QuickSort Step 1: Partition

1. Choose a pivot.
2. Place that pivot in the right spot.
3. Pivot the rest of the array.
QuickSort

<table>
<thead>
<tr>
<th>85</th>
<th>24</th>
<th>63</th>
<th>45</th>
<th>17</th>
<th>31</th>
<th>96</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>17</td>
<td>24</td>
<td>31</td>
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<td>96</td>
<td>85</td>
</tr>
</tbody>
</table>

How fast is QuickSort?

- $T(n) = T(\text{lower}) + T(\text{upper}) + \Theta(n)$
- If lower and upper are both size $n/2$?
  
  $$T(n) = 2T\left(\frac{n}{2}\right) + n = \Theta(n \log n)$$

- What if we select a pivot uniformly at random?

- What if we could find a median in $\Theta(n)$...
Average Case Analysis of QuickSort

Suppose
- All permutations equally likely
- All $n$ values are distinct (for simplicity)
- Define $S_1, S_2, \ldots, S_n$ as sorted order.

Let $P_{i,j}$ be probability we compare $S_i$ and $S_j$.

$$P_{i,j} = \frac{\# \text{yes}}{\# \text{total}} = \frac{2}{j-i+1}$$

Expected number of comparisons

$$E \left( \sum_{i=1}^{n} \sum_{j=i+1}^{n} X_{i,j} \right) = \sum_{i=1}^{n} \sum_{j=i+1}^{n} E(X_{i,j})$$

$X_{i,j}$: IRV

$$= \sum_{i=1}^{n} \sum_{j=i+1}^{n} \frac{2}{j-i+1}$$

$$= \sum_{i=1}^{n} \frac{n-i+1}{k}$$

$$< \sum_{i=1}^{n} \sum_{k=1}^{2} \frac{1}{k}$$

is $\Theta(n \log n)$
The Selection Problem

- Given a list $S$ and numeric $k$
- Want: if we sorted $S$, what is $S_k$?
- Brute force:
  - Sort $S$ in $\Theta(n \log n)$
  - Return $S_k$
- Can we do better?

Randomized Selection

```
quickSelect(S, k)
    If $n$ is small, brute force and return.
    Pick a random $x \in S$ and put rest into:
    $L$, elements smaller than $x$
    $G$, elements greater than $x$
    if $k \leq |L|$ then
        return quickSelect($L$, $k$)
    else if $k == |L| + 1$ then
        return $x$
    else
        return quickSelect($G$, $k-|L|-1$)
```
Randomized Selection

- What is the worst-case running time?
- What would cause that bad time?
- Estimate the expected running time?
  
  *Hint: on average, the pivot is the median.*

\[
T(n) = T\left(\frac{n}{2}\right) + n
\]

is \( \Theta(n) \)