CompSci 260P
Spring 2023 Lecture 10:
Intro to Greedy
Interval Problems and Proofs
Two possible algorithms (four on handout):

1. Sign up for the class that begins earliest.

![Diagram showing two intervals with one selected for least amount of time.]

2. Sign up for the class that meets for the least amount of time.
Two more algorithms (four on handout):

- Sign up for the class that conflicts with the fewest other classes.
- Sign up for the class that ends earliest.
Interval Scheduling Problem (proof)

Correct Algorithm:

▶ Sign up for the class that ends earliest.
▶ Remove it and all overlapping classes from the set of available classes.
▶ Repeat this process until no classes remain.

Claim: There is an optimal solution that includes the first-ending class.

Proof of Claim: Suppose all optimal solutions do not. Select an arbitrary optimal solution OPT.

\[ \text{OPT}' = \text{OPT} - \text{first ending} \]
Proof of Correctness

- We began with an arbitrary optimal set \( \text{OPT} \)
  - Its first element was not first-ending.
  - We removed that one.
  - We added our first one: the first-ending.
  - This forms a set we’ll call \( \text{OPT}' \)

**Claim**: \( \text{OPT}' \) is an optimal solution.

- Is it the same size as every optimal solution?
  - Yes; zero net change

- Is it a valid solution?
  - First end of input
  - First end of \( \text{OPT} \)
  - Rest of \( \text{OPT} \) (and \( \text{OPT}' \))
Proof of Correctness

- We proved that an optimal solution that includes the first-ending class.
- What does the full proof look like?

\textit{Induction (formally).}

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Prove relevant claim.
Interval Coloring

- $n$ groups requested to use a study room
- Group $i$ would like to use it from $s_i$ to $f_i$.
- Cannot put overlapping in same room
- Cannot reject a group.
- Minimize number of distinct rooms assigned
- Why is your algorithm correct?
Interval Coloring Solution

**Algorithm:** When a group arrives, give lowest numbered room currently free.

► How many rooms will this use?

**Maximum overlap**

► Could any solution use fewer? Why or why not?

No, at time of max overlap, you'd have rejected a group or made a shared room.
After Lecture Follow-Up

- Write solutions to today’s problems
- Lecture covered the idea
- Written communication ≠ spoken presentation
- Share and critique with study group