1. Freshly energized after finishing the dynamic programming quiz, a friend of yours in CompSci 161 has decided to open a new pizza chain called Algorithmic Pizza. Your friend is interested in servicing a very long stretch of road which currently has no pizza parlors. There are \( n \) houses located at various places on this road, and we want to make sure that each house is within 5 miles of an Algorithmic Pizza. Give an algorithm that will achieve this goal with the fewest possible pizza parlors. Prove that your algorithm achieves the optimal solution; that is, that no alternate arrangement can “cover” every house while using fewer pizza parlors.
2. Business is booming for Algorithmic Pizza. You are the manager at a specific store, and you have been flooded with $n$ different pizza orders. Making a pizza is divided into two stages: (1) preparation, wherein the chef tosses the dough, and applies the sauce, cheese, and toppings, and (2) baking. Order $i$ takes $p_i$ time in stage 1, and $b_i$ time in stage 2. You only have a single chef, so only a single pizza can be in the preparation stage at any given time. You have a large oven, however, so you can bake any or all of the pizzas simultaneously.

Give an algorithm that will choose an order in which to begin preparing the pizzas so that the last pizza comes out of the oven as soon as possible. Note that due to different baking times, the last pizza into the oven is not necessarily the last one out. Prove that your algorithm achieves the best possible finish time.