What have we seen?

- $\mathbb{N} = \{0, 1, 2, \ldots\}$
- $|\mathbb{N}| = \aleph_0$
- There are this many odds, integers ($\mathbb{Z}$)
- There are this many triples of naturals
- There are this many rational numbers
And now \( \mathbb{R} \)

- \( \delta(k, x) : k \)th digit of \( x \) after decimal point
  - ex: \( \delta(2, \pi) = 4 \)
- I claim \( f : \mathbb{N} \to \mathbb{R} \) is bijective.
- To disprove: find \( r \in \mathbb{R} \) s.t. \( \forall y \in \mathbb{N} : f(y) = r \)

\( \text{Is} \quad f(0) = 6 \? \quad \text{If so,} \quad \lfloor r \rfloor = 7 \quad \text{else,} \quad \lfloor r \rfloor = 6 \)

for all \( k \in \mathbb{N}, k \geq 1 \)?

set \( S(k, r) = 7 \) if \( S(k, y) = 6 \) else set = 6
Cantor’s Theorem

Claim: $|\mathcal{P}(A)| > |A|

Suppose FSOC it is not. Then there is some $f : A \rightarrow 2^A$ that is surjective.

\[
\text{def } B = \{x \in A \mid x \notin f(x)\} \\
B \subseteq A \text{ so } B \in \mathcal{P}(A) \text{ and } B \notin 2^A
\]

\[
\exists b \ f(b) = B ? \\
\text{is } b \in B? \\
\text{Yes? } \rightarrow \leftarrow \\
\text{No? } \rightarrow \leftarrow
\]
CompSci 162
Spring 2023 Lecture 3:
Formal Languages, Automata
Five Languages

$L_1 = \{a, abb, aaaa\}$ <small>finite</small>

$L_2 = \{a^n | n \in \mathbb{N} \text{ is prime} \}$

$L_3 = \{b^n a^n b^m | n, m \in \mathbb{N} \text{ and } n \equiv m \pmod{3} \}$

$L_4 = \text{The set of all } w \in \Sigma^* \text{ with at most three } a\text{'s} \}$

$L_5 = \{a^n | n \in \mathbb{N} \text{ and } \exists x, y, z \in \mathbb{N} - \{0\} \text{ such that } x^n + y^n = z^n \} = \{a, aa\}$

**Question 9** Which of these languages are infinite?
Question 10

Are all languages (over a given alphabet) defined by some string in a suitable meta-language?

See discussion
Question 11

What happens when input is “1101”?  

$q_1$  

$q_2$  

$q_3$  

Start  

1  

0  

1  

0, 1  

0  

0, 1  

$igcirc$ = “accept” state  

$igcirc$ = “reject” state (implicit)  

This machine accepts 1101
Formal Definition

$Q = \text{set of states}$

$q_0 = \text{start state}$

$q = \{q_1, q_2, q_3\}$

$q_0 = q_1$

F

$\delta : Q \times \Sigma \rightarrow Q$:

\begin{array}{c|c|c}
q_1 & 0 & q_1 \\
q_2 & 0 & q_2 \\
q_3 & 0 & q_2 \\
\end{array}$
Formal Definition of Computation

1. \( r_0 = q_0 \)
2. \( \delta(r_i, w_{i+1}) = r_{i+1} \) for \( i = 0 \ldots n - 1 \)
3. \( r_n \in F \)

Input = “1101”

---

**Comment:**

- **Accept!**
Question 13

Design a DFA over the language $\Sigma = \{a, b\}$ that accepts all strings with an odd number of instances of the letter $a$.

\[
\text{hint: comment your "code"!}
\]

\[
\text{// } q_1: \text{ even \# as } \\
\text{// } q_2: \text{ odd \# as}
\]