CompSci 162  Unit 3 Diagnostic Exam 3  Spring 2023
DO NOT OPEN EXAM UNTIL INSTRUCTED TO DO SO
SILENCE AND STORE ALL ELECTRONICS
Format Style: Answer underneath the question

This is a diagnostic exam intended to help you evaluate your readiness for the real exam.

The following rules apply to you, whether you think they do or not. Read and understand them; failure to abide by these rules, or directions given by course staff during the exam, may result in disciplinary action, including but not limited to a failing grade in the class.

• This exam is solely for students enrolled in this lecture. Anyone not enrolled in this lecture may not take an exam.

• You may not open the exam or begin writing until the instructor has explicitly given you permission to do so.

• Keep your UCI ID readily accessible during the test. Proctors may request to see it.

• This exam is closed book, closed notes, and is individual effort. Once course staff begin passing out exams, you may not communicate with anyone other than proctors for any reason, nor may you have electronics, including calculators watches and phones, available to you during the test for any reason. YOU DO NOT NEED A CALCULATOR!

• If you leave your seat during the test for any reason, your instructor may collect it and deem you to have turned it in. Do not ask proctors for an exemption to this, they are not authorized to grant such.

• If you are still seated at 10:35, you may not leave your seat until explicitly dismissed by the instructor. Leaving after 10:35 and before being dismissed may result in a grade penalty.

• If you believe a question is ambiguous, write at least two reasonable interpretations and indicate clearly which one you will be using. Then answer your question with that assumption. Unless your interpretation makes the problem much more trivial than intended, we will grade your response as if one of us had made that clarification.

• The purpose of the real exam is to evaluate how well you understand the material presented in the course. It is an academic integrity violation to do anything that subverts the goals of this assessment including, but not limited to, not doing your own work or submitting that of anyone else.

• We will only grade responses marked in the space provided for each question.
Scratch paper. Nothing you write on this side of the page will be graded.
1. (4 points) Describe a Turing Machine that makes the following computation. The input tape contains a string in \{a, b\}^*. When the machine terminates, the string contains the same number of each letter that appeared initially, but now the string is of the form \(a^j b^k\).

You do not need to, nor should you, draw the schematic for the machine; a description is fine, as long as one could reasonably implement it from your description.
Scratch paper. Nothing you write on this side of the page will be graded.
2. (4 points) Show that the following is decidable:

Let \( L_2 = \{ \langle M, w \rangle : M \text{ never moves the R/W head left when computing on } w \} \). Show that \( L_2 \) is decidable.

In lecture, we showed the following problems are decidable; you may use any of these, without proof, in your answer if you wish.

- \( A_{DFA} = \{ \langle B, w \rangle : B \text{ is a DFA that accepts input string } w \} \)
- \( A_{NFA} = \{ \langle B, w \rangle : B \text{ is an NFA that accepts input string } w \} \)
- \( A_{REX} = \{ \langle R, w \rangle : R \text{ is a regular expression that generates string } w \} \)
- \( E_{DFA} = \{ \langle A \rangle : A \text{ is a DFA and } L(A) = \emptyset \} \)
  
  Recall that \( L(A) \) refers to “the language of \( A \)” ; that is, the set of strings that will cause \( A \) to accept.

- \( EQ_{DFA} = \{ \langle A, B \rangle : A \text{ and } B \text{ are DFAs and } L(A) = L(B) \} \)
- \( E_{CFG} = \{ \langle G \rangle : G \text{ is a CFG and } L(G) = \emptyset \} \)
- \( A_{LBA} = \{ \langle M, w \rangle : M \text{ is an LBA that accepts string } w \} \)
Scratch paper. Nothing you write on this side of the page will be graded.
3. (2 points) Show that the following problem is undecidable:

Let \(L_3 = \{\langle M_1, M_2 \rangle : M_2 \text{ is the Turing Machine with the fewest number of states that accepts the same language as } M_1\}\)

*Hint:* consider the languages that you know are undecidable. For which one can you find the smallest Turing Machine that is in that language?

In lecture, we showed the following problems are undecidable; you may use any of these, without proof, in your answer if you wish.

- \(A_{TM} = \{\langle M, w \rangle : M \text{ is a Turing Machine and } M \text{ accepts } w\}\)
- \(HALT_{TM} = \{\langle M, w \rangle : M \text{ is a Turing Machine and } M \text{ halts on input } w\}\)
- \(E_{TM} = \{\langle M \rangle : M \text{ is a Turing Machine and } L(M) = \emptyset\}\)
  
  Recall that \(L(M)\) refers to “the language of \(M\)” ; that is, the set of strings that, if they are on the tape at the start of computation, will cause \(M\) to accept.
- \(EQ_{TM} = \{\langle M_1, M_2 \rangle : M_1 \text{ and } M_2 \text{ are Turing Machines and } L(M_1) = L(M_2)\}\)
- \(E_{LBA} = \{\langle M \rangle : M \text{ is an LBA where } L(M) = \emptyset\}\)

You **may not** use Rice’s Theorem for this problem. If you do not know what that is, do not worry, we did not cover it in class.