CS171, Intro to AI — Final Exam — Fall 2016

Name:____

ID #:________________________________ **Seat (Row, #):**__________________________

The exam will begin on the next page. **Please DO NOT turn the page until told.**

When you are told to begin the exam, please check first to make sure you have all **10** pages, numbered 1–10 at the bottom of each page. If your exam is incomplete, we will supply a new copy. The exam is printed on both sides; remember to work **both sides** of each page.

The exam is **closed notes, closed book**. No calculators, cell phones or electronics.

Please turn off cell phones now.

Please clear your desk entirely, except for a pen, pencil, eraser, a blank piece of paper (for scratch use) and an optional water bottle. Please write your name and ID # on the blank piece of paper and **turn it in with your exam.**

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1. (12 points) For each of the following terms on the left, write in the letter corresponding to the best answer or the correct definition on the right.

C	Syntax	B	Chain of inference rule conclusions leading to a desired sentence.
I	Semantics	C	Specifies all the sentences in a language that are well formed.
L	Entailment	D	Describes a sentence that is true in all models.
J	Sound	E	Stands for a proposition that can be true or false.
K	Complete	F	Represented as a canonical conjunction of disjunctions.
E	Propositional Symbol	G	Possible world that assigns TRUE or FALSE to each proposition.
D	Valid	H	Describes a sentence that is false in all models.
M	Satisfiable	I	Defines truth of each sentence with respect to each possible world.
H	Unsatisfiable	J	An inference procedure that derives only entailed sentences.
B	Proof	K	An inference procedure that derives all entailed sentences.
G	Model	L	The idea that a sentence follows logically from other sentences.
F	Conjunctive Normal Form	M	Describes a sentence that is true in some model.

2. (10 points) **Conversion to CNF.** Convert this Propositional Logic wff (well-formed formula) to Conjunctive Normal Form and simplify. Show your work (correct result without work = 0 pts).

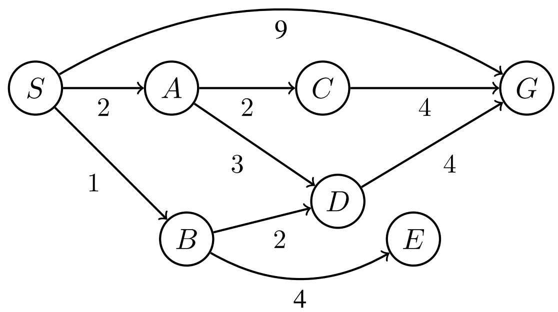
$$(P \Leftrightarrow Q) \Rightarrow (\neg Q \wedge R)$$

(one valid solution; if you failed to produce the correct answer, partial credit will be awarded up to 5 points, 1 point per useful conversion.)

$$((P \wedge Q) \vee (\neg P \wedge \neg Q)) \Rightarrow (\neg Q \wedge R)$$
$$\neg((P \wedge Q) \vee (\neg P \wedge \neg Q)) \vee (\neg Q \wedge R)$$
$$(\neg(P \wedge R) \wedge \neg(\neg P \wedge \neg Q)) \vee (\neg Q \wedge R)$$
$$((\neg P \vee \neg Q) \wedge (P \vee Q)) \vee (\neg Q \wedge R)$$
$$(\neg P \vee \neg Q \vee \neg Q) \wedge (P \vee Q \vee \neg Q) \wedge (\neg P \vee \neg Q \vee R) \wedge (P \vee Q \vee R)$$
$$(\neg P \vee \neg Q) \wedge (\neg P \vee \neg Q \vee R) \wedge (P \vee Q \vee R)$$
$$(\neg P \vee \neg Q) \wedge (P \vee Q \vee R)$$

3. (12 points) **Heuristic search.** Execute "Tree Search" through this graph (i.e., do not remember visited nodes). Step costs are given next to each arc. Heuristic values are given in the table (as $h(n) = X$). The successors of each node are indicated by the arrows out of that node. Successors are returned in left-to-right (equivalent: alphabetical) order.

