pointed out by Miller—is that the subject recodes the stimulus into a smaller number of chunks before storing it in short-term memory. If ten items can be recoded as two chunks, then ten items can be retained. In the other experiments where "too much" appears to be retained in short-term memory, the times allowed the subjects permit them in fact to fixate the excess of items in long-term memory.

I shall cite just two examples from the literature. N. C. Waugh and D. A. Norman report experiments, their own and others', that show that only the first two of a sequence of items is retained reliably across interruption, but with some residual retention of the remaining items.[11] Computation of the fixation times available to the subjects in these experiments shows that a transfer rate to long-term memory of one chunk per five seconds would explain most of the residuals. (This explanation is entirely consistent with the theoretical model that Waugh and Norman themselves propose.)

Roger Shepard has reported that subjects shown a very long sequence of photographs—mostly landscapes—can remember which of these they have seen (when asked to choose from a large set) with high reliability.[12] When we note that the task is a recognition task, requiring storage only of differentiating cues, and that the average time per item was about six seconds, the phenomenon becomes entirely understandable—indeed predictable—within the framework of the theory that we are proposing.

THE ORGANIZATION OF MEMORY

I have by no means exhausted the list of experiments I could cite in support of the fixation parameter and the short-term capacity parameter and in support of the hypothesis that

[11]N. C. Waugh and D. A. Norman, "Primary Memory," *Psychological Review, 72*(1965):89–104.

[12]Roger N. Shepard, "Recognition Memory for Words, Sentences, and Pictures," *Journal of Verbal Learning and Verbal Behavior, 6*(1957):156–163.

these parameters are the principal, and almost only, characteristics of the information-processing system that are revealed, or could be revealed, by these standard psychological experiments.

This does not imply that there are not other parameters, and that we cannot find experiments in which they are revealed and from which they can be estimated. What it does imply is that we should not look for great complexity in the laws governing human behavior, in situations where the behavior is truly simple and only its environment is complex.

In our laboratory we have found that mental arithmetic tasks, for instance, provide a useful environment for teasing out other possible parameters. Work that Dansereau has been carrying forward shows that the times required for elementary arithmetic operations and for fixation of intermediate results account for only part—perhaps one-half—of the total time for performing mental multiplications of four digits by two. Much of the remaining time appears to be devoted to retrieving numbers from the memory where they have been temporarily fixated, and "placing" them in position in short-term memory where they can be operated upon. We hope, through Dansereau's work, to arrive at estimates for these new parameters and at some understanding of the processes that underlie them.[13]

Stimulus Chunking

I should like now to point to another kind of characteristic of the inner system—more "structural" and also less quantitative—that is revealed in certain experiments. Memory is generally conceived to be organized in an "asso-

[13]The first results of this work, relating to the over-all time requirements of the mental arithmetic tasks, are reported in Donald F. Dansereau and Lee W. Gregg, "An Information Processing Analysis of Mental Multiplication," *Psychonomic Science, 6*(1966):71–72. The story of the parameters of memory is discussed in more detail and brought up to date in *Models of Thought*, chapters 2.2 and 2.3.

ciative" fashion, but it is less clear just what that term is supposed to mean. One thing it means is revealed by McLean and Gregg. They gave subjects lists to learn—specifically 24 letters of the alphabet in scrambled order. They encouraged, or induced, chunking of the lists by presenting the letters either one at a time, or three, four, six, or eight on a single card. In all of the grouped conditions, subjects learned in about half the time required in the one-at-a-time condition.[14]

McLean and Gregg also sought to ascertain whether the learned sequence was stored in memory as a single long list or as a hierarchized list of chunks, each of which was a shorter list. They determined this by measuring how subjects grouped items temporally when they recited the list, and especially when they recited it backwards. The results were clear: the alphabets were stored as sequences of short subsequences; the subsequences tended to correspond to chunks presented by the experimenter, or sublengths of those chunks; left to his own devices, the subject tended to prefer chunks of three or four letters. (Recall the role of chunks of this length in the experiments on effects of meaningfulness in rote learning.)

Visual Memory

The materials in the McLean-Gregg experiments were strings of symbols. We might raise similar questions regarding the form of storage of information about two-dimensional visual stimuli.[15] In what sense do memory and thinking represent the visual characteristics of stimuli? I do

[14]R. S. McLean and L. W. Gregg, "Effects of Induced Chunking on Temporal Aspects of Serial Recitation," *Journal of Experimental Psychology, 74*(1967):455–459.

