\[
\{a^i b^j c^k : i, j, k \geq 0 \text{ and } i == j \text{ OR } i == k\}
\]
Also from lecture 9

\[ \{ww^R : w \in \{a, b\}^*\} \]
CompSci 162
Spring 2023 Lecture 10:
Equivalence of CFGs and PDAs
What is this lecture?

- Unit 1: RegEx / DFA / NFA equivalences

- Today, CFGs and PDAs:
  - A proof in two parts
    - Every CFL is accepted by some PDA
    - Every language accepted by a PDA is CF
Every CFL is accepted by some PDA

- But first an example
- Recall the grammar:
  \[ S \rightarrow aSa \mid bSb \mid c \text{ for } \{wcw^R : w \in \{a, b\}^*\} \]
- Define a PDA based on this grammar.

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Stack:

- \(S\)
- \(\#Sa\)
- \(\#Sba\)
- \(\#Sbbba\)
- \(\#bbba\)

---

Transition diagram:

- From state 0:
  - Transition on \(\varepsilon, S\) to 0
  - Transition on \(\varepsilon, a, \varepsilon\) to 1
  - Transition on \(b, b, \varepsilon\) to 2
  - Transition on \(c, c, \varepsilon\) to 2

- From state 1:
  - Transition on \(\varepsilon, S, aSa\) to 1
  - Transition on \(\varepsilon, S, bSb\) to 1
  - Transition on \(\varepsilon, S, c\) to 0

- From state 2:
  - Transition on \(a, a, \varepsilon\) to 1
  - Transition on \(b, b, \varepsilon\) to 1
  - Transition on \(c, c, \varepsilon\) to 1
General Method

Given $G = (V, \Sigma, R, S)$, create PDA:

Can always do with 3 states for each $A \rightarrow X$ (each rule) $X \in (\text{non-terminals and terminals and } \epsilon)$

Push start symbol

for all $a \in \Sigma$
Every language accepted by PDA is CF

For all pairs of states \( p, q \), create \( A_{pq} \)

Simplifying Assumptions:

1. Only one accept state

2. PDA empties stack before accepting

3. Each transition: exactly one push XOR one pop

\[ \text{generates all strings that take you from } p \rightarrow q \]