition is not a tautology or a contradiction (eg. $p \lor q$)
- *Logical Equivalence:* When compound propositions have the same truth values in all possible cases (truth tables are the same)
  - When two propositions are equivalent
  - Notated: $p \equiv q$
    or $p \Leftrightarrow q$

Not a logical connective

Some laws you should know…

- Logical Equivalences

Equivalence	Name
$p \land \mathbf{T} \equiv p$ $p \lor \mathbf{F} \equiv p$	Identity Laws
$p \lor \mathbf{T} \equiv \mathbf{T}$ (always true) $p \land \mathbf{F} \equiv \mathbf{F}$ (always false)	Domination Laws
$p \lor p \equiv p$ $p \land p \equiv p$	Idempotent Laws
$p \lor \neg p \equiv \mathbf{T}$ $p \land \neg p \equiv \mathbf{F}$	Negation Laws
$\neg(\neg p) \equiv p$	Double negation Law

Some laws you should know… (2)

- Logical Equivalences

Equivalence	Name
$p \lor q \equiv q \lor p$ $p \land q \equiv q \land p$	Commutative Laws
$(p \lor q) \lor r \equiv p \lor (q \lor r)$ $(p \land q) \land r \equiv p \land (q \land r)$	Associative Laws
$p \lor (q \land r) \equiv (p \lor q) \land (p \lor r)$ $p \land (q \lor r) \equiv (p \land q) \lor (p \land r)$	Distributive Laws
$\neg(p \land q) \equiv \neg p \lor \neg q$ $\neg(p \lor q) \equiv \neg p \land \neg q$	De Morgan's Laws
$p \lor (p \land q) \equiv p$ $p \land (p \lor q) \equiv p$	Absorption Laws

Showing Equivalence

De Morgan's Law #1: $\neg(p \lor q) \equiv \neg p \land \neg q$

Note: These are NOT the same symbols

p	q	$p \lor q$	$\neg(p \lor q)$	$\neg p$	$\neg q$	$\neg p \land \neg q$
T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T

LHS

RHS

Check to see that all of the truth values are equivalent

More Logical Equivalences

Involving Conditional Statements
$p \to q \equiv \neg p \lor q$
$p \to q \equiv \neg q \to \neg p$
$p \lor q \equiv \neg p \to q$
$p \land q \equiv \neg(p \to \neg q)$
$(p \to q) \land (p \to r) \equiv p \to (q \land r)$
$(p \to q) \lor (p \to r) \equiv p \to (q \lor r)$
$(p \to q) \land (q \to r) \equiv (p \lor q) \to r$
$(p \to q) \lor (q \to r) \equiv (p \land q) \to r$

Note: These are NOT the same symbols

Involving Bi-Conditional Statements
$p \leftrightarrow q \equiv (p \to q) \land (q \to p)$
$p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$
$p \leftrightarrow q \equiv (p \land q) \lor \neg p \land \neg q$
$\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$

Showing Equivalence

Let's show the first equivalence with a truth table: $p \to q \equiv \neg p \vee q$

p	q	$p \to q$	$\neg p$	$\neg p \vee q$
T	T	T	F	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

LHS

RHS

We have to evaluate $\neg p \vee q$

... all of the ... equivalent

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Showing Equivalence

- We can use Logical Equivalences we already know to show new equivalences
- Show $\neg(p \to q) \equiv p \wedge \neg q$

1. We want to convert to ∨s or ∧s

$\neg(p \to q) \equiv \neg(p \to q)$

$\equiv \neg(\neg p \vee q)$ by the previous example

$\equiv \neg(\neg p) \wedge \neg q$ by the 2nd De Morgan's law

$\equiv p \wedge \neg q$ by the double negation law

2. We want to convert to ∧s

3. We want to get to p

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Announcements

HOMEWORK – Due Thursday

- **SECTION 1.1:** 2,5,7,9,11,15,23,27,33 (d,e,f)
- **SECTION 1.2:** 3, 7,9,11,15,17,23,29,35

QUIZ – THURSDAY

- **Will cover sections 1.1, 1.2**