   For each search strategy below, show the order in which nodes are expanded (i.e., to expand a node means that its children are generated), from start node S and ending with the goal node (G) that is found. Show the path from start to goal, or write "None". Give the cost of the path found.



$h(S) = 6$
$h(A) = 0$
$h(B) = 6$
$h(C) = 4$
$h(D) = 1$
$h(E) = 10$
$h(G) = 0$

   (a) (2 points) **Breadth first search**

   Order of node expansion: S(G)

   Path found: SG Cost of path found: 9

   (b) (2 points) **Uniform cost search**

   Order of node expansion: SBADCED(G)

   Path found: SBDG Cost of path found: 7

   (c) (2 points) **Greedy (best-first) search**

   Order of node expansion: S(G)

   Path found: SG Cost of path found: 9

   (d) (2 points) **A* search**

   Order of node expansion: SADBD(G)

   Path found: SBDG Cost of path found: 7

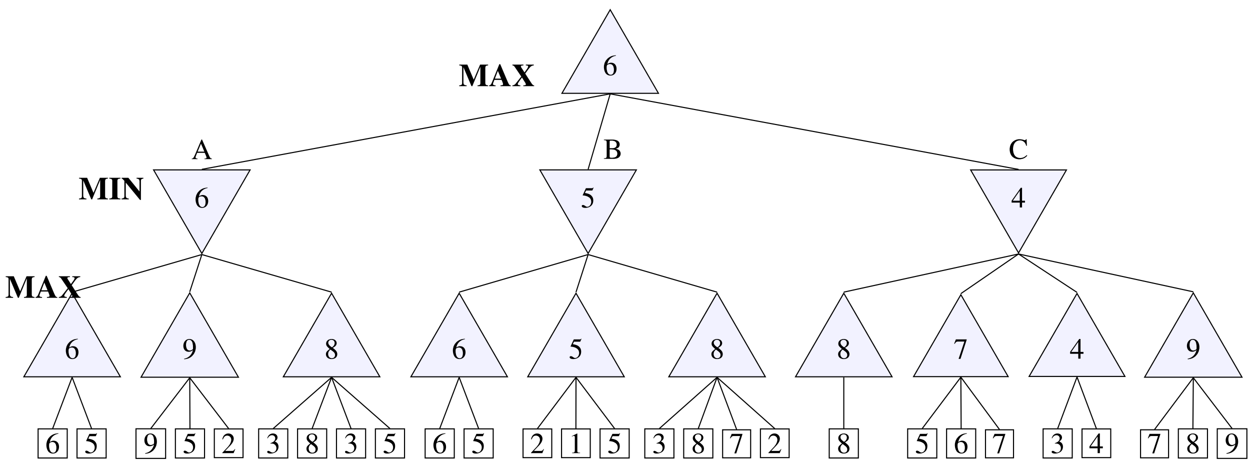
   (e) (2 points) **Admissible.** Is the heuristic $h(n)$ admissible? (Y=Yes, N=No): Y

   (f) (2 points) **Consistent.** Is the heuristic $h(n)$ consistent? (Y=Yes, N=No): N

4. (14 points) **Minimax search** The game tree below illustrates a position reached in the game. Process the tree left-to-right. It is Max's turn to move. At each leaf node is the estimated score returned by the heuristic static evaluator.

   (a) (5 points) Fill in each blank triangle with the proper mini-max search value.