[15]The letters in the stimuli of the McLean-Gregg experiment are, of course, also two-dimensional visual stimuli. Since they are familiar chunks, however, and can be immediately recognized and recoded, there is no reason to suppose that their two-dimensional character plays any role in the subject's behavior in the experiment. Again this is "obvious," but only if we already have a general theory of how stimuli are processed "inside."

not wish to revive the debate on "imageless thought"—certainly not in the original form that debate took. But perhaps the issue can now be made more operational than it was at the turn of the century.

As I enter into this dangerous ground, I am comforted by the thought that even the most fervent opponents of mentalism have preceded me. I quote, for example, from B. F. Skinner's *Science and Human Behavior* (1952, p. 266):

> A man may see or hear "stimuli which are not present" on the pattern of the conditioned reflexes: he may see X, not only when X is present, but when any stimulus which has frequently accompanied X is present. The dinner bell not only makes our mouth water, it makes us see food.

I do not know exactly what Professor Skinner means by "seeing food," but his statement gives me courage to say what an information-processing theory might mean by it. I shall describe in a simplified form one kind of experiment that has been used to throw light on the question. Suppose we allow a subject to memorize the following visual stimulus—a magic square:

4	9	2
3	5	7
8	1	6

Now we remove the stimulus and ask the subject a series of questions about it, timing his answers. What numeral lies to the right of 3, to the right of 1? What numeral lies just below 5? What numeral is diagonally above and to the right of 3? The questions are not all of the same difficulty—in fact I have arranged them in order of increasing difficulty and would expect a subject to take substantially longer to answer the last question than the first.

Why should this be? If the image stored in memory were isomorphic to a photograph of the stimulus, we should expect no large differences in the times required to answer the different questions. We must conclude that the stored image is organized quite differently from a photograph. An alternative hypothesis is that it is a list structure—a hypothesis that is consistent, for example, with the data from the McLean-Gregg experiment and that is much in the spirit of information-processing models of cognition.

For example, if what was stored were a list of lists: "TOP," "MIDDLE," "BOTTOM," where "TOP" is 4–9–2, "MIDDLE" is 3–5–7, and "BOTTOM" is 8–1–6; the empirical results would be easy to understand. The question "What numeral lies to the right of 3?" is answered by searching down lists. The question "What numeral lies just below 5?" is answered, on the other hand, by matching two lists, item by item—a far more complex process than the previous one.

There is no doubt, of course, that a subject could *learn* the up–down relations or the diagonal relations as well as the left–right relations. An EPAM-like theory would predict that it would take the subject about twice as long to learn both left–right and up–down relations as the former alone. This hypothesis can be easily tested, but, to the best of my knowledge, it has not been.

Evidence about the nature of the storage of "visual" images, pointing in the same direction as the example I have just given, is provided by A. de Groot's well-known experiments on chess perception.[16] De Groot put chess positions—taken from actual games—before subjects for, say, five seconds; then he removed the positions and asked the subjects to reconstruct them. Chess grandmasters and masters could reconstruct the positions (with perhaps 20 to 24 pieces on the board) almost without error, while duffers were able to locate hardly any of the pieces correctly, and the performance of players of intermediate skill fell somewhere between masters and duffers. But the remarkable fact was that, when masters and grandmasters were shown other chessboards with the same numbers of pieces *arranged at random,* their abilities to reconstruct the boards were no better than the duffers' with the boards from actual games, while the duffers performed as well or poorly as they had before.

[16]Adriaan D. de Groot, "Perception and Memory versus Thought: Some Old Ideas and Recent Findings," in B. Kleinmuntz (ed.), *Problem Solving* (New York: Wiley, 1966), pp. 19–50. See also the work by Chase and Simon reported in chapters 6.4 and 6.5 of *Models of Thought.*

What conclusion shall we draw from the experiment? The data are inconsistent with the hypothesis that the chess masters have some special gift of visual imagery—or else why the deterioration of their performance? What the data suggest strongly is that the information about the board is stored in the form of *relations* among the pieces, rather than a "television scan" of the 64 squares. It is inconsistent with the parameters proposed earlier—seven chunks in short-term memory and five seconds to fixate a chunk—to suppose that anyone, even a grandmaster, can store 64 pieces of information (or 24) in ten seconds. It is quite plausible that he can store (in short-term and long-term memory) information about enough relations (supposing each one to be a familiar chunk) to permit him to reproduce the board of figure 4:

