What we have seen is $\mathcal{NP}$-complete

- Boolean Satisfiability
- 3-SAT
- Independent Set
- Vertex Cover

Test will include a list and definitions
## Set Cover

<table>
<thead>
<tr>
<th>Set Number</th>
<th>Elements</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>A B</td>
</tr>
<tr>
<td>2</td>
<td>A C D E</td>
</tr>
<tr>
<td>3</td>
<td>B C F I</td>
</tr>
<tr>
<td>4</td>
<td>D G</td>
</tr>
<tr>
<td>5</td>
<td>E H</td>
</tr>
<tr>
<td>6</td>
<td>I J</td>
</tr>
<tr>
<td>7</td>
<td>F G H J</td>
</tr>
</tbody>
</table>

Prove Set Cover is in NP (left as to-do at home)
Idea For Reduction

\[
\text{Vertex-Cover}(G, k) \\
\{ \\
\text{Create } |V| \text{ empty sets } E \\
\text{Let } E \text{ be elements for each edge } e=(u,v) \text{ add } e \text{ to sets } u,v. \\
\text{return SetCover}(sets, k) \\
\}
\]
CompSci 162
Spring 2023 Lecture 22:
Graph Coloring is $\mathcal{NP}$-complete
3-Color is in \(NP\)

**DTM:**

Certificate: mapping \(V \rightarrow \{\text{blue, gold, pink}\}\)

Verifier:

1. ensure every vertex mapped validly

2. for each \(e = (u,v)\)
   
   if \(\text{color}(u) = \text{color}(v)\)
   
   reject

3. accept
Use **3-Color** to get a truth value assignment on \( n \) variables

- Remember, all \( 2^n \) TVAs should be possible.
- Running time polynomial plus call to **3-Color**
Amend the $3$-COLOR usage to get a satisfying truth value assignment on $n$ variables. For each clause $A \lor B \lor C$, add as follows:

- Must be neutral if $A, B$ false
- Must be neutral if $C$ false
Hamiltonian Path Problem

$G$ has a path that visits each vertex exactly once?

(Start/End may or may not be specified)
Creating a truth value assignment

- Design a graph so that any Hamiltonian Path in it will correspond to a truth value assignment on \( n \) variables. Do not (yet) worry about *satisfying* assignments.
- How many Hamiltonian Paths are in your graph?
- What does any given Hamiltonian Path mean as a truth-value assignment?
Creating a truth value assignment

\[ X_1 \quad \overleftarrow{L \Rightarrow R : T} \quad \overrightarrow{R \Rightarrow L : F} \]

\[ X_2 \]

\[ \ldots \]

\[ X_n \]