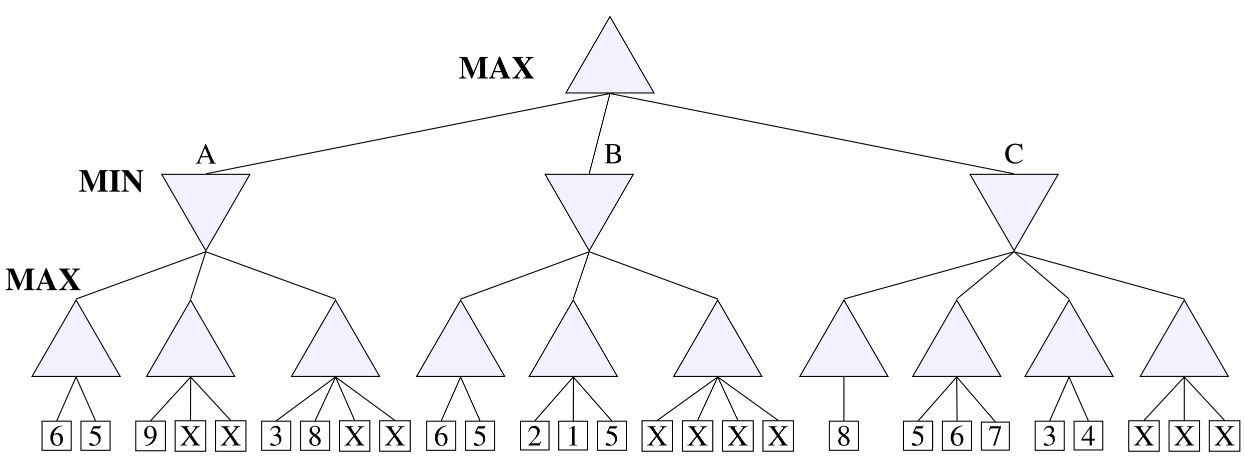
   (b) (2 points) What is MAX's best move (write A or B or C)? : A



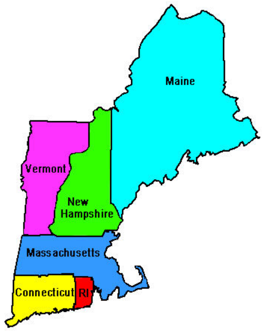
Alpha-Beta Pruning Search. This is the same tree as above. You do not need to indicate the branch node values again. Cross out each leaf node that will be pruned by Alpha-Beta Pruning.

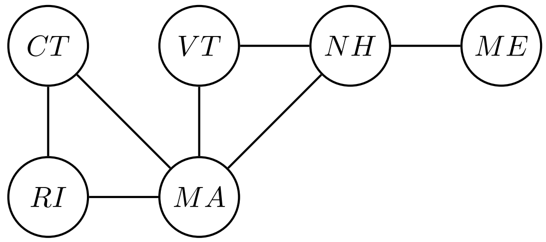
   (c) (5 points) Mark **"X"** in each box corresponding to a *pruned* leaf node.

   (d) (2 points) What score does Max expect to achieve? : 6



5. (10 points) **Constraint Satisfaction** You are a map-coloring robot assigned to color this New England USA map. Adjacent regions must be colored a different color (R=Red, B=Blue, G=Green). The constraint graph is shown.





(a) (2 points) FORWARD CHECKING. Cross out all values that would be eliminated by Forward Checking, after variable MA has just been assigned value R as shown:

CT	RI	MA	VT	NH	ME
G B	G B	R	G B	G B	R G B

(b) (2 points) ARC CONSISTENCY. CT and RI have been assigned values, but no constraint propagation has been done. Cross out all values that would be eliminated by Arc Consistency (AC-3 in your book).

CT .	RI .	MA	VT	NH	ME
R	G	B	R G	R G	R G B

(c) (2 points) MINIMUM-REMAINING-VALUES HEURISTIC. Consider the assignment below. RI is assigned and constraint propagation has been done. List all unassigned variables that might be selected by the Minimum-Remaining-Values (MRV) Heuristic: CT, MA

CT	RI	MA	VT	NH	ME
R B	G	R B	R G B	R G B	R G B

(d) (2 points) DEGREE HEURISTIC. Consider the assignment below. (It is the same assignment as in problem c above.) RI is assigned and constraint propagation has been done. List all unassigned variables that might be selected by the Degree Heuristic: MA

