Lec 17
Undecidability

\[ A_{TM} = \{ \langle H, w \rangle : M \text{ is TM, and } M \text{ accepts } w \} \]

Turing - recognizable

How many TMs exist?
- finite \( \Sigma \)
- \( \Sigma^* \) countable

\[ A_{TM} \text{ is undecidable} \]

Aside: Quine program

\[ \text{FSDC: F} \text{<br>is a function that for } \langle M, w \rangle \]
\[ H(\langle M, w \rangle) \rightarrow H \text{ accepts if } M \text{ accepts } w \]
\[ H \text{ rejects if } M \text{ rejects or loop forever} \]

build D, input: \( \langle M \rangle \)
1. Run \( H(\langle M, w \rangle) \)
2. If \( H \) accepts, loop forever
   - else, \( D \) accepts

Review

- Church-Turing Thesis
- High level describe Turing Machines
  - and subroutines
  - Turing - recognizable, Turing-decidable

Set of languages?
\[ \Sigma^* \text{? } S_0, S_1, \ldots \]

For any \( S \in \Sigma^* \) binary encoded (bits)

How many \( \phi \) large binary sequences?

\[ \beta \leftrightarrow 2^N \leftrightarrow \text{strictly larger than } N \]

I.e. each \( b \in \beta \) is a subset of natural \#s

So uncountably infinite

Conceptual

Where is \( D \)?

<table>
<thead>
<tr>
<th>( \langle M_1 \rangle )</th>
<th>( \langle M_2 \rangle )</th>
<th>( \langle M_3 \rangle )</th>
<th>( \langle M_4 \rangle )</th>
<th>...</th>
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</thead>
<tbody>
<tr>
<td>reject</td>
<td>accept</td>
<td>loop forever</td>
<td>accept</td>
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Contradiction: \( H \) cannot exist

\[ ???! \]
$T_{\text{recog}}$ and $co-T_{\text{recog}}$

$\overline{A_{TM}}$