1. Black has castled on the K's side, with a fianchettoed K's bishop defending the K's Knight.
2. White has castled on the Q's side, with his Queen standing just before his King.
3. A Black pawn on his K5 and a White pawn on his Q5 are attacked and defended by their respective K's and Q's Knights, the White Queen also attacking the Black pawn on the diagonal.
4. White's Q-Bishop attacks the Knight from KN5.
5. The Black Queen attacks the White K's position from her QN3.
6. A Black pawn stands on its QB4.
7. A White pawn on K3 blocks that advance of the opposing Black pawn.
8. Each side has lost a pawn and a Knight.
9. White's K-Bishop stands on K2.

Pieces not mentioned are assumed to be in their starting positions. Since some of the relations as listed are complex, I

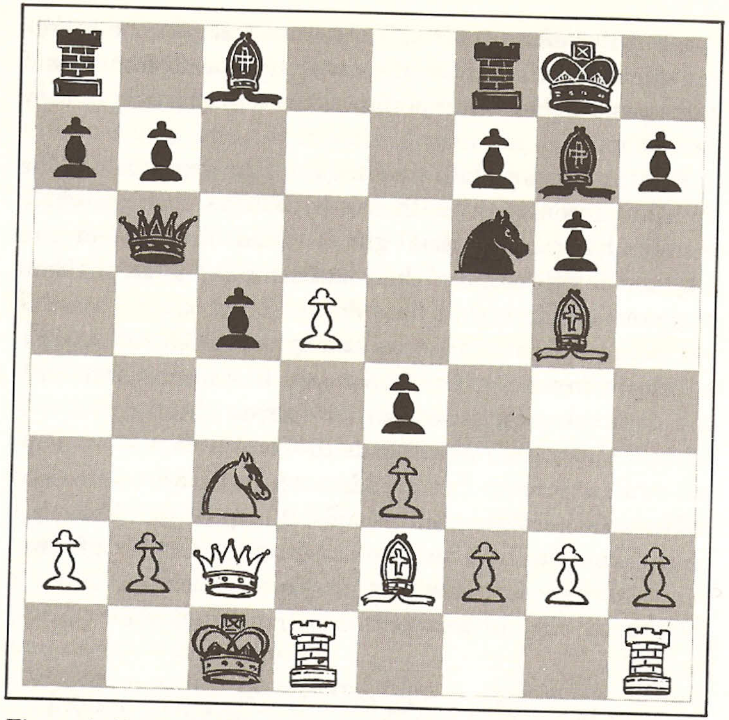

Figure 4. Chess position used in memory experiment.

shall have to provide reasons for considering them unitary "chunks." I think most strong chess players would regard them as such. Incidentally I wrote down these relations from my own memory of the position, in the order in which they occurred to me. Eye-movement data for an expert chess player looking at this position tend to support this analysis of how the relations are analyzed and stored.[17] The eye-movement data exhibit with especial clarity the relations 3 and 5.

[17]O. K. Tikhomirov and E. D. Poznyanskaya, "An Investigation of Visual Search as a Means of Analysing Heuristics," English translation from *Voprosy psikhologii,* 1966, vol. 12, in *Soviet Psychology,* 2(Winter 1966–1967):3–15. See also *Models of Thought,* chapters 6.2 and 6.3.

The implication of this discussion of visual memory for my main theme is that many of the phenomena of visualization do not depend in any detailed way upon underlying neurology but can be explained and predicted on the basis of quite general and abstract features of the organization of memory—features which are essentially the same ones that were postulated in order to build information-processing theories of rote learning and of concept attainment phenomena.

Specifically, we are led to the hypothesis that memory is an organization of list structures (lists whose components can also be lists), which include descriptive components (two-termed relations) and short (three-element or four-element) component lists. A memory having this form of organization appears to have the right properties to explain phenomena relating to storage of visual and auditory stimuli as well as "symbolic" stimuli.

PROCESSING NATURAL LANGUAGE

A theory of human thinking cannot and should not avoid reference to that most characteristic cognitive skill of human beings—the use of language. How does language fit into the general picture of cognitive processes that I have been sketching and into my general thesis that psychology is a science of the artificial?

Historically the modern theory of transformational linguistics and the information-processing theory of cognition were born in the same matrix—the matrix of ideas produced by the development of the modern digital computer, and in the realization that, though the computer was embodied in hardware, its soul was a program. One of the initial professional papers on transformational linguistics and one of the initial professional papers on information-processing psychology were presented, the one after the other, at a meeting at MIT in September 1956.[18] Thus the

[18]N. Chomsky, "Three Models for the Description of Language,"

house can be designed from the outside in or from the inside out.[12]

Alternatives are also open, in organizing the design process, as to how far development of possible subsystems will be carried before the over-all coordinating design is developed in detail, or vice-versa, how far the over-all design should be carried before various components, or possible components, are developed. These alternatives of design are familiar to architects. They are familiar also to composers, who must decide how far the architectonics of a musical structure will be evolved before some of the component musical themes and other elements have been invented. Computer programmers face the same choices, between working downward from executive routines to subroutines or upward from component subroutines to a coordinating executive.

A theory of design will include principles—most of which do not yet exist—for deciding such questions of precedence and sequence in the design process.

Process as a Determinant of Style

When we recall that the process will generally be concerned with finding a satisfactory design, rather than an optimum design, we see that sequence and the division of labor between generators and tests can affect not only the efficiency with which resources for designing are used but also the nature of the final design as well. What we ordinarily call "style" may stem just as much from these decisions about the design process as from alternative emphases on the goals to be realized through the final design. An architect who designs buildings from the outside in will arrive at quite different buildings from one who designs from the inside out, even though both of them might agree on the characteristics that a satisfactory building should possess.

[12]I am indebted to John Grason for many ideas on the topic of this section. J. Grason, "Fundamental Description of a Floor Plan Design Program," EDRA1, *Proceedings of the First Environmental Design Association Conference,* H. Sanoff and S. Cohn (eds.), North Carolina State University, 1970.

When we come to the design of systems as complex as cities, or buildings, or economies, we must give up the aim of creating systems that will optimize some hypothesized utility function, and we must consider whether differences in style of the sort I have just been describing do not represent highly desirable variants in the design process rather than alternatives to be evaluated as "better" or "worse." Variety, within the limits of satisfactory constraints, may be a desirable end in itself, among other reasons, because it permits us to attach value to the search as well as its outcome—to regard the design process as itself a valued activity for those who participate in it.

We have usually thought of city planning as a means whereby the planner's creative activity could build a system that would satisfy the needs of a populace. Perhaps we should think of city planning as a valuable creative activity in which many members of a community can have the opportunity of participating—if we have wits to organize the process that way. I shall have more to say on these topics in the next chapter.

However that may be, I hope I have illustrated sufficiently that both the shape of the design and the shape and organization of the design process are essential components of a theory of design. These topics constitute the sixth item in my proposed curriculum in design:

6. *The organization of complex structures and its implication for the organization of design processes.*

REPRESENTATION OF THE DESIGN

I have by no means surveyed all facets of the emerging science of design. In particular I have said little about the influence of problem representation on design. Although the importance of the question is recognized today, we have little systematic knowledge about it. I shall cite one example, to make clear what I mean by "representation."

Here are the rules of a game, which I shall call number scrabble. The game is played by two people with nine cards—let us say the ace through the nine of hearts. The cards are placed in a row, face up, between the two players. The players draw alternately, one at a time, selecting any one of the cards that remain in the center. The aim of the game is for a player to make up a "book," that is, a set of exactly three cards whose spots add to 15, before his opponent can do so. The first player who makes a book wins; if all nine cards have been drawn without either player making a book, the game is a draw.

What is a good strategy in this game? How would you go about finding one? If the reader has not already discovered it for himself, let me show how a change in representation will make it easy to play the game well. The magic square here, which I introduced in the third chapter, is made up of the numerals from 1 through 9.

4	9	2
3	5	7
8	1	6

Each row, column, or diagonal adds to 15, and every triple of these numerals that add to 15 is a row, column, or diagonal of the magic square. From this, it is obvious that "making a book" in number scrabble is equivalent to getting "three in a row" in the game of tic-tac-toe. But most people know how to play tic-tac-toe well, hence can simply transfer their usual strategy to number scrabble.[13]

[13]Number scrabble is not the only isomorph of tic-tac-toe. John A. Michon has described another, JAM, which is the dual of tic-tac-toe in the sense of projective geometry. That is, the rows, columns, and diagonals of tic-tac-toe become points in JAM, and the squares of the former become line segments joining the points. The game is won by "jamming" all the segments through a point—a move consists of seizing or jamming a single segment. Other isomorphs of tic-tac-toe are known as well.

Problem Solving as Change in Representation

That representation makes a difference is a long-familiar point. We all believe that arithmetic has become easier since Arabic numerals and place notation replaced Roman numerals, although I know of no theoretic treatment that explains why.[14]

That representation makes a difference is evident for a different reason. All mathematics exhibits in its conclusions only what is already implicit in its premises, as I mentioned in a previous chapter. Hence all mathematical derivation can be viewed simply as change in representation, making evident what was previously true but obscure.

This view can be extended to all of problem solving—solving a problem simply means representing it so as to make the solution transparent.[15] If the problem solving could actually be organized in these terms, the issue of representation would indeed become central. But even if it cannot—if this is too exaggerated a view—a deeper understanding of how representations are created and how they contribute to the solution of problems will become an essential component in the future theory of design.

Spatial Representation

Since much of design, particularly architectural and engineering design, is concerned with objects or arrangements in real Euclidean two-dimensional or three-dimensional space, the representation of space and of things in space will necessarily be a central topic in a science of design. From our previous discussion of visual perception, it should be clear that "space" inside the head of the designer or the memory of a computer may have very different properties from a picture on paper or a three-dimensional model.

These representational issues have already attracted the

[14]My colleague, Allen Newell, has been investigating this question. I shall not try to anticipate his answer.

[15]Saul Amarel, "On the Mechanization of Creative Processes," IEEE *Spectrum* 3(April 1966):112–114.

attention of those concerned with computer-aided design—the cooperation of human and computer in the design process. As a single example, I may mention Ivan Sutherland's SKETCHPAD program, which allows geometric shapes to be represented and conditions to be placed on these shapes in terms of constraints, to which they then conform.[16]

Geometric considerations are also prominent in the attempts to automate completely the design, say, of printed or etched circuits, or of buildings. Grason, for example, in a system for designing house floor plans, constructs an internal representation of the layout that helps one decide whether a proposed set of connections among rooms, selected to meet design criteria for communication, and so on, can be realized in a plane.[17]

The Taxonomy of Representation

An early step toward understanding any set of phenomena is to learn what kinds of things there are in the set—to develop a taxonomy. This step has not yet been taken with respect to representations. We have only a sketchy and incomplete knowledge of the different ways in which problems can be represented and much less knowledge of the significance of the differences.

In a completely pragmatic vein we know that problems can be described verbally, in natural language. They often can be described mathematically, using standard formalisms of algebra, geometry, set theory, analysis, or topology. If the problems relate to physical objects, they (or their solutions) can be represented by floor plans, engineering drawings, renderings, or three-dimensional models. Prob-

[16] I. E. Sutherland, "SKETCHPAD, A Man-Machine Graphical Communication System," *Proceedings, AFIPS Spring Joint Computer Conference, 1963* (Baltimore: Spartan Books), pp. 329–346.

[17] See also C. E. Pfefferkorn, "The Design Problem Solver: A System for Designing Equipment or Furniture Layouts," in C. M. Eastman (ed.), *Spatial Synthesis in Computer-Aided Building Design* (London: Applied Science Publishers, 1975).

lems that have to do with actions can be attacked with flow charts and programs.

Other items most likely will need to be added to the list, and there may exist more fundamental and significant ways of classifying its members. But even though our classification is incomplete, and perhaps superficial, we can begin to build a theory of the properties of these representations. A number of topics in the growing theories of machines and of programming languages may give us some notion of the directions that a theory of representations—at least on its more formal side—may take.[18] These topics can also provide, at the beginning, some of the substance for the final subject in our program on the theory of design:

7. *Alternative representations for design problems.*

SUMMARY—TOPICS IN THE THEORY OF DESIGN

My main goal in this chapter has been to show that there already exist today a number of components of a theory of design and a substantial body of knowledge, theoretical and empirical, relating to each. As we draw up our curriculum in design—in the science of the artificial—to take its place by the side of natural science in the whole engineering curriculum, it includes at least the following topics:

THE EVALUATION OF DESIGNS

1. Theory of evaluation: utility theory, statistical decision theory
2. Computational methods:
   a. Algorithms for choosing *optimal* alternatives such as linear programming computations, control theory, dynamic programming

[18] By way of example, see Marvin L. Minsky, *Computation: Finite and Infinite Machines* (Englewood Cliffs, N.J.: Prentice-Hall, 1967), and Kenneth E. Iverson, *A Programming Language* (New York: Wiley, 1962).