CT	RI	MA	VT	NH	ME
R B	G	R B	R G B	R G B	R G B

(e) (2 points) MIN-CONFLICTS HEURISTIC. Consider the complete but inconsistent assignment below. MA has just been selected to be assigned a new value during local search for a complete and consistent assignment. List all values that could be chosen for MA by the Min-Conflicts Heuristic.R

CT	RI	MA	VT	NH	ME
B	G	?	G	G	B

6. (10 points) **The Horned And Magical Unicorn.** If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is either immortal or a mammal, then it is horned. The unicorn is magical if it is horned. Prove that the unicorn is both horned and magical. Use these propositional variables ("immortal" = "not mortal"):

$Y =$ unicorn is mYthical	$R =$ unicorn is moRtal	$M =$ unicorn is a maMmal
$H =$ unicorn is Horned	$G =$ unicorn is maGical	

You have translated your goal sentence,"horned and magical," into $(H \land G)$, so the negated goal is:

$$(\neg H \lor \neg G)$$

You have translated the English sentences into a propositional logic Knowledge Base (KB):

$(\neg Y \lor \neg R)$	$(Y \lor R)$	$(Y \lor M)$
$(R \lor H)$	$(\neg M \lor H)$	$(\neg H \lor G)$

Produce a resolution proof, using KB and the negated goal, that the unicorn is horned and magical. Repeatedly choose two clauses, write one clause in the first blank space on a line, and the other clause in the second. Apply resolution to them. Write the resulting clause in the third blank space, and insert it into the knowledge base. Continue until you produce ( ). If you cannot produce ( ), then you have made a mistake. The shortest proof I know of is only six lines. It is OK to use more lines, if your proof is correct.

(one valid solution, if you failed to produce the correct proof, partial credit will be awarded up to 5 points, 1 point per useful conversion.)

Resolve $\neg H \lor \neg G$ with $\neg H \lor G$ to produce $\neg H$

Resolve $\neg H$ with $\neg M \lor H$ to produce $\neg M$

Resolve $\neg M$ with $Y \lor M$ to produce Y

Resolve Y with $\neg Y \lor \neg R$ to produce $\neg R$

Resolve $\neg H$ with $R \lor H$ to produce R

Resolve $\neg R$ with R to produce $()$

7. (10 points) **English to FOL Conversion.** For each English sentence below, write the FOL sentence that best expresses its intended meaning. Use Cat(x) for "x is a cat," Mouse(x) for "x is mouse," and Chases(x, y) for "x chases y." The first one is done for you as an example.

   (a) (0 points) "Every cat chases every mouse."

   $$\forall x \, \forall y \, \big[Cat(x) \land Mouse(y)\,\big] \Rightarrow Chases(x, y)$$

   (b) (2 points) "For every cat, there is a mouse that the cat chases."

   $$\forall x \exists y \quad Cat(x) \Rightarrow Mouse(y) \land Chages(x, y)$$

   (c) (2 points) "There is a cat who chases every mouse."

   $$\exists x \forall y \quad Cat(x) \land \big[Mouse(y) \Rightarrow Chases(x, y)\big]$$

   (d) (2 points) "Some cat chases some mouse."

   $$\exists x \exists y \quad Cat(x) \land Mouse(y) \land Chases(x, y)$$

   (e) (2 points) "There is a mouse that every cat chases."

   $$\exists y \forall x \quad Mouse(y) \land \big[Cat(x) \Rightarrow Chases(x, y)\big]$$

   (f) (2 points) "For every mouse, there is a cat who chases that mouse."

   $$\forall y \exists x \quad Mouse(y) \Rightarrow Cat(x) \land Chases(x, y)$$

8. (10 points) **Probability.** Consider the following full joint distribution for Boolean variables A, B, C:

A	B	C	P(a,b,c)
t	t	t	0.08
t	t	f	0.12
t	f	t	0.24
t	f	f	0.36
f	t	t	0.04
f	t	f	0.06
f	f	t	0.04
f	f	f	0.06

Calculate the following probabilities (write a number from the interval [0,1]):

(a) (1 point) P(A = f) = .2

(b) (1 point) P(B = t) = .3

(c) (1 point) P(B = t, C = t) = .12

(d) (1 point) P(A = f, C = t) = .08

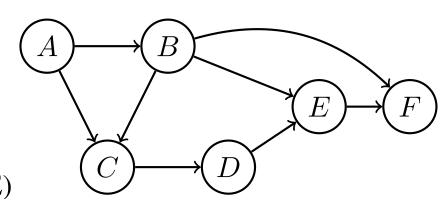
(e) (1 point) P(A = t | B = t) = 2/3

(f) (1 point) P(C = f | B = t) = 3/5

(g) (1 point) Are A and B independent of each other? (Y=Yes, N=No): N

(h) (1 point) Are B and C independent of each other? (Y=Yes, N=No): Y

(i) (2 points) Give the factored conditional probability expression corresponding to this Bayesian network:



P(A) P(B|A) P(C |AB) P(D|C) P(E|BD) P(F|BE)

9. (12 points) **Naïve Bayes** You are a robot in an animal shelter, and must learn to discriminate Dogs from Cats. You choose to learn a Naïve Bayes classifier. You are given the following (noisy) examples:

Example	**Sound**	**Fur**	**Color**	**Class**
Example #1	Meow	Coarse	Brown	Dog
Example #2	Bark	Fine	Brown	Dog
Example #3	Bark	Coarse	Black	Dog
Example #4	Bark	Coarse	Black	Dog
Example #5	Meow	Fine	Brown	Cat
Example #6	Meow	Coarse	Black	Cat
Example #7	Bark	Fine	Black	Cat
Example #8	Meow	Fine	Brown	Cat

Recall that Bayes' rule allows you to rewrite the conditional probability of the class given the attributes as the conditional probability of the attributes given the class. As usual, α is a normalizing constant that makes the probabilities sum to one.

$$P(\text{Class} \mid \text{Sound, Fur, Color}) = \alpha\, P(\text{Sound, Fur, Color} \mid \text{Class})\, P(\text{Class})$$

(a) (4 points) Now assume that the attributes (Sound, Fur, and Color) are conditionally independent given the Class. Rewrite the expression above, using this assumption of conditional independence (i.e., rewrite it as a Naïve Bayes Classifier expression).

$$\alpha\, P(\text{Sound} \mid \text{Class})\, P(\text{Fur} \mid \text{Class})\, P(\text{Color} \mid \text{Class})\, P(\text{Class})$$

(b) (4 points) Fill in numerical values for the following expressions. Leave your answers as common fractions (e.g., 1/4, 3/5).

P(Dog)=1/2

P(Cat)=1/2

P(Sound=Meow | Class=Dog)=1/4

P(Sound=Meow | Class=Cat)=3/4

P(Sound=Bark | Class=Dog)=3/4

P(Sound=Bark | Class=Cat)=1/4

P(Fur=Coarse | Class=Dog)=3/4

P(Fur=Coarse | Class=Cat)=1/4

P(Fur=Fine | Class=Dog)=1/4

P(Fur=Fine | Class=Cat)=3/4

P(Color=Brown | Class=Dog)=1/2

P(Color=Brown | Class=Cat)=1/2

P(Color=Black | Class=Dog)=1/2

P(Color=Black | Class=Cat)=1/2

(c) (4 points) Consider a new example (Sound=Bark $\wedge$ Fur=Coarse $\wedge$ Color=Brown). Write these class probabilities as the product of α and common fractions from above.

P(Class=Dog | Sound=Bark $\wedge$ Fur=Coarse $\wedge$ Color=Brown) = 9/10

P(Class=Cat | Sound=Bark $\wedge$ Fur=Coarse $\wedge$ Color=Brown) = 1/10