CompSci 161
Winter 2023 Lecture 17:
Greedy Algorithms:
Interval Scheduling
Two possible algorithms (four on handout):

- Sign up for the class that begins earliest.

- Sign up for the class that meets for the least amount of time.
Unweighted Interval Scheduling Problem

Two more algorithms (four on handout):

- Sign up for the class that conflicts with the fewest other classes.

- Sign up for the class that ends earliest.

(ALG Better: top 4)
Interval Scheduling Problem (proof)

Correct Algorithm:

▶ Sign up for the class that ends earliest.
▶ Remove it and all overlapping classes from the set of available classes.
▶ Repeat this process until no classes remain.

Claim: There is an optimal solution that includes the first-ending class.
Proof of Claim: Suppose all optimal solutions do not. Select an arbitrary optimal solution OPT.

OPT': OPT, remove the first ending from OPT and add the first ending from input.
Proof of Correctness

- We began with an arbitrary optimal set OPT
  - Its first element was not first-ending.
  - We removed that one
  - We added our first one: the first-ending.
  - This forms a set we’ll call OPT’

- **Claim**: OPT’ is an optimal solution.
  - Is it the same size as every optimal solution?
    - Yes (maybe say why)
  - Is it a valid solution?
    - Rest of OPT is after dotted line
    - First ending from OPT
"How To Prove It"
by Daniel Velleman (spelling?)
Proof of Correctness

- We proved that an optimal solution exists that includes the first-ending class.
- What does the full proof look like?