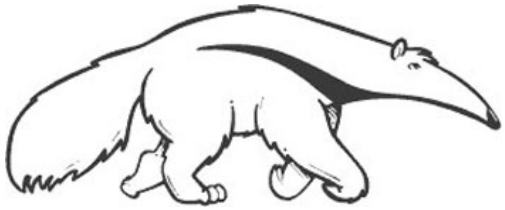
Games and Adversarial Search

CS171, Fall 2016

Introduction to Artificial Intelligence

Prof. Alexander Ihler



Types of games

	Deterministic:	Chance:
Perfect Information:	chess, checkers, go, othello	backgammon, monopoly
Imperfect Information:	battleship, Kriegspiel	Bridge, poker, scrabble, …

- Start with deterministic, perfect info games (easiest)
- Not considered:
  - Physical games like tennis, ice hockey, etc.
  - But, see "robot soccer", http://www.robocup.org/

Typical assumptions

- Two agents, whose actions alternate
- Utility values for each agent are the opposite of the other
  - "Zero-sum" game; this creates adversarial situation
- Fully observable environments

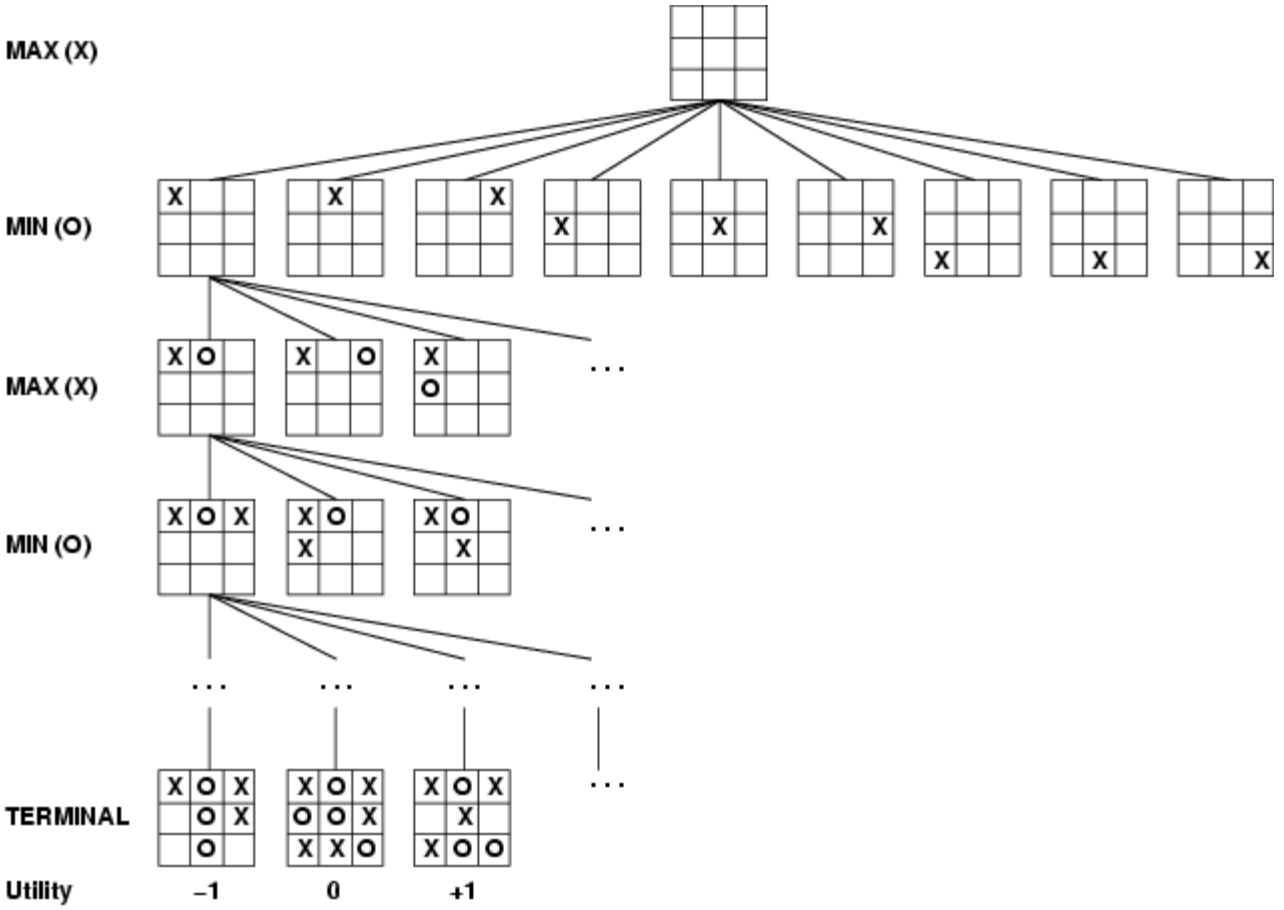
- In game theory terms:
  - Deterministic, turn-taking, zero-sum, perfect information

- Generalizes: stochastic, multiplayer, non zero-sum, etc.

- Compare to e.g., Prisoner's Dilemma" (R&N pp. 666-668)
  - Non-turn-taking, Non-zero-sum, Imperfect information

Game Tree (tic-tac-toe)

- All possible moves at each step



- How do we search this tree to find the optimal move?

Search versus Games

- Search: no adversary
  - Solution is (heuristic) method for finding goal
  - Heuristics & CSP techniques can find optimal solution
  - Evaluation function: estimate cost from start to goal through a given node
  - Examples: path planning, scheduling activities, …

- Games: adversary
  - Solution is a strategy
    - Specifices move for every possible opponent reply
  - Time limits force an approximate solution
  - Evaluation function: evaluate “goodness” of game position
  - Examples: chess, checkers, Othello, backgammon

Games as search

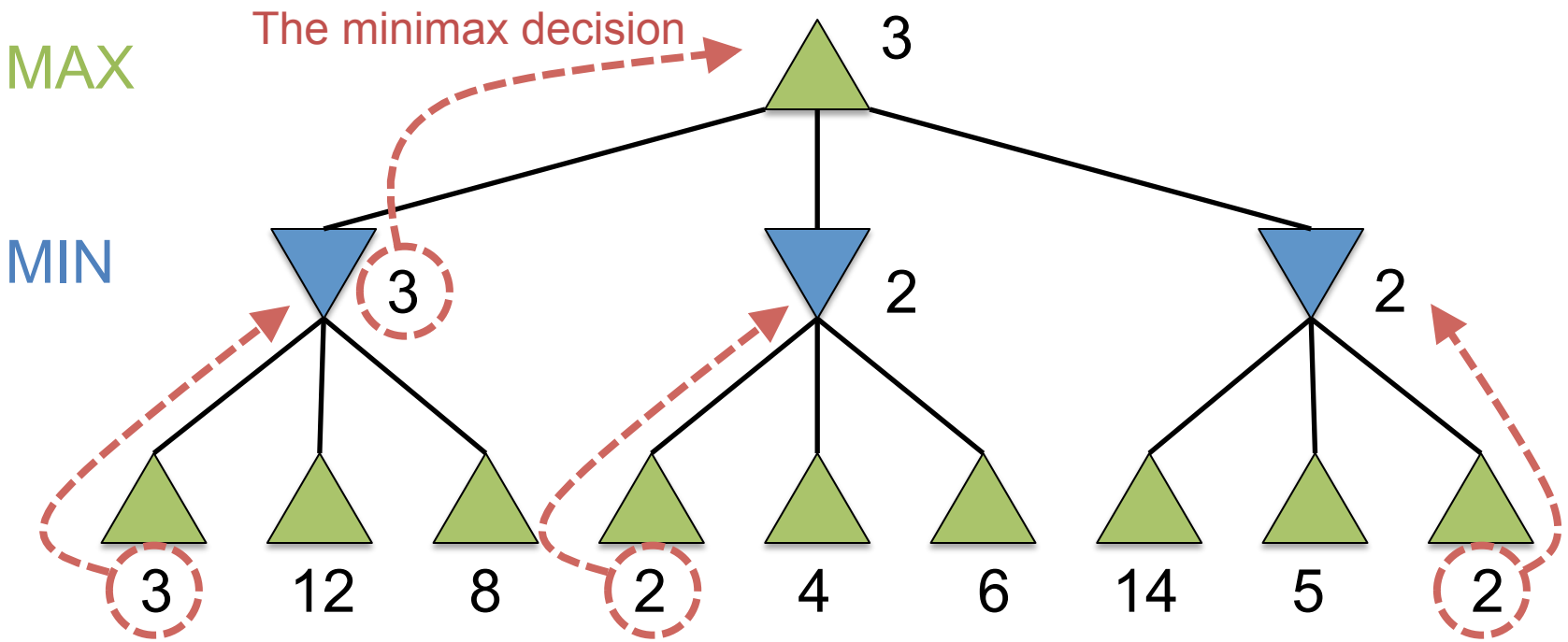
- Two players, “MAX” and “MIN”
- MAX moves first, & take turns until game is over
  - Winner gets reward, loser gets penalty
  - “Zero sum”: sum of reward and penalty is constant
- Formal definition as a search problem:
  - Initial state: set-up defined by rules, e.g., initial board for chess
  - Player(s): which player has the move in state s
  - Actions(s): set of legal moves in a state
  - Results(s,a): transition model defines result of a move
  - Terminal-Test(s): true if the game is finished; false otherwise
  - Utility(s,p): the numerical value of terminal state s for player p
    - E.g., win (+1), lose (-1), and draw (0) in tic-tac-toe
    - E.g., win (+1), lose (0), and draw (1/2) in chess
- MAX uses search tree to determine “best” next move

Min-Max: an optimal procedure

- Designed to find the optimal strategy & best move for MAX:

1. Generate the whole game tree to leaves
2. Apply utility (payoff) function to leaves
3. Back-up values from leaves toward the root:
   - a Max node computes the max of its child values
   - a Min node computes the min of its child values
4. At root: choose move leading to the child of highest value

Two-ply Game Tree



Minimax maximizes the utility of the worst-case outcome for MAX

Recursive min-max search

```
mmSearch(state)
  return argmax( [ minValue( apply(state,a) ) for each action a ] )
```

Simple stub to call recursion f'ns

```
maxValue(state)
  if (terminal(state)) return utility(state);
  v = -infty
  for each action a:
    v = max( v,  minValue( apply(state,a) ) )
 return v
```

If recursion limit reached, eval position

Otherwise, find our best child:

```
minValue(state)
  if (terminal(state)) return utility(state);
  v = infty
  for each action a:
    v = min( v,  maxValue( apply(state,a) ) )
 return v
```

If recursion limit reached, eval position

Otherwise, find the worst child:

Properties of minimax

- Complete? Yes (if tree is finite)
- Optimal?
  - Yes (against an optimal opponent)
  - Can it be beaten by a suboptimal opponent? (No – why?)
- Time? $O(b^m)$
- Space?
  - O(bm) (depth-first search, generate all actions at once)
  - O(m) (backtracking search, generate actions one at a time)

Game tree size

- Tic-tac-toe
  - B $\approx$ 5 legal actions per state on average; total 9 plies in game
    - "ply" = one action by one player; "move" = two plies
  - 5^9 = 1,953,125
  - 9! = 362,880 (computer goes first)
  - 8! = 40,320 (computer goes second)
  - Exact solution is quite reasonable

- Chess
  - B $\approx$ 35 (approximate average branching factor)
  - D $\approx$ 100 (depth of game tree for "typical" game)
  - $B^d = 35^{100} \approx 10^{154}$ nodes!!!
  - Exact solution completely infeasible

It is usually impossible to develop the whole search tree.

Cutting off search

- One solution: cut off tree before game ends
- Replace
  - Terminal(s) with Cutoff(s) – e.g., stop at some max depth
  - Utility(s,p) with Eval(s,p) – estimate position quality

- Does it work in practice?
  - $B^m = 10^6$, $b = 35 \Rightarrow m = 4$
  - 4-ply lookahead is a poor chess player
  - 4-ply $\approx$ human novice
  - 8-ply $\approx$ typical PC, human master
  - 12-ply $\approx$ Deep Blue, Kasparov

Static (Heuristic) Evaluation Functions

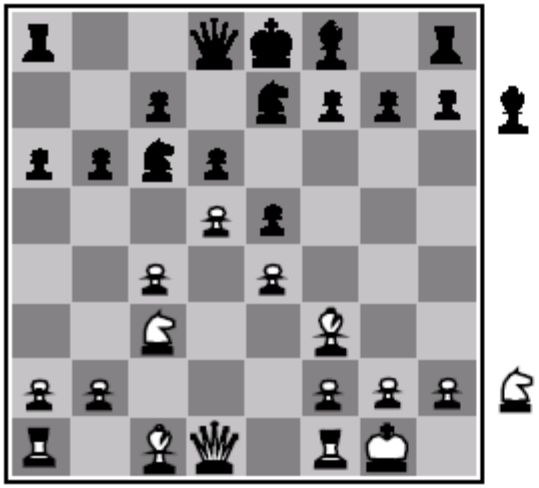
- An Evaluation Function:
  - Estimate how good the current board configuration is for a player.
  - Typically, evaluate how good it is for the player, and how good it is for the opponent, and subtract the opponent's score from the player's.
  - Often called "static" because it is called on a static board position
  - Ex: Othello: Number of white pieces - Number of black pieces
  - Ex: Chess: Value of all white pieces - Value of all black pieces

- Typical value ranges:

  $[-\infty, \infty]$ (loss/win) or $[-1 , +1]$ or $[0 , 1]$

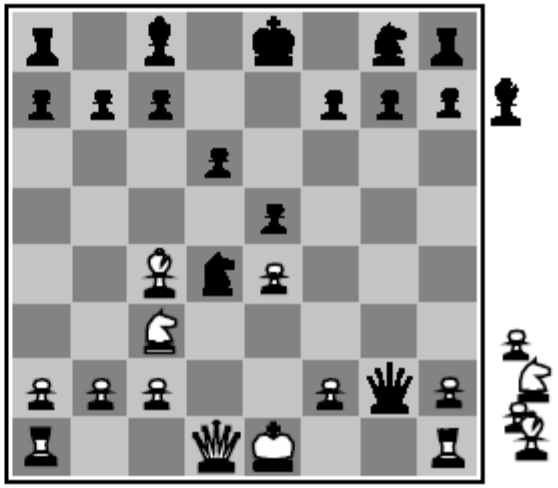
- Board evaluation: X for one player $\Rightarrow$ -X for opponent
  - Zero-sum game: scores sum to a constant

Evaluation functions



Black to move

White slightly better



White to move

Black winning

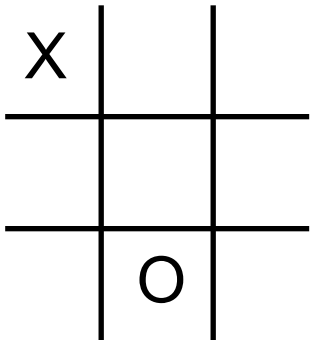
For chess, typically *linear* weighted sum of features

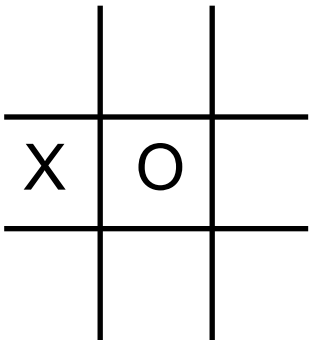
$$Eval(s) = w_1 f_1(s) + w_2 f_2(s) + \ldots + w_n f_n(s)$$

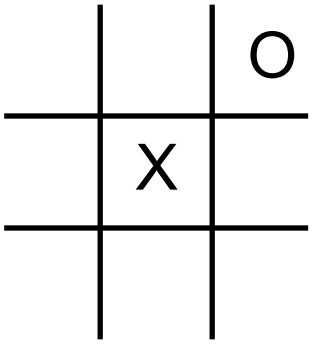
e.g., $w_1 = 9$ with
$f_1(s) =$ (number of white queens) – (number of black queens), etc.

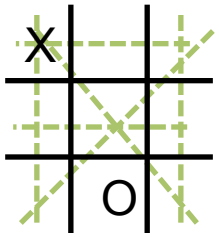
Applying minimax to tic-tac-toe

- The static heuristic evaluation function:
  - Count the number of possible win lines

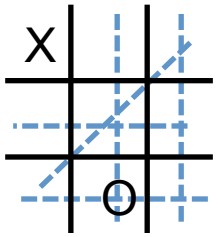








X has 6 possible win paths



O has 5 possible win paths

X has 4 possible wins
O has 6 possible wins

E(n) = 4 – 6 = -2

X has 5 possible wins
O has 4 possible wins

E(n) = 5 – 4 = 1

E(s) = 6 – 5 = 1

Minimax values (two ply)

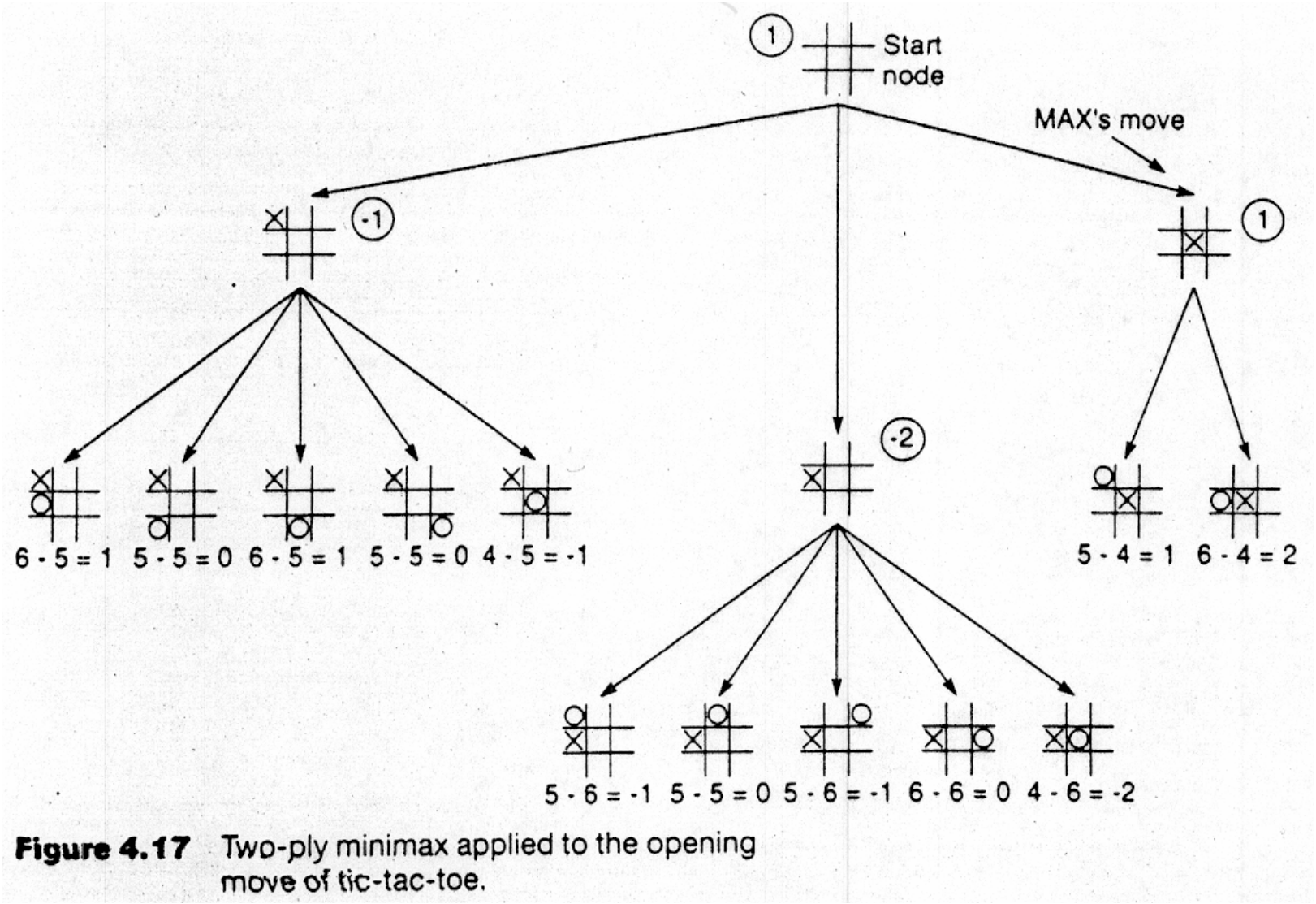


Figure 4.17 Two-ply minimax applied to the opening move of tic-tac-toe.

Minimax values (two ply)

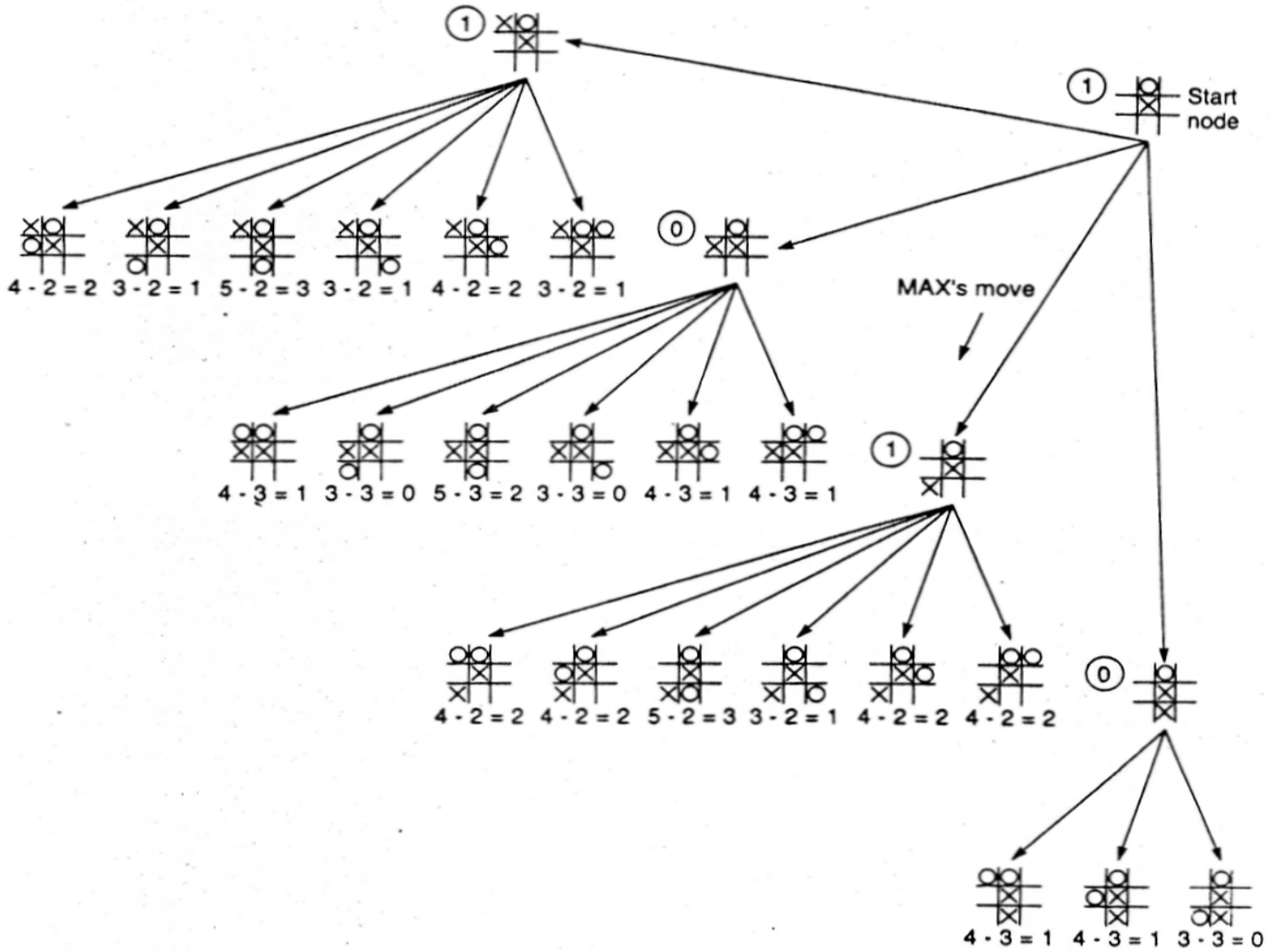


Figure 4.18 Two-ply minimax applied to **X**'s second move of tic-tac-toe.

Minimax values (two ply)

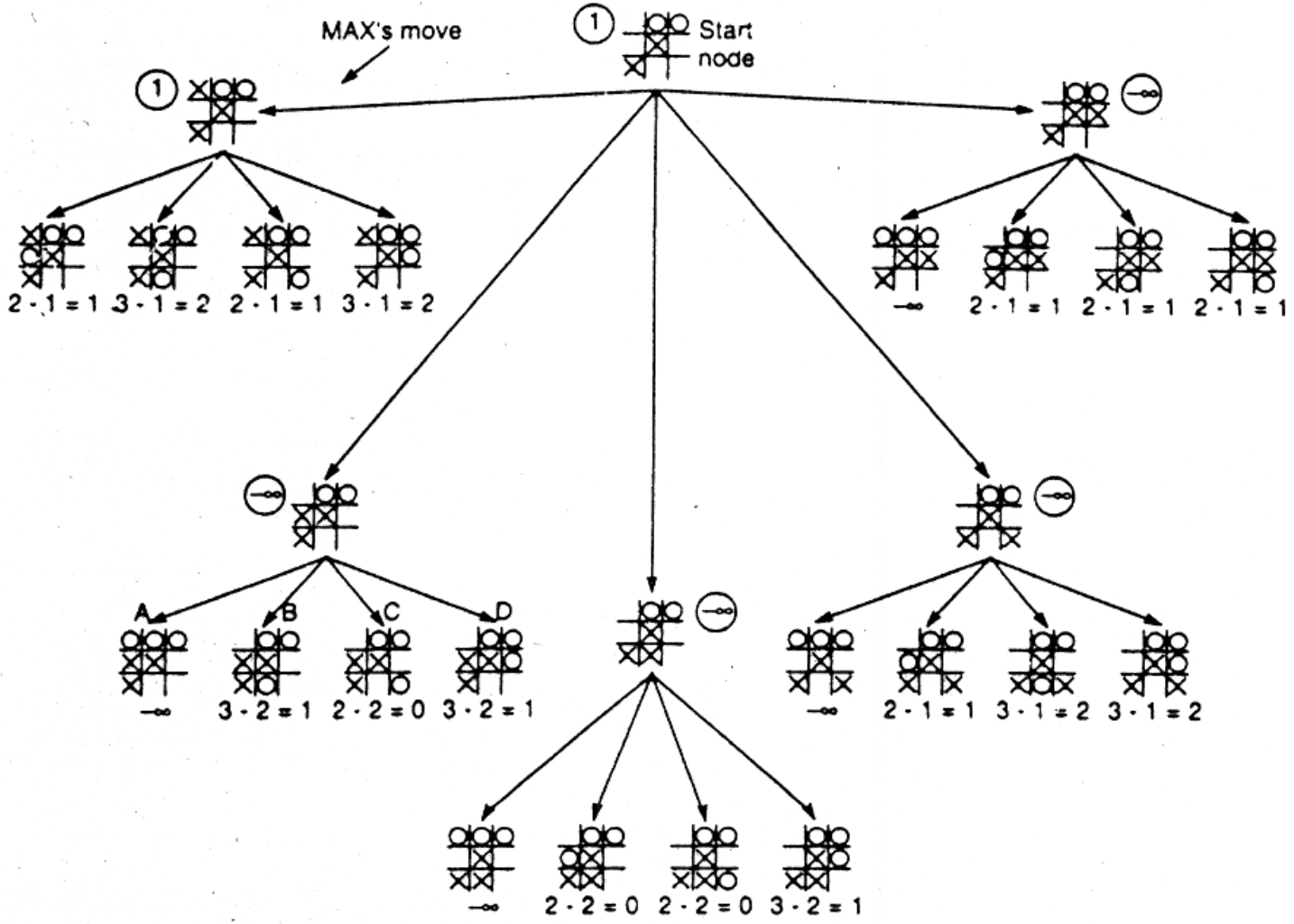
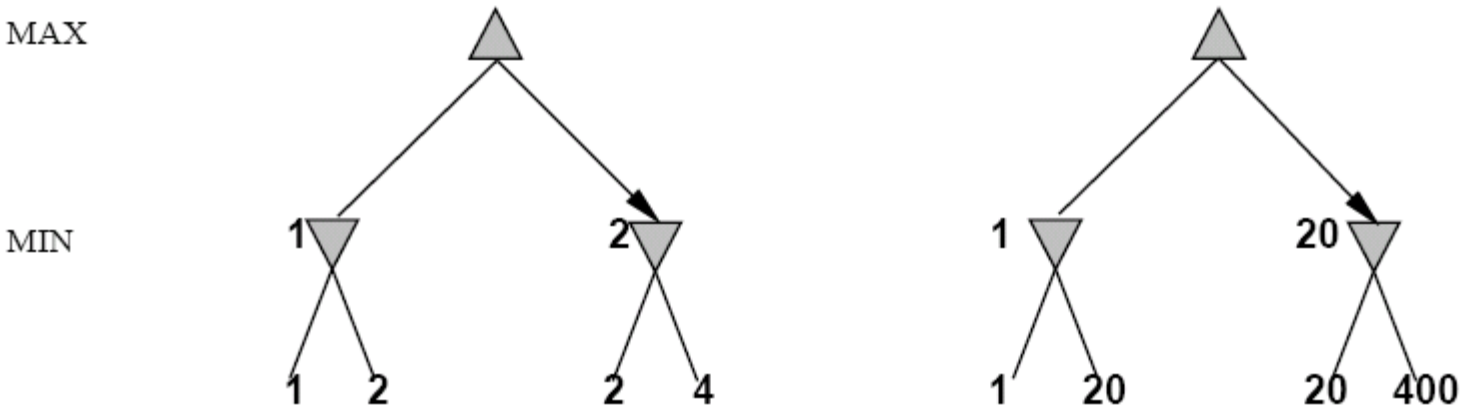


Figure 4.19 Two-ply minimax applied to **X**'s move near end game.

Digression: Exact values don't matter



Behaviour is preserved under any *monotonic* transformation of Eval

Only the order matters:

payoff in deterministic games acts as an *ordinal utility* function

Iterative deepening

- In real games, there is usually a time limit T to make a move

- How do we take this into account?
- Minimax cannot use "partial" results with any confidence, unless the full tree has been searched
  - Conservative: set small depth limit to guarantee finding a move in time < T
  - But, we may finish early – could do more search!

- In practice, iterative deepening search (IDS) is used
  - IDS: depth-first search with increasing depth limit
  - When time runs out, use the solution from previous depth
  - With alpha-beta pruning (next), we can sort the nodes based on values from the previous depth limit in order to maximize pruning during the next depth limit $\Rightarrow$ search deeper!

Limited horizon effects

- The Horizon Effect
  - Sometimes there's a major "effect" (such as a piece being captured) which is just "below" the depth to which the tree has been expanded.
  - The computer cannot see that this major event could happen because it has a "limited horizon".
  - There are heuristics to try to follow certain branches more deeply to detect such important events
  - This helps to avoid catastrophic losses due to "short-sightedness"

- Heuristics for Tree Exploration
  - Often better to explore some branches more deeply in the allotted time
  - Various heuristics exist to identify "promising" branches
  - Stop at "quiescent" positions – all battles are over, things are quiet
  - Continue when things are in violent flux – the middle of a battle

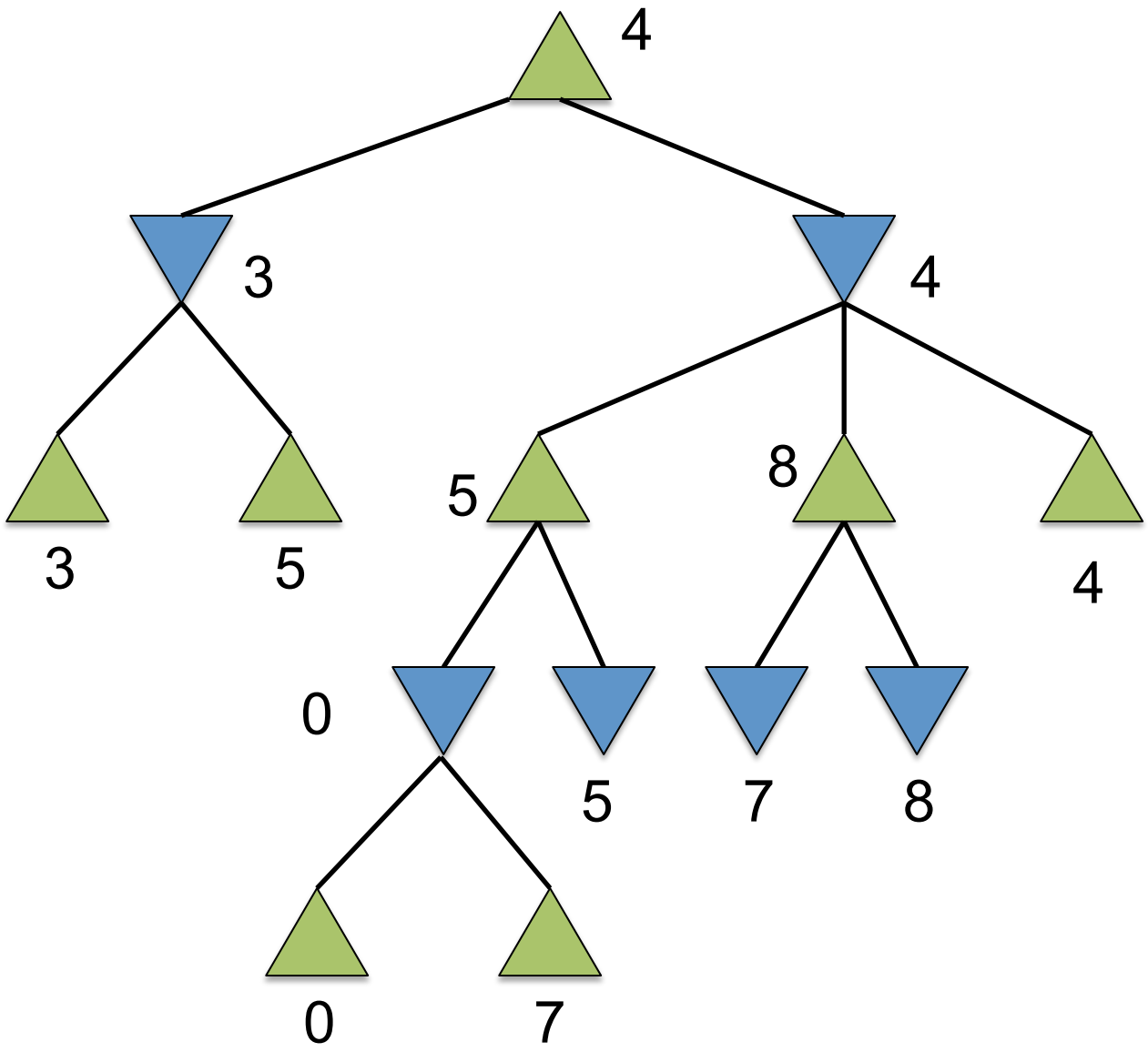
Selectively deeper game trees

MAX
(Computer's move)

MIN
(Opponent's move)

MAX
(Computer's move)

MIN
(Opponent's move)



Eliminate redundant nodes

- On average, each board position appears in the search tree approximately $10^{150} / 10^{40} = 10^{100}$ times
  - Vastly redundant search effort
- Can't remember all nodes (too many)
  - Can't eliminate all redundant nodes
- Some short move sequences provably lead to a redundant position
  - These can be deleted dynamically with no memory cost
- Example:

  1. P-QR4 P-QR4; 2. P-KR4 P-KR4

  leads to the same position as

  1. P-QR4 P-KR4; 2. P-KR4 P-QR4

Summary

- Game playing as a search problem
- Game trees represent alternate computer / opponent moves
- Minimax: choose moves by assuming the opponent will always choose the move that is best for them
  - Avoids all worst-case outcomes for Max, to find the best
  - If opponent makes an error, Minimax will take optimal advantage (after) & make the best possible play that exploits the error
- Cutting off search
  - In general, it's infeasible to search the entire game tree
  - In practice, Cutoff-Test decides when to stop searching
  - Prefer to stop at quiescent positions
  - Prefer to keep searching in positions that are still in flux
- Static heuristic evaluation function
  - Estimate the quality of a given board configuration for MAX player
  - Called when search is cut off, to determine value of position found

Games & Adversarial Search: Alpha-Beta Pruning

CS171, Fall 2016

Introduction to Artificial Intelligence

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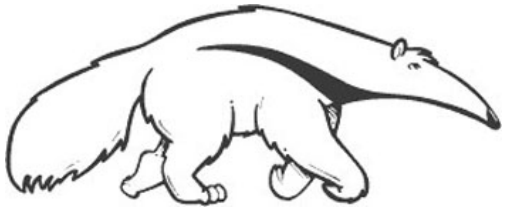

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Alpha-Beta pruning

- Exploit the “fact” of an adversary
- If a position is provably bad
  - It’s no use searching to find out just how bad
- If the adversary can force a bad position
  - It’s no use searching to find the good positions the adversary won’t let you achieve
- Bad = not better than we can get elsewhere

Pruning with Alpha/Beta

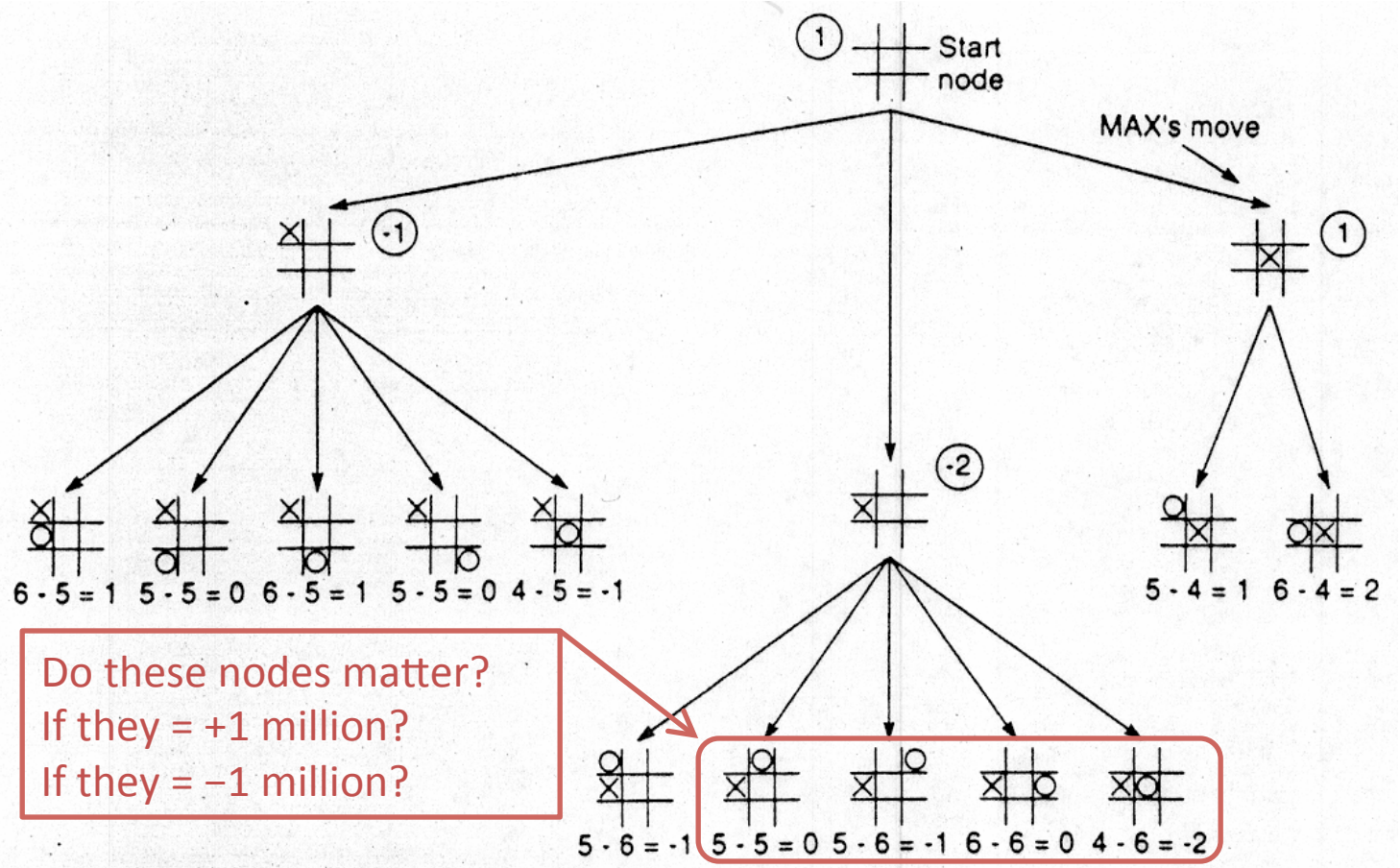
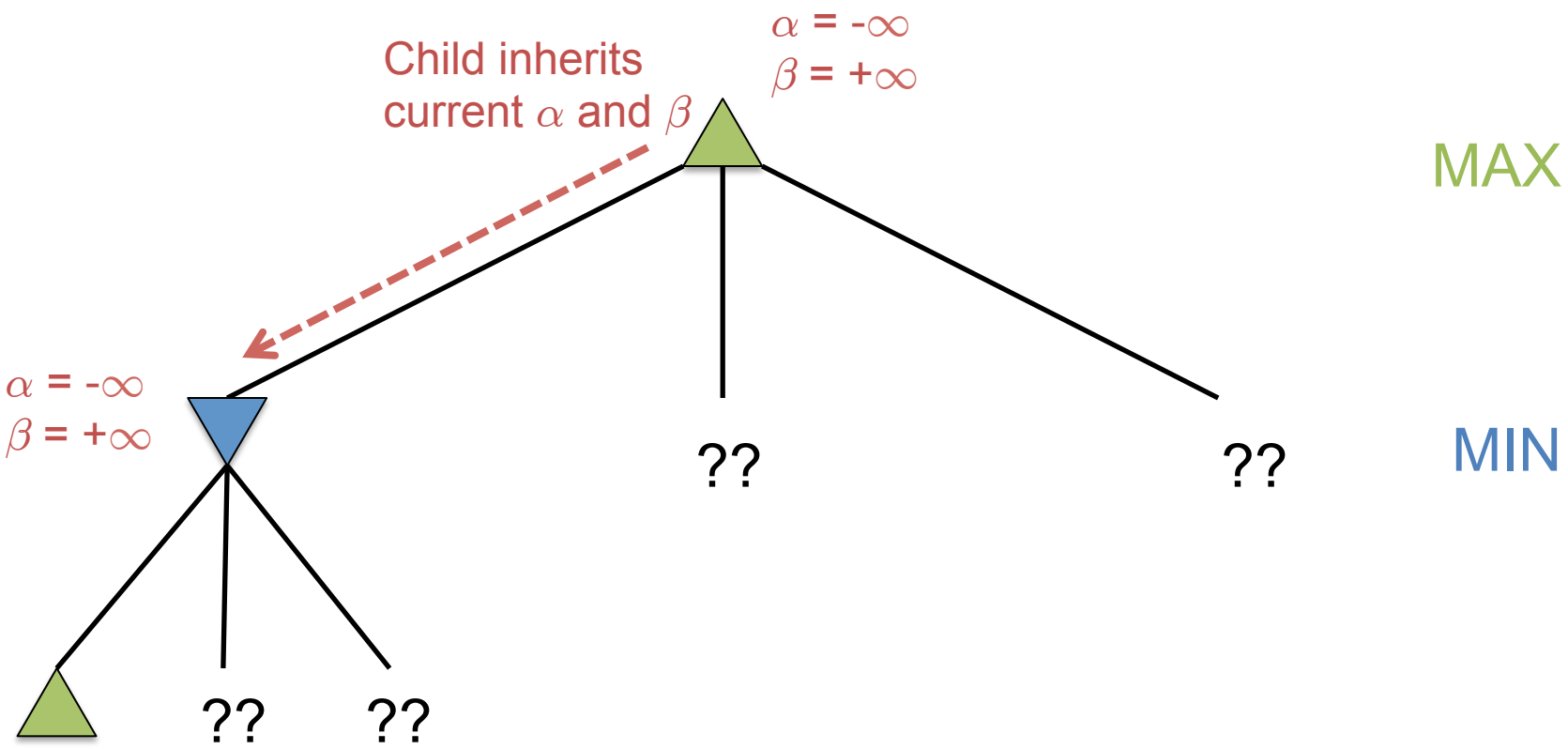


Figure 4.17 Two-ply minimax applied to the opening move of tic-tac-toe.

Alpha-Beta Example

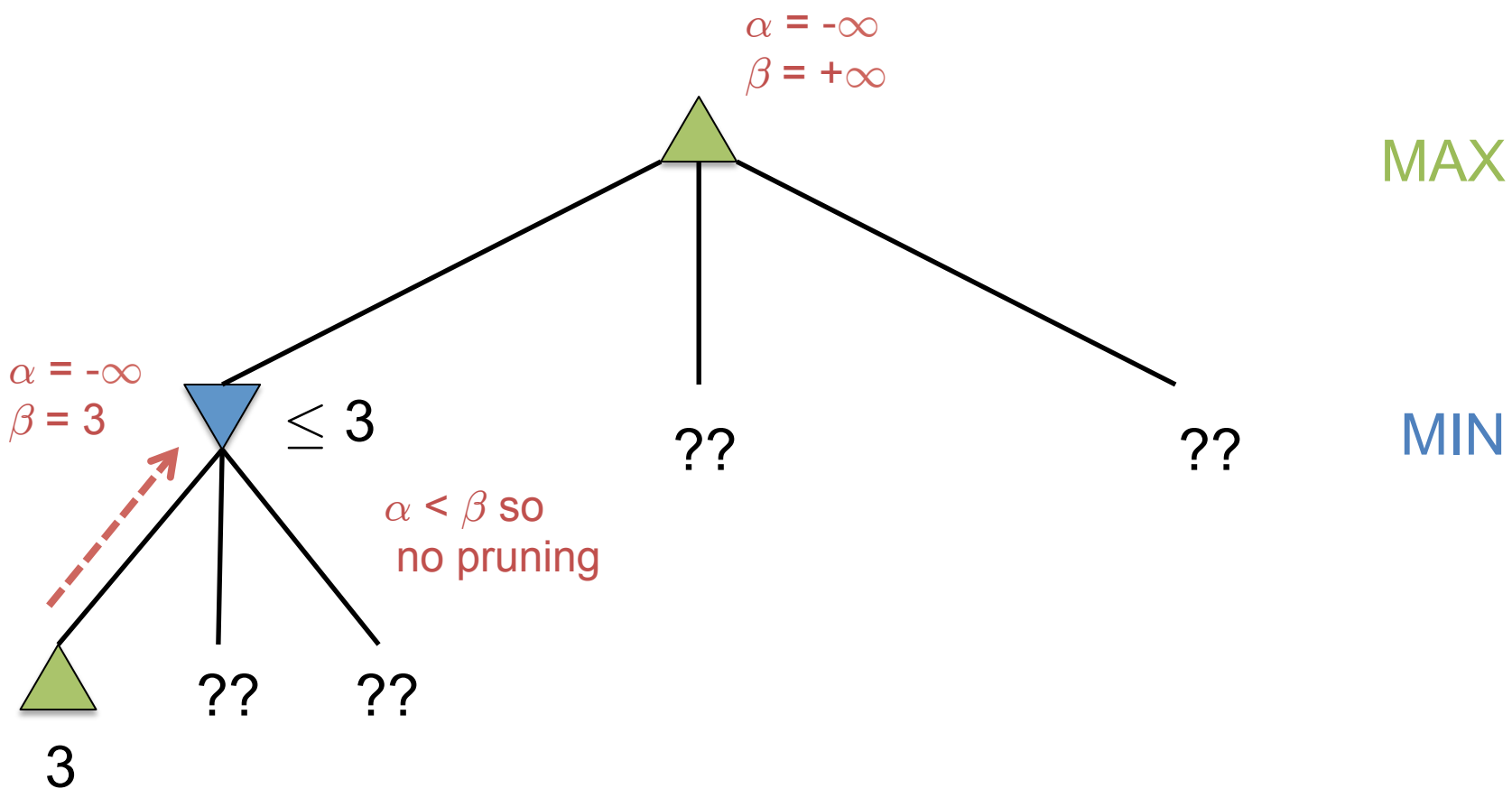
Initially, possibilities are unknown: range (α=-∞, β=+∞)

Do a depth-first search to the first leaf.



Alpha-Beta Example

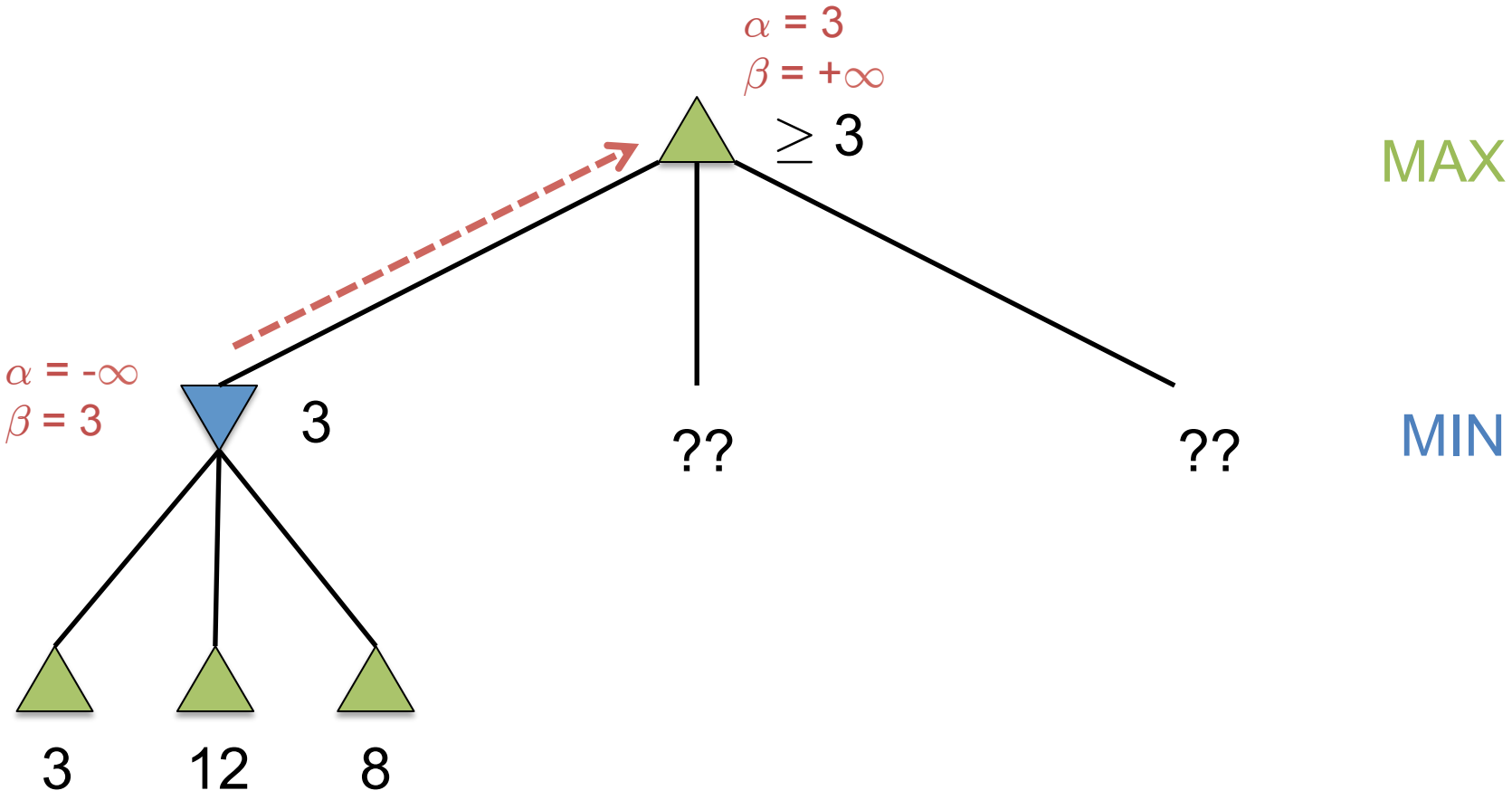
See the first leaf, after MIN's move: MIN updates β



Alpha-Beta Example

See remaining leaves; value is known

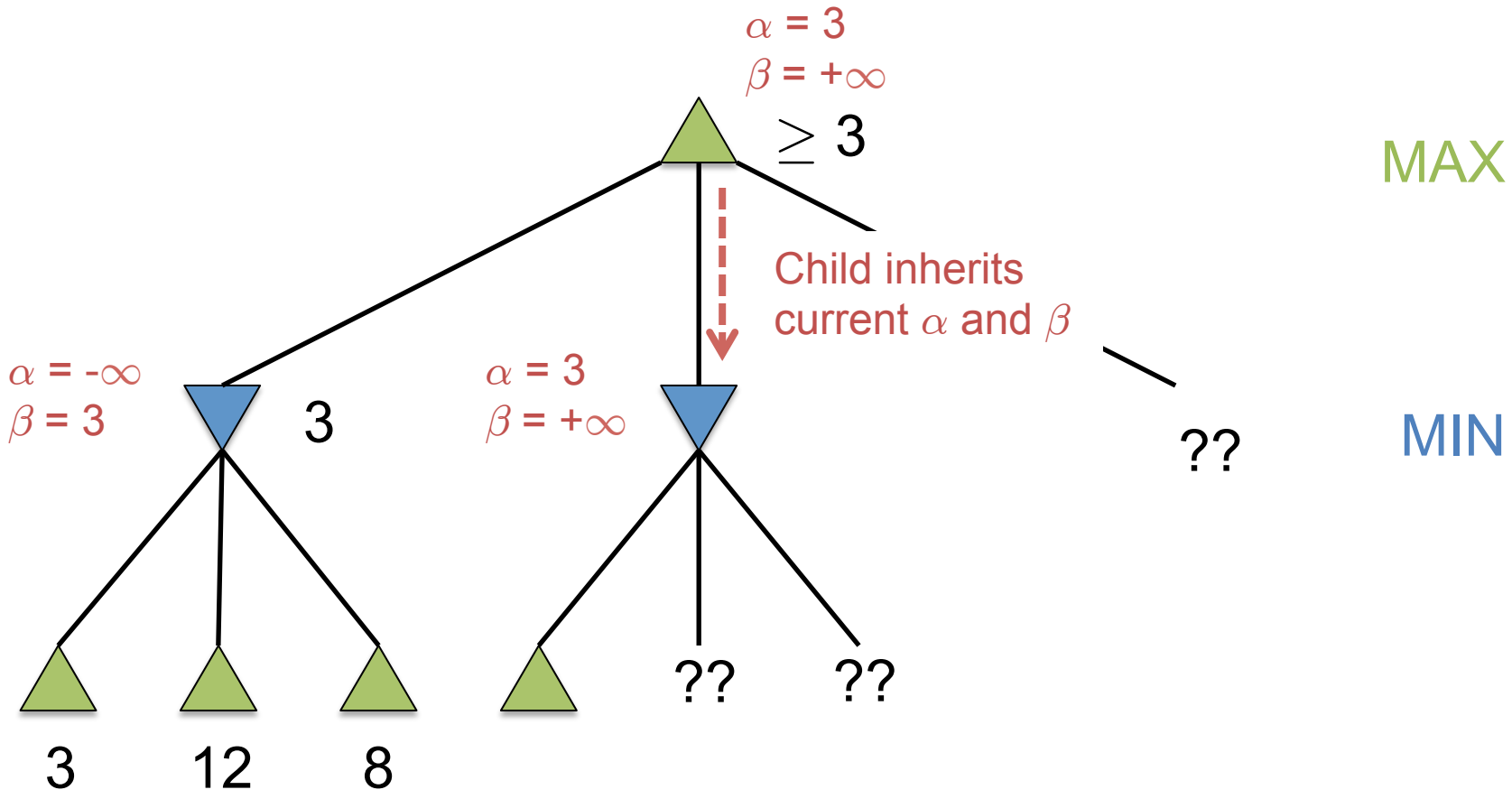
Pass outcome to caller; MAX updates α



Alpha-Beta Example

Continue depth-first search to next leaf.

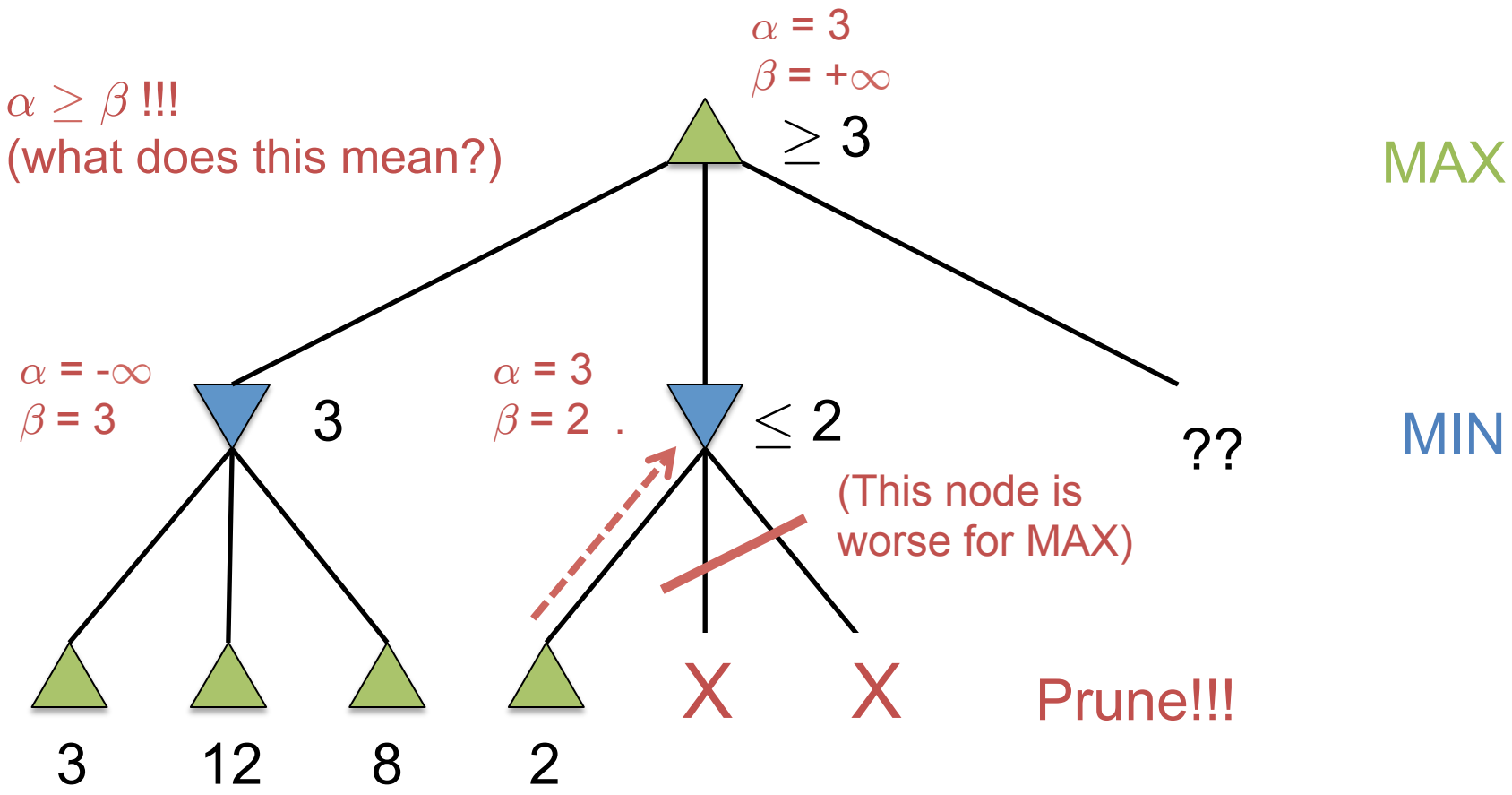
Pass α, β to descendants



Alpha-Beta Example

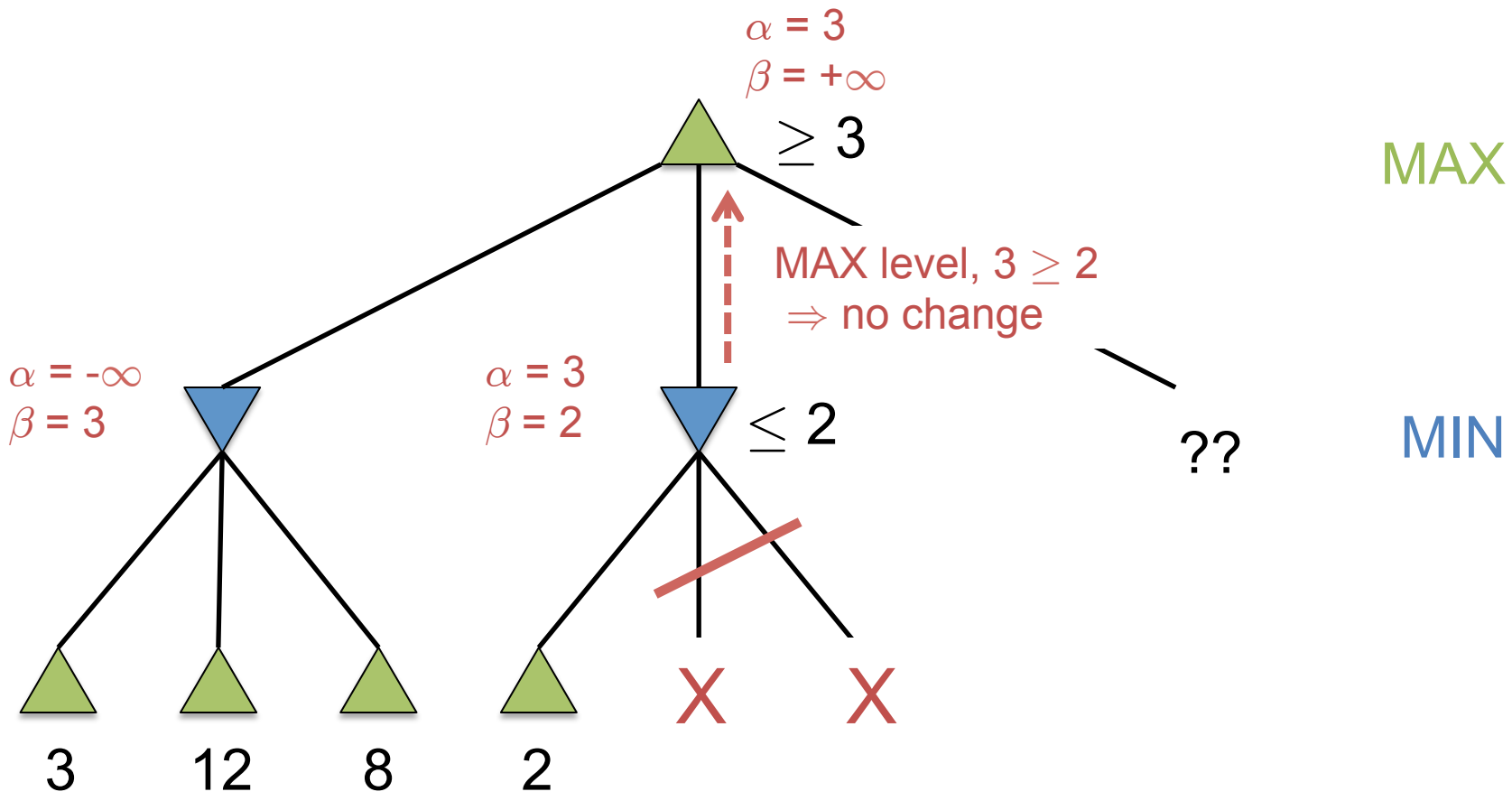
Observe leaf value; MIN's level; MIN updates \beta

Prune – play will never reach the other nodes!



Alpha-Beta Example

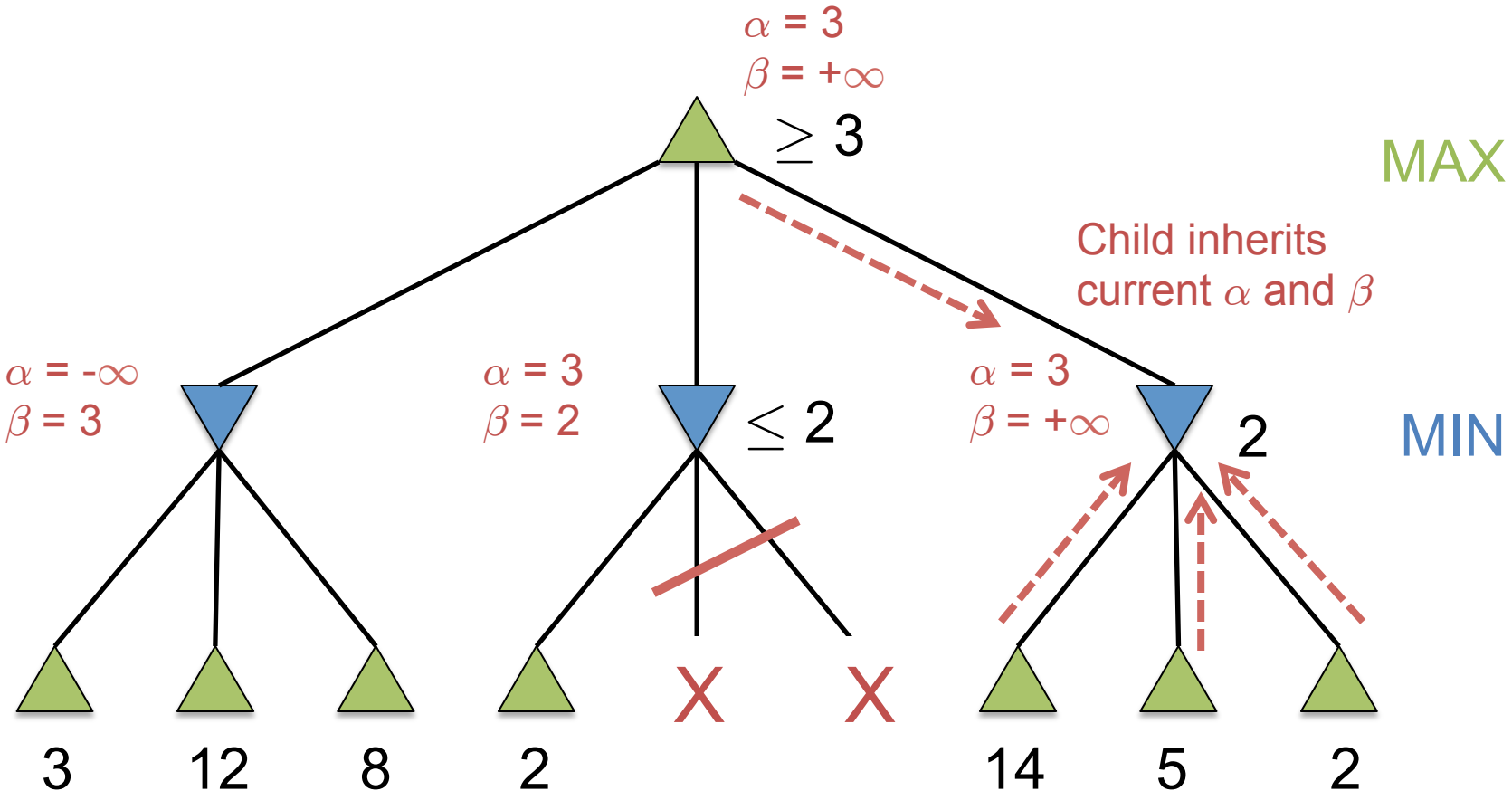
Pass outcome to caller & update caller:



Alpha-Beta Example

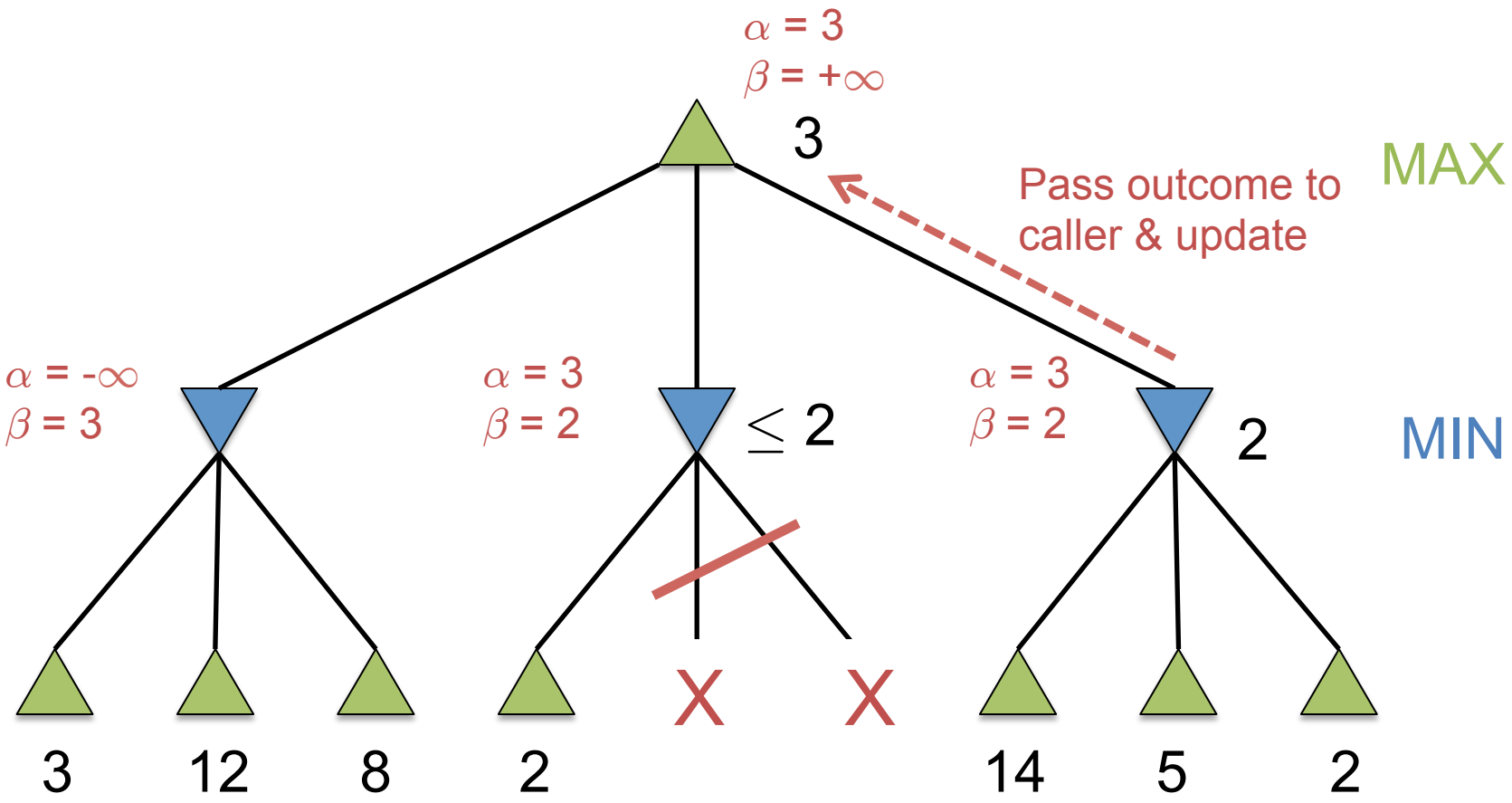
Continue depth-first exploration…

No pruning here; value is not resolved until final leaf.



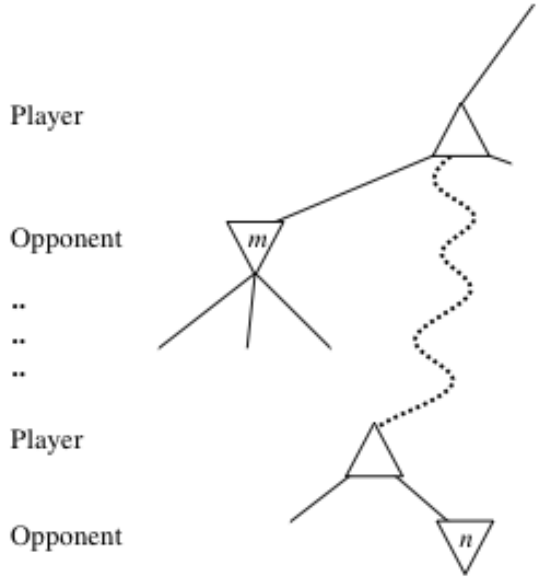
Alpha-Beta Example

Value at the root is resolved.



General alpha-beta pruning

- Consider a node n in the tree:



- If player has a better choice at
  - Parent node of n
  - Or, any choice further up!
- Then n is never reached in play

- So:
  - When that much is known about n, it can be pruned

Recursive α-β pruning

```
abSearch(state)
  alpha, beta, a  = -infty, +infty, None
  for each action a:
     alpha, a = max( (alpha,a) , (minValue( apply(state,a), alpha, beta), a) )
  return a
```

Simple stub to call recursion f' ns
Initialize alpha, beta; no move found
Score each action; update alpha & best action

```
maxValue(state, al, be)
  if (cutoff(state)) return eval(state);
  for each action a:
     al = max( al,  minValue( apply(state,a), al, be)
     if (al ≥ be)  return  +infty
  return al
```

If recursion limit reached, eval heuristic

Otherwise, find our best child:
If our options are too good, our min ancestor will never let us come this way
Otherwise return the best we can find

```
minValue(state, al, be)
  if (cutoff(state)) return eval(state);
  for each action a:
     be = min( be,  maxValue( apply(state,a), al, be)
     if (al ≥ be)  return  -infty
  return be
```

If recursion limit reached, eval heuristic

Otherwise, find the worst child:
If our options are too bad, our max ancestor will never let us come this way
Otherwise return the worst we can find

Effectiveness of α-β Search

- Worst-Case
  - Branches are ordered so that no pruning takes place. In this case alpha-beta gives no improvement over exhaustive search

- Best-Case
  - Each player's best move is the left-most alternative (i.e., evaluated first)
  - In practice, performance is closer to best rather than worst-case

- In practice often get $O(b^{(d/2)})$ rather than $O(b^d)$
  - This is the same as having a branching factor of sqrt(b),
    - since $(\text{sqrt}(b))^d = b^{(d/2)}$ (i.e., we have effectively gone from b to square root of b)
  - In chess go from b ~ 35 to b ~ 6
    - permiting much deeper search in the same amount of time

Iterative deepening

- In real games, there is usually a time limit T to make a move

- How do we take this into account?
- Minimax cannot use "partial" results with any confidence, unless the full tree has been searched
  - Conservative: set small depth limit to guarantee finding a move in time < T
  - But, we may finish early – could do more search!

- Added benefit with Alpha-Beta Pruning:
  - Remember node values found at the previous depth limit
  - Sort current nodes so that each player's best move is left-most child
  - Likely to yield good Alpha-Beta Pruning $\Rightarrow$ better, faster search
  - Only a heuristic: node values will change with the deeper search
  - Usually works well in practice

Comments on alpha-beta pruning

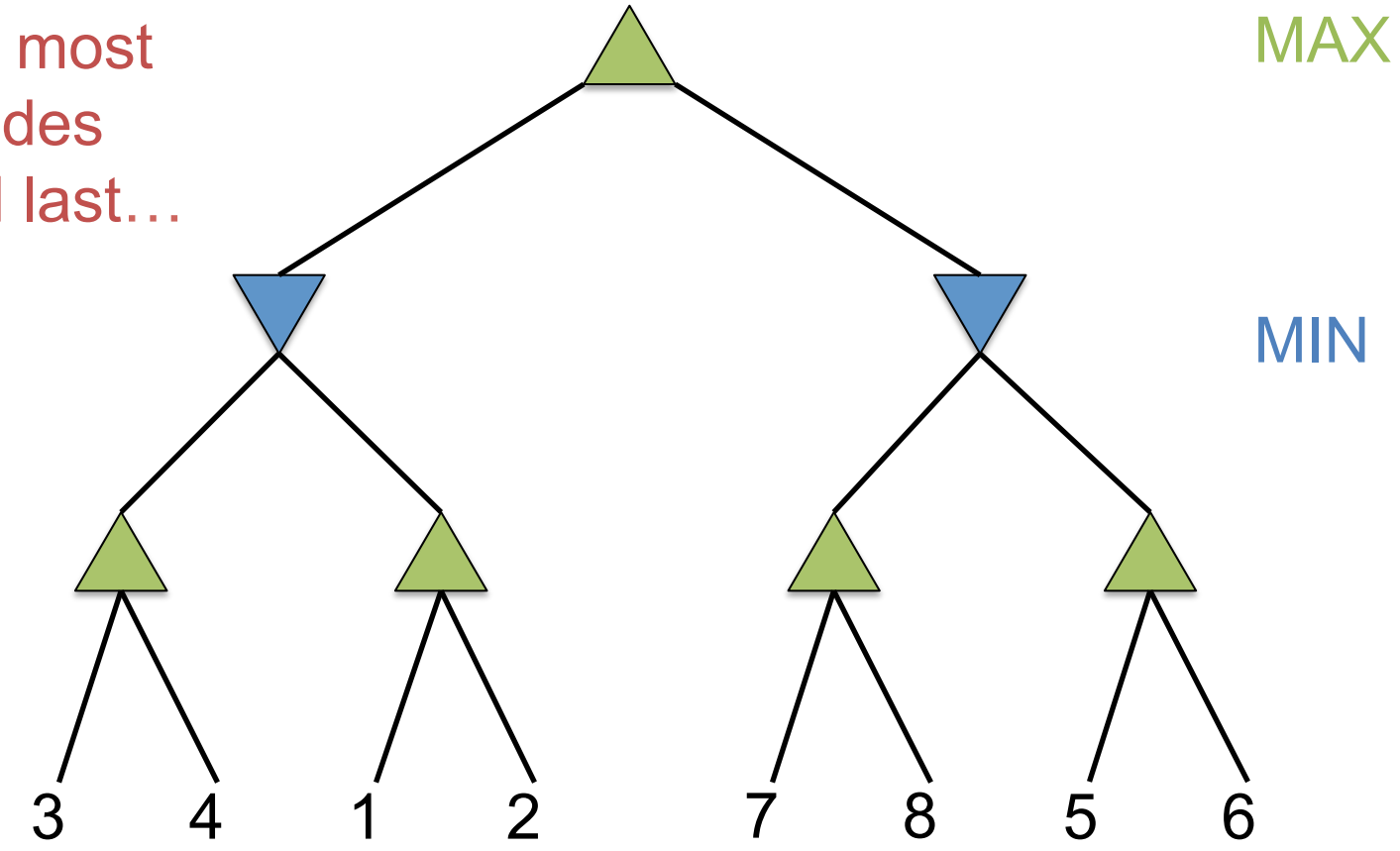
- Pruning does not affect final results
- Entire subtrees can be pruned
- Good move ordering improves pruning
  - Order nodes so player's best moves are checked first
- Repeated states are still possible
  - Store them in memory = transposition table

Iterative deepening reordering

Which leaves can be pruned?

None!

because the most favorable nodes are explored last…

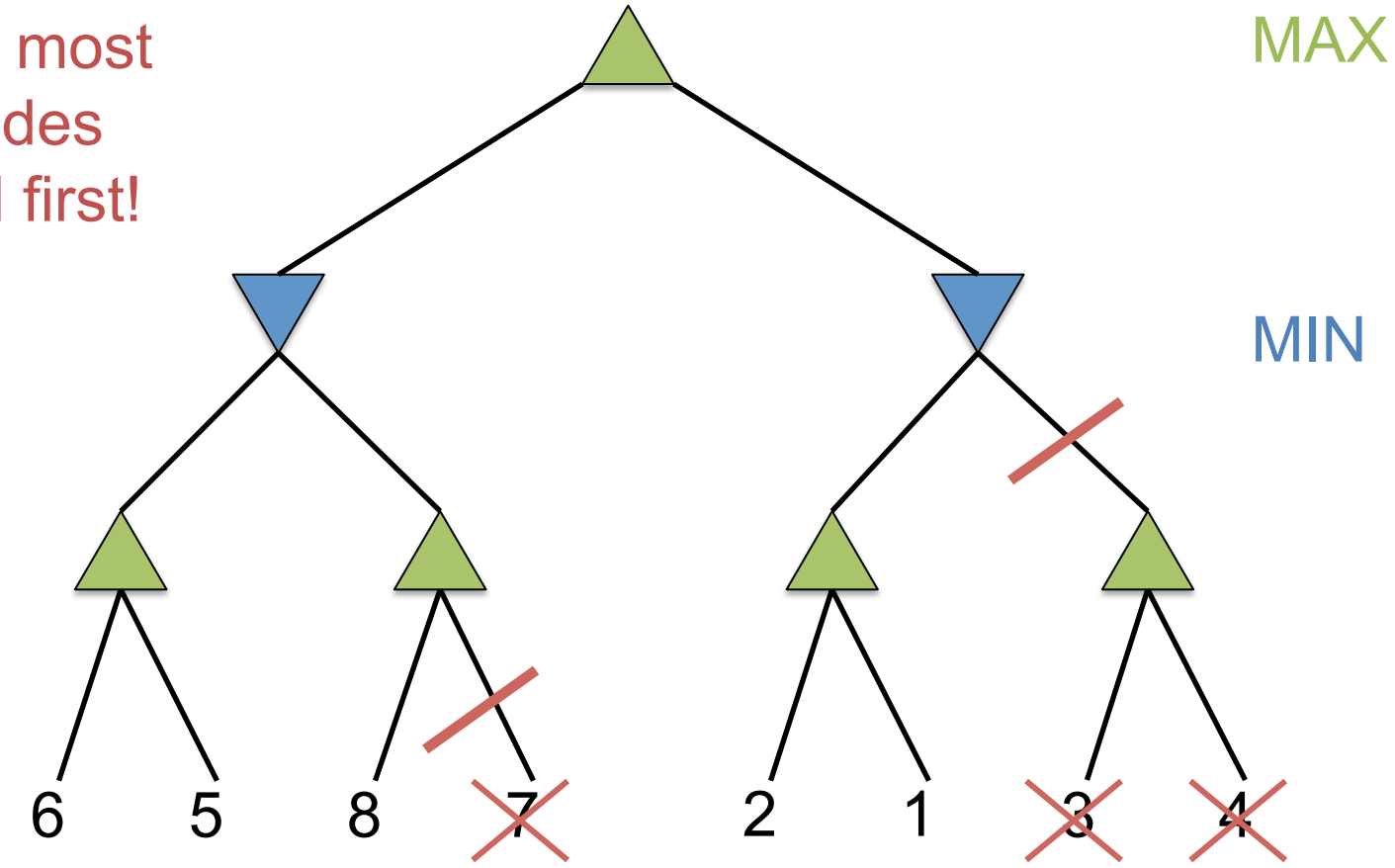


Iterative deepening reordering

Different exploration order: now which leaves can be pruned?

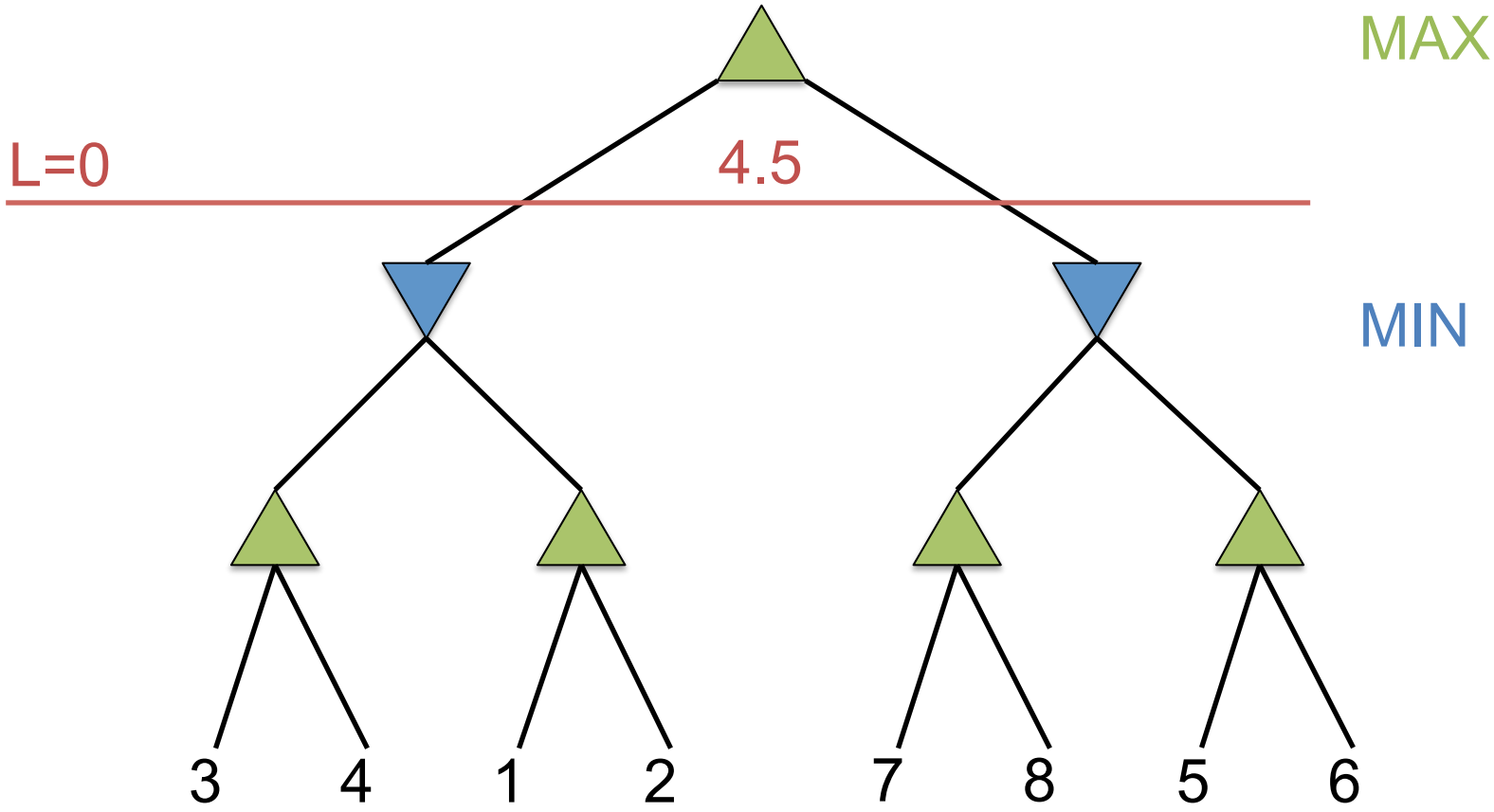
Lots!

because the most favorable nodes are explored first!



Iterative deepening reordering

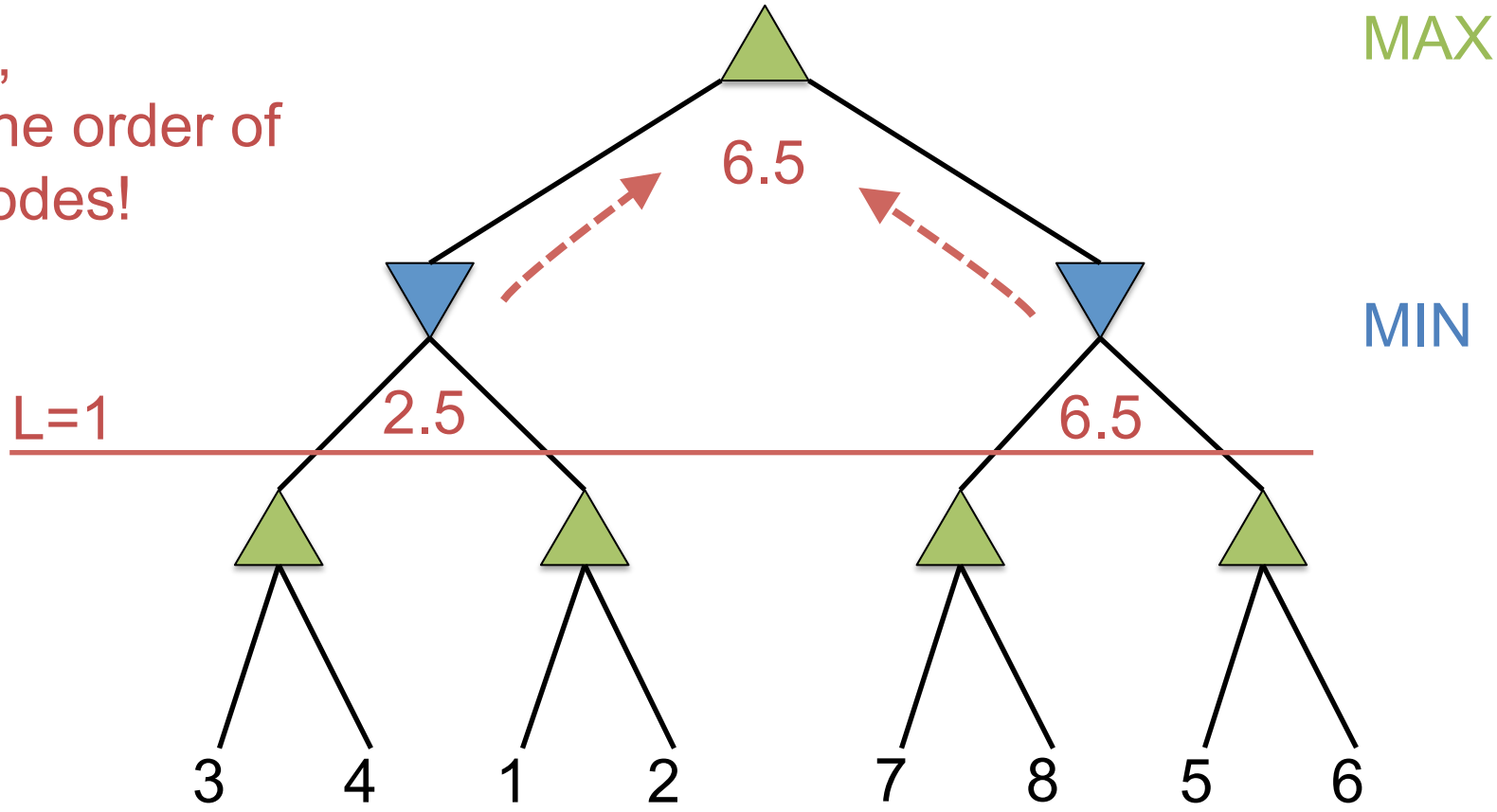
Order with no pruning; use iterative deepening approach.
Assume node score is the average of leaf values below.



Iterative deepening reordering

Order with no pruning; use iterative deepening approach.
Assume node score is the average of leaf values below.

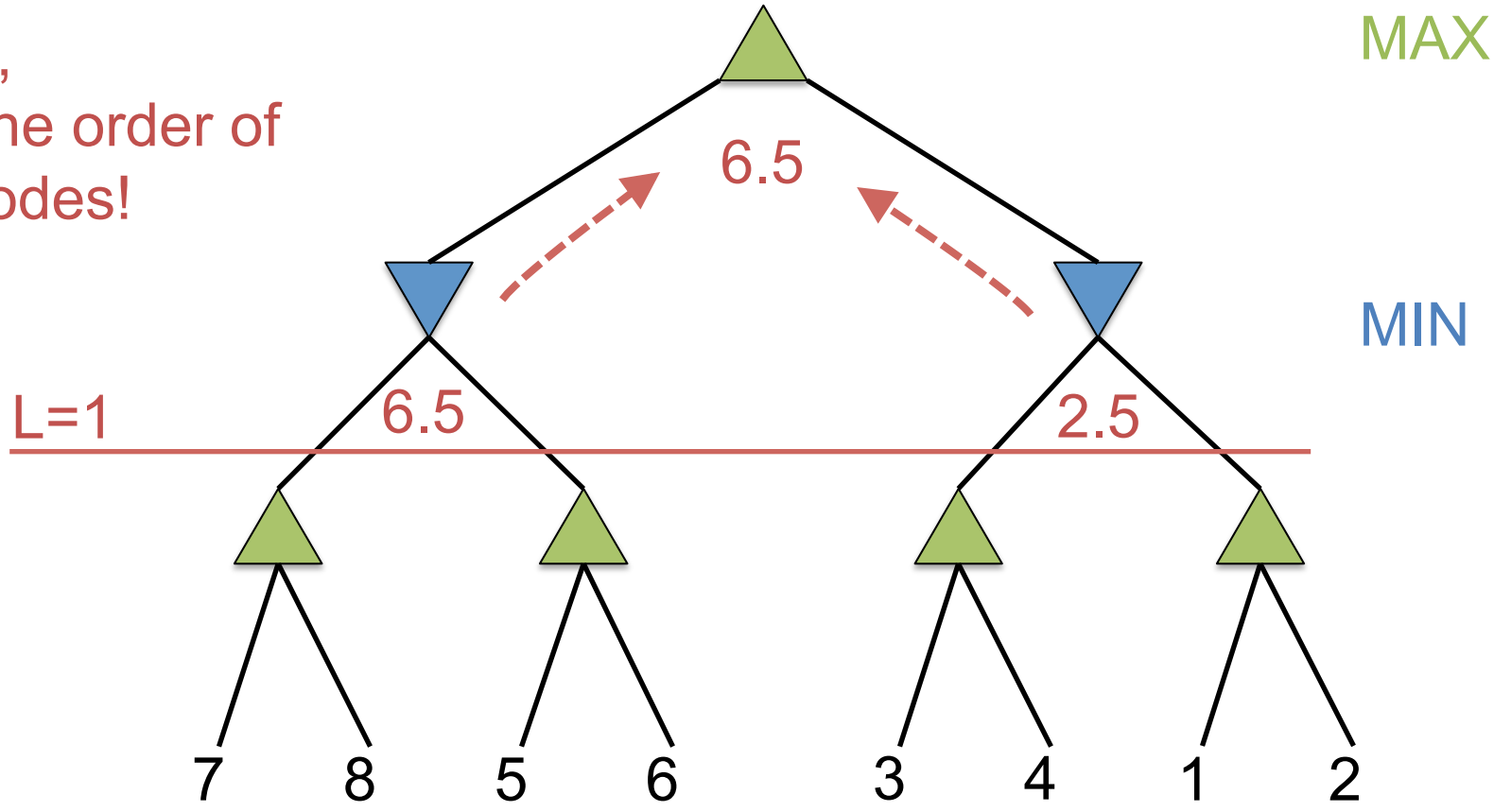
For L=2,
switch the order of
these nodes!



Iterative deepening reordering

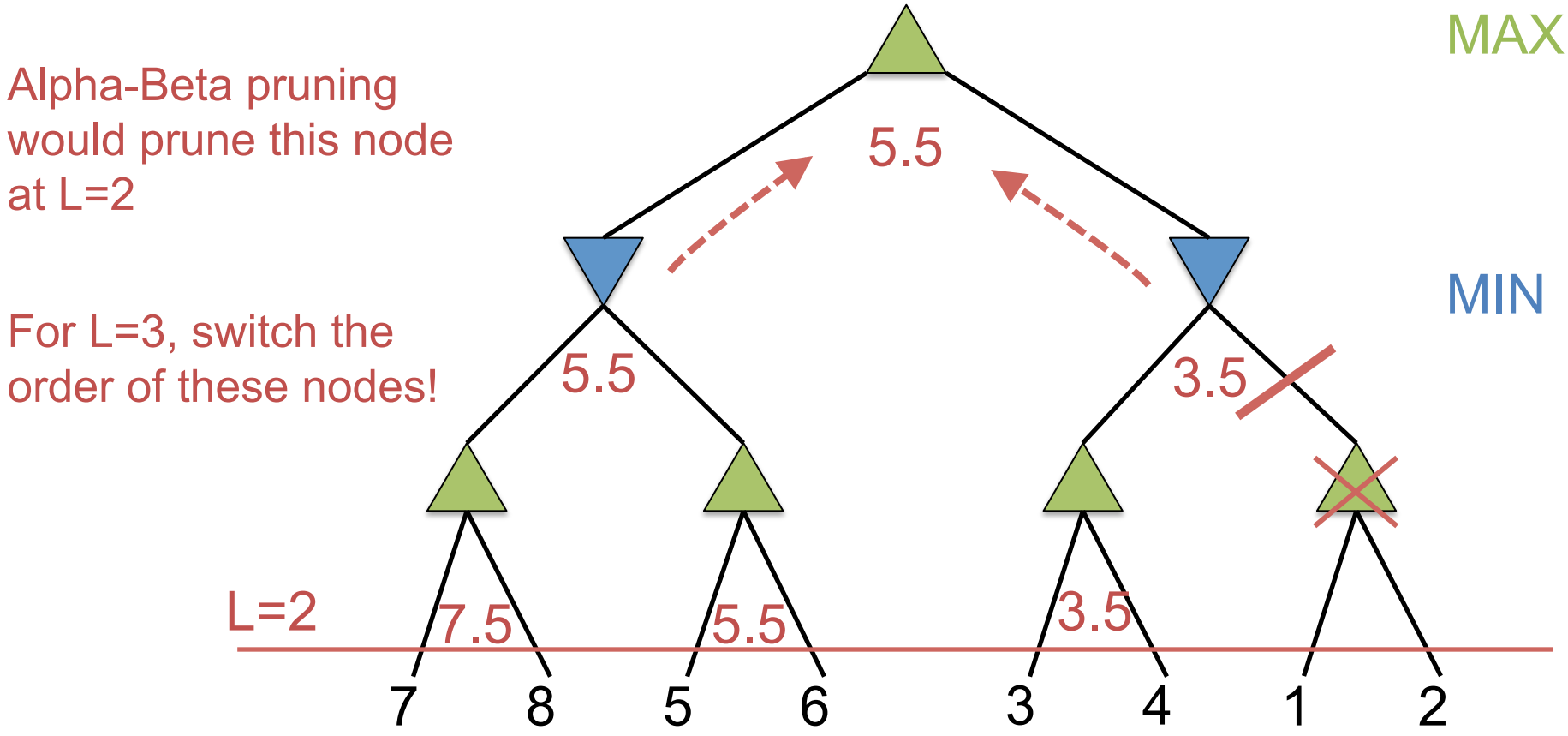
Order with no pruning; use iterative deepening approach.
Assume node score is the average of leaf values below.

For L=2,
switch the order of
these nodes!



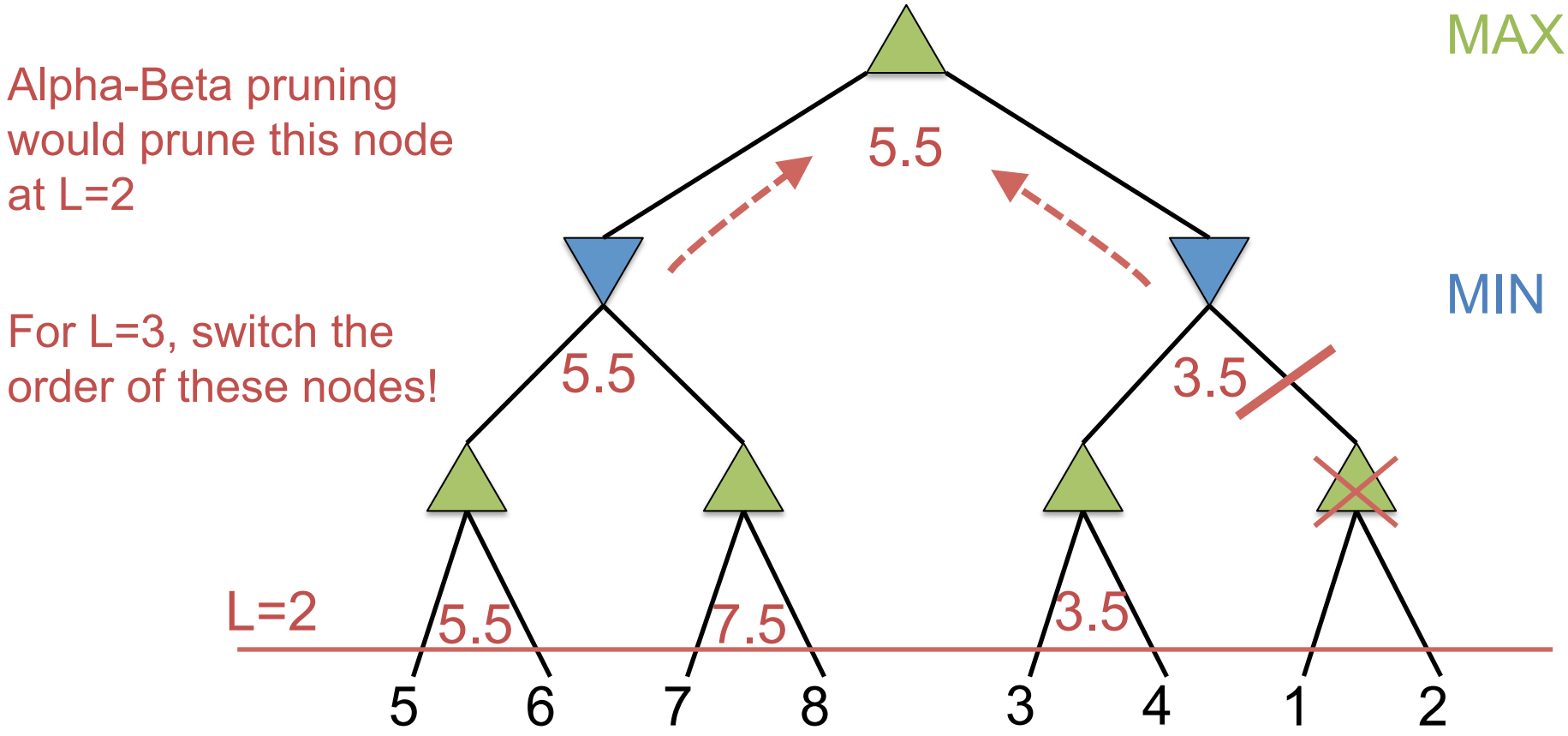
Iterative deepening reordering

Order with no pruning; use iterative deepening approach.
Assume node score is the average of leaf values below.



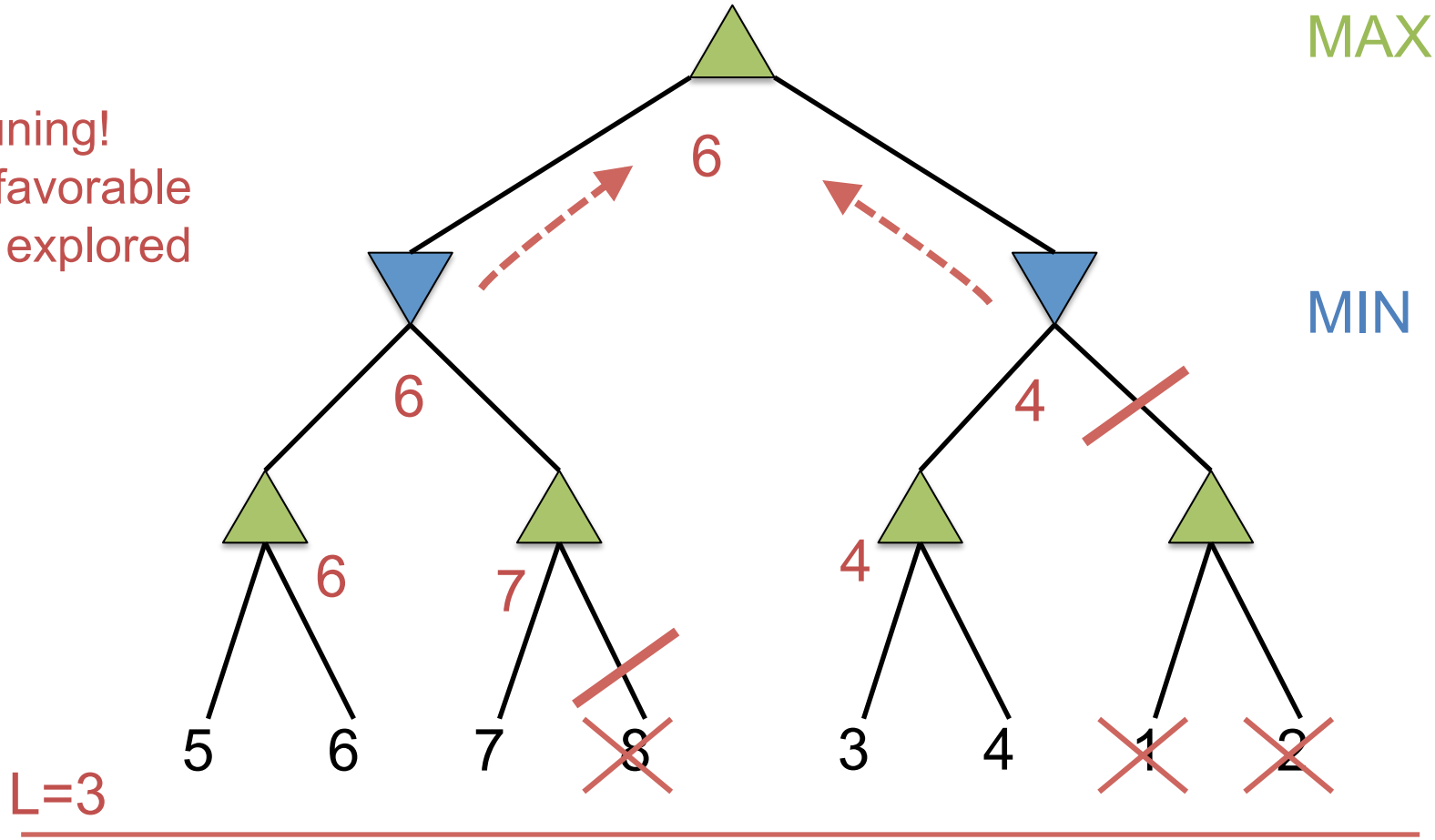
Iterative deepening reordering

Order with no pruning; use iterative deepening approach.
Assume node score is the average of leaf values below.



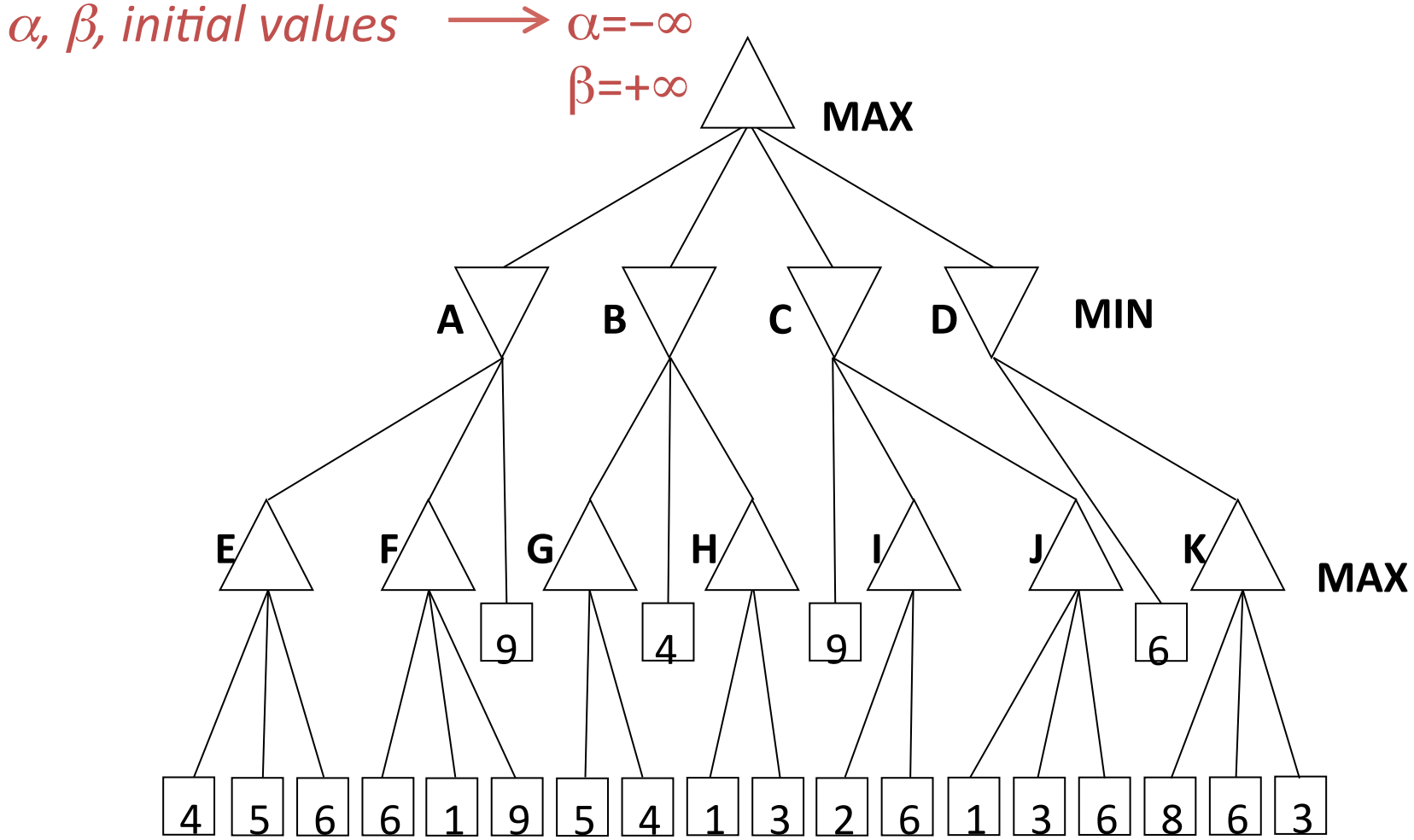
Iterative deepening reordering

Order with no pruning; use iterative deepening approach.
Assume node score is the average of leaf values below.



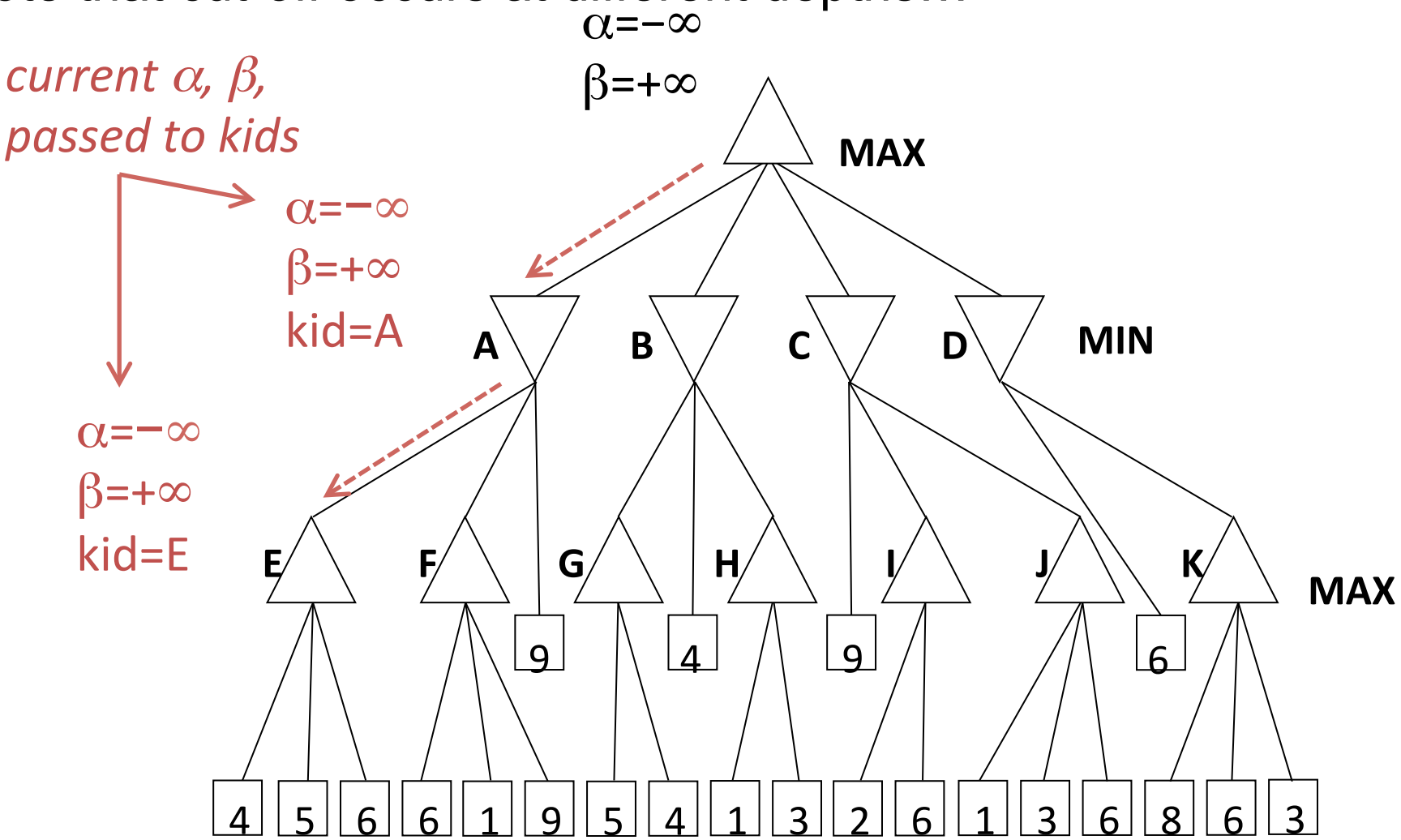
Longer Alpha-Beta Example

Branch nodes are labelel A..K for easy discussion

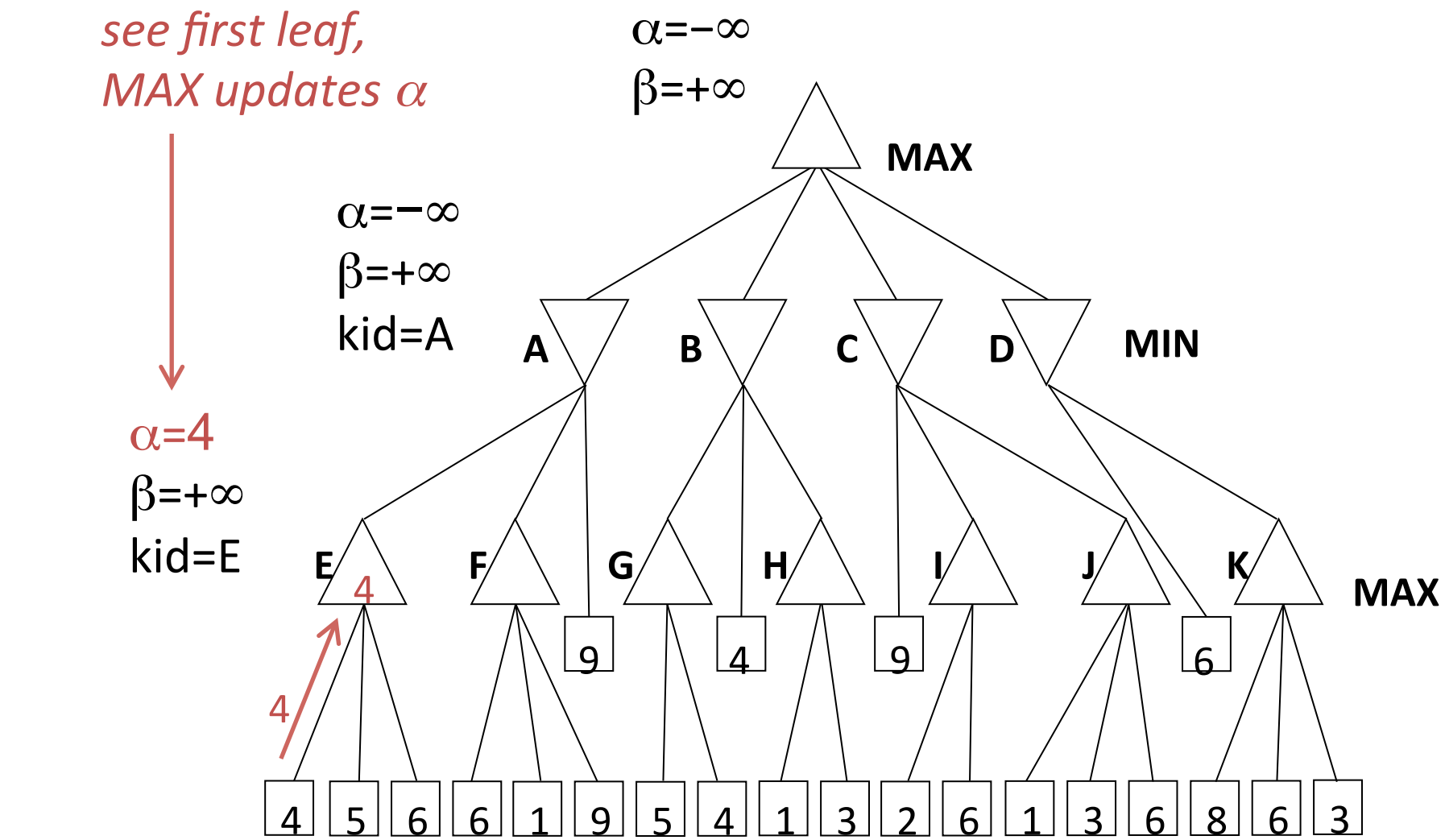


Longer Alpha-Beta Example

Note that cut-off occurs at different depths…

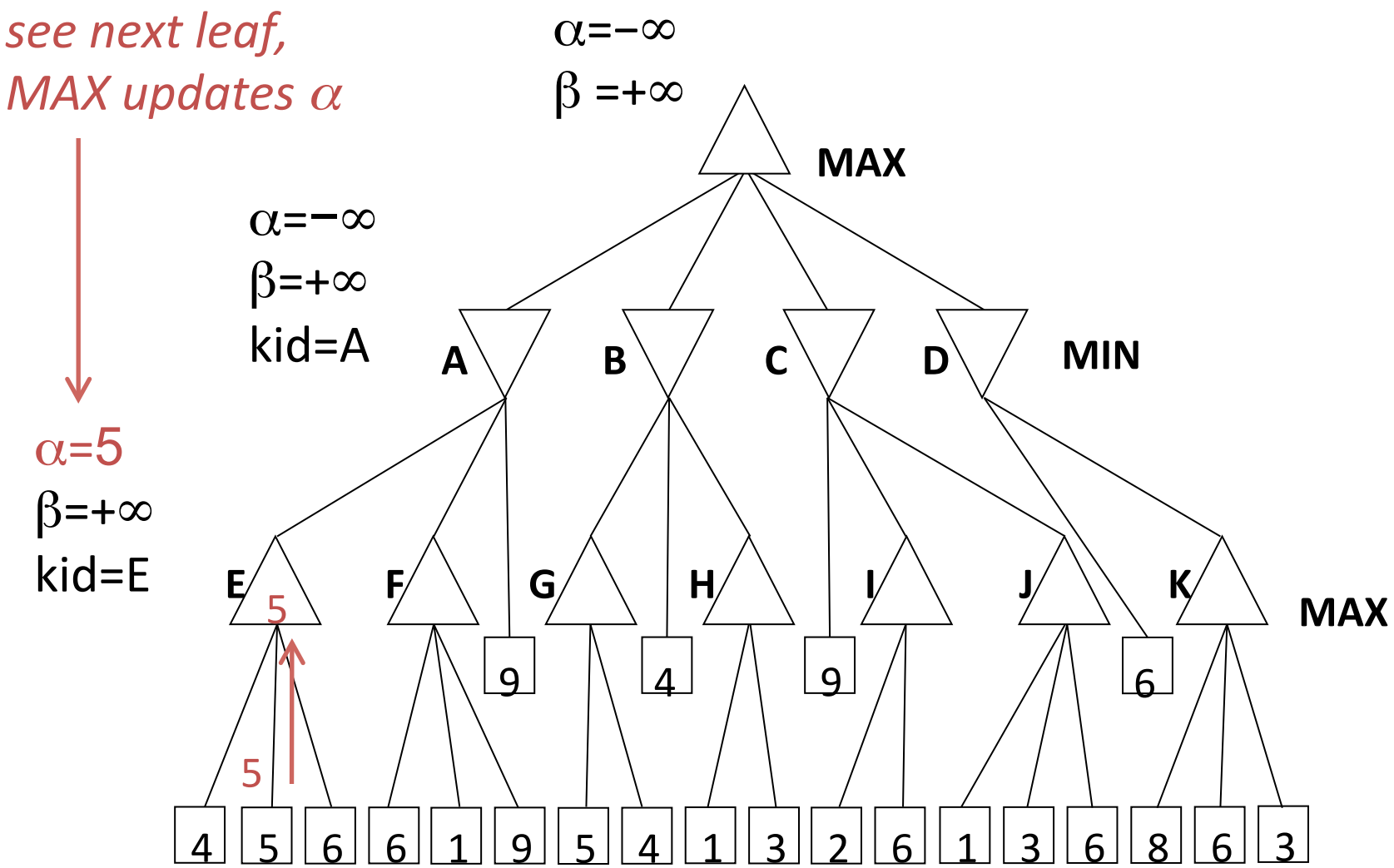


Longer Alpha-Beta Example



We also are running MiniMax search and recording node values within the triangles, without explicit comment.

Longer Alpha-Beta Example



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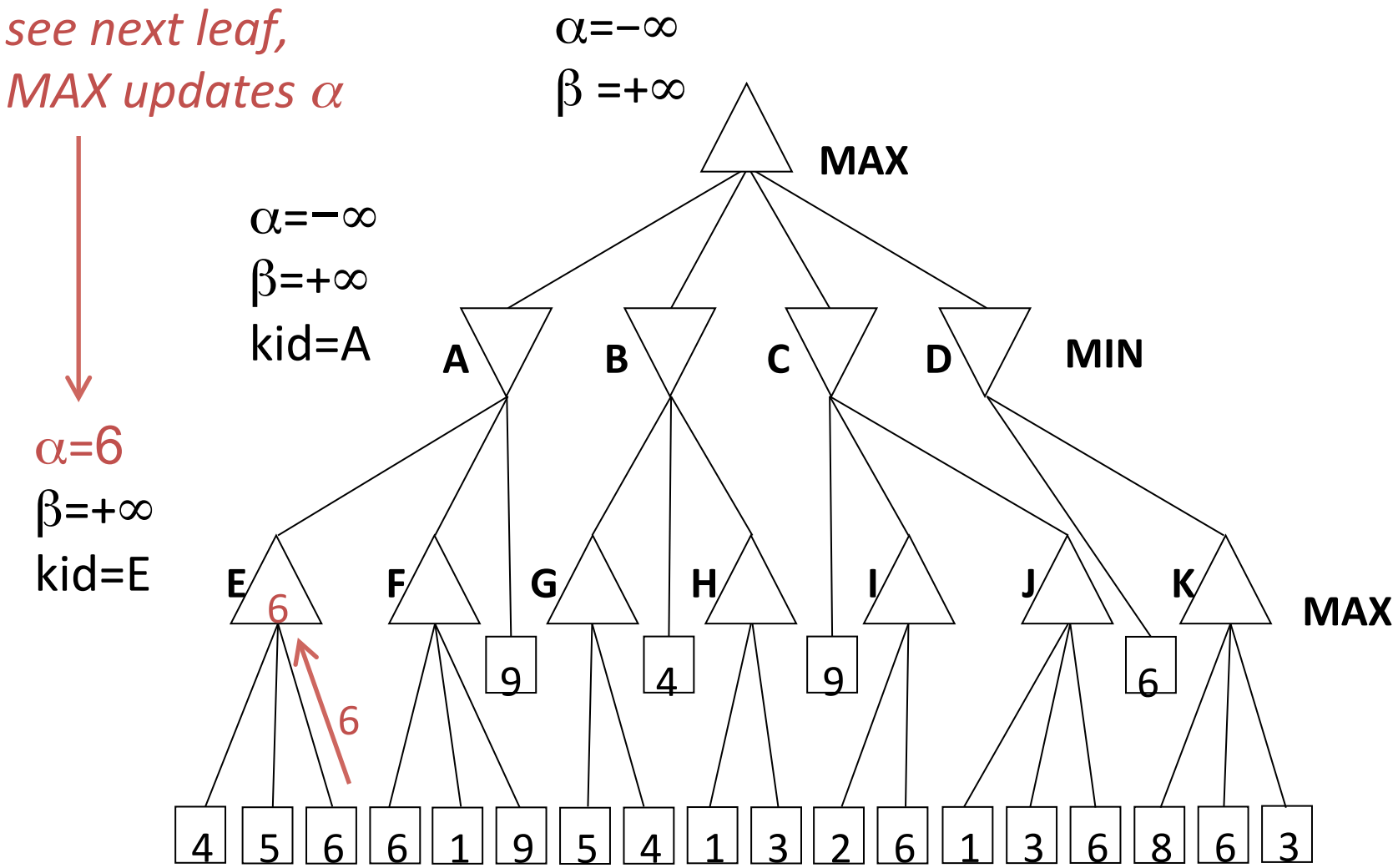

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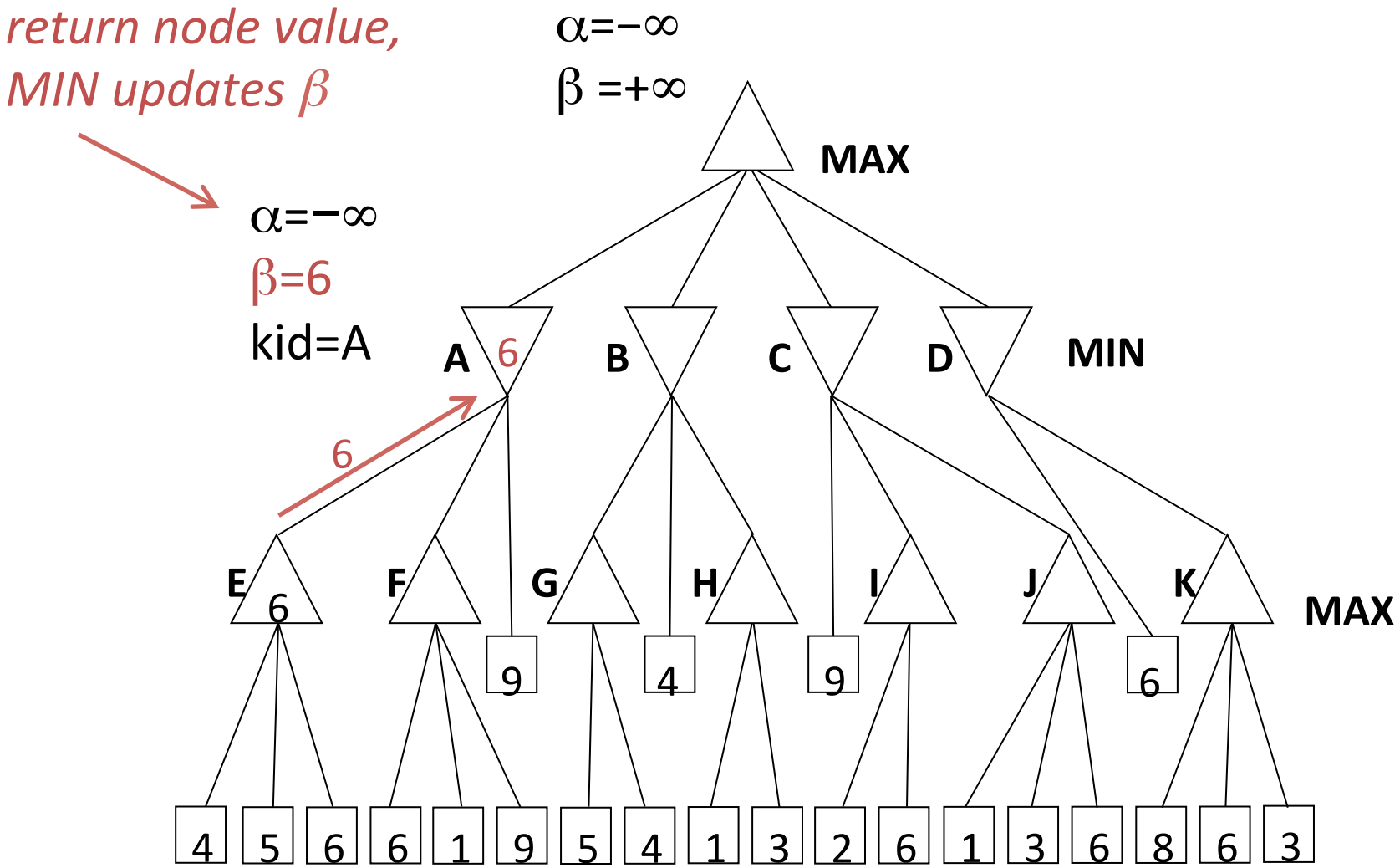

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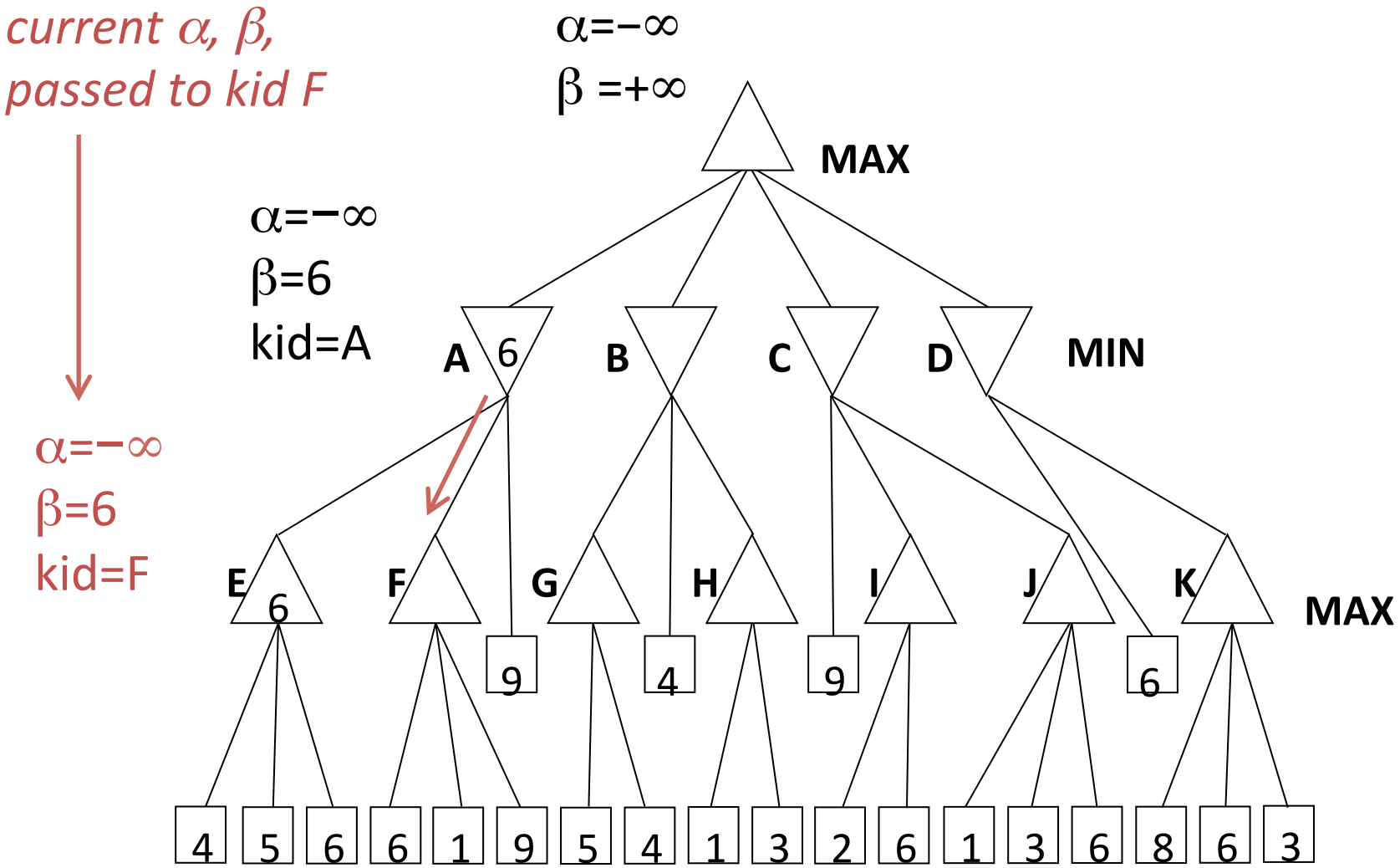

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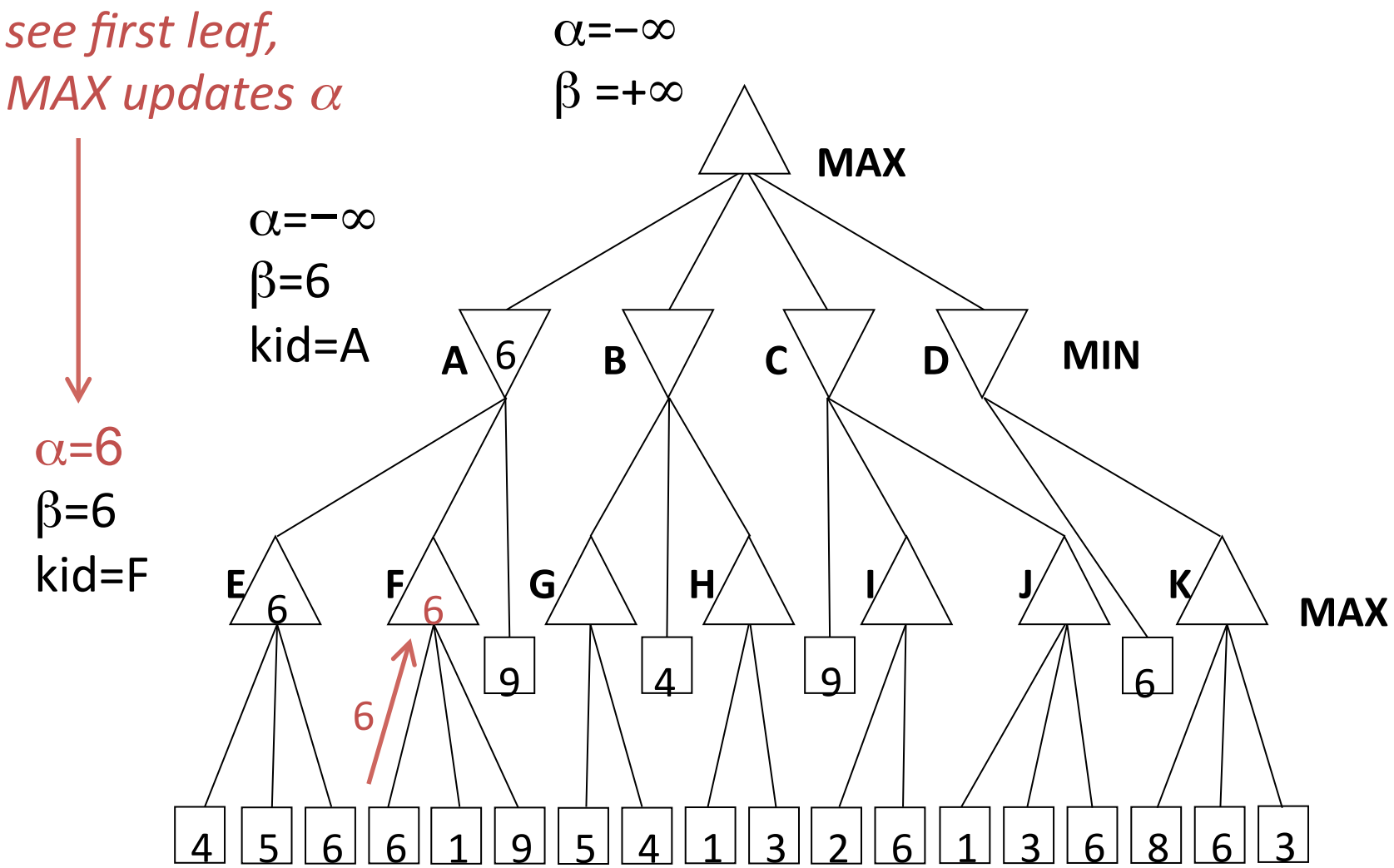

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