Lecture 10: Hash Tables III: Cuckoo Hash Tables
Example Insertion into Cuckoo Table

\[
\begin{array}{|c|c|c|c|c|c|c|}
\hline
x & 12 & 26 & 92 & 23 & 28 & 94 & 15 \\
\hline
h_0(x) & 1 & 4 & 4 & 1 & 6 & 6 & 4 \\
\hline
h_1(x) & 1 & 2 & 8 & 2 & 2 & 8 & 1 \\
\hline
\end{array}
\]

Eviction sequence:

0 1 2 3 4 5 6 7 8 9 10

12 26 92 28

15 28 26 23 ... 94
Visualizing Cuckoo Hashing

<table>
<thead>
<tr>
<th>Key</th>
<th>12</th>
<th>26</th>
<th>92</th>
<th>23</th>
<th>28</th>
<th>94</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h_0$</td>
<td>1</td>
<td>4</td>
<td>4</td>
<td>1</td>
<td>6</td>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>$h_1$</td>
<td>1</td>
<td>2</td>
<td>8</td>
<td>2</td>
<td>2</td>
<td>8</td>
<td>1</td>
</tr>
</tbody>
</table>

Two cycles in same component

What if
Relevant Graph Theory

**Theorem:** Let $G$ be an undirected connected graph. We can direct the edges of $G$ such that no vertex has more than one incoming edge if and only if $G$ has at most one cycle.

**Acyclic?** Pick $v_1$, direct away

**One cycle?**
1. Direct cycle
2. Direct away from cycle

2+? NO. Shared vertex? NO?
Relevant Graph Theory

**Theorem**: Let $G$ be an undirected connected graph. We can direct the edges of $G$ such that no vertex has more than one incoming edge if and only if $G$ has at most one cycle.

**Importance:**
- Connected component has at most one cycle?
- What about when a second cycle?
About Cuckoo Tables

- Find and Delete, always $O(1)$
- Insertion facts, not proven in ICS 46:
  - Low probability we need a rebuild.
  - Very few expected rebuilds in large table.
  - Long eviction cycles rare too
  - Expected time $O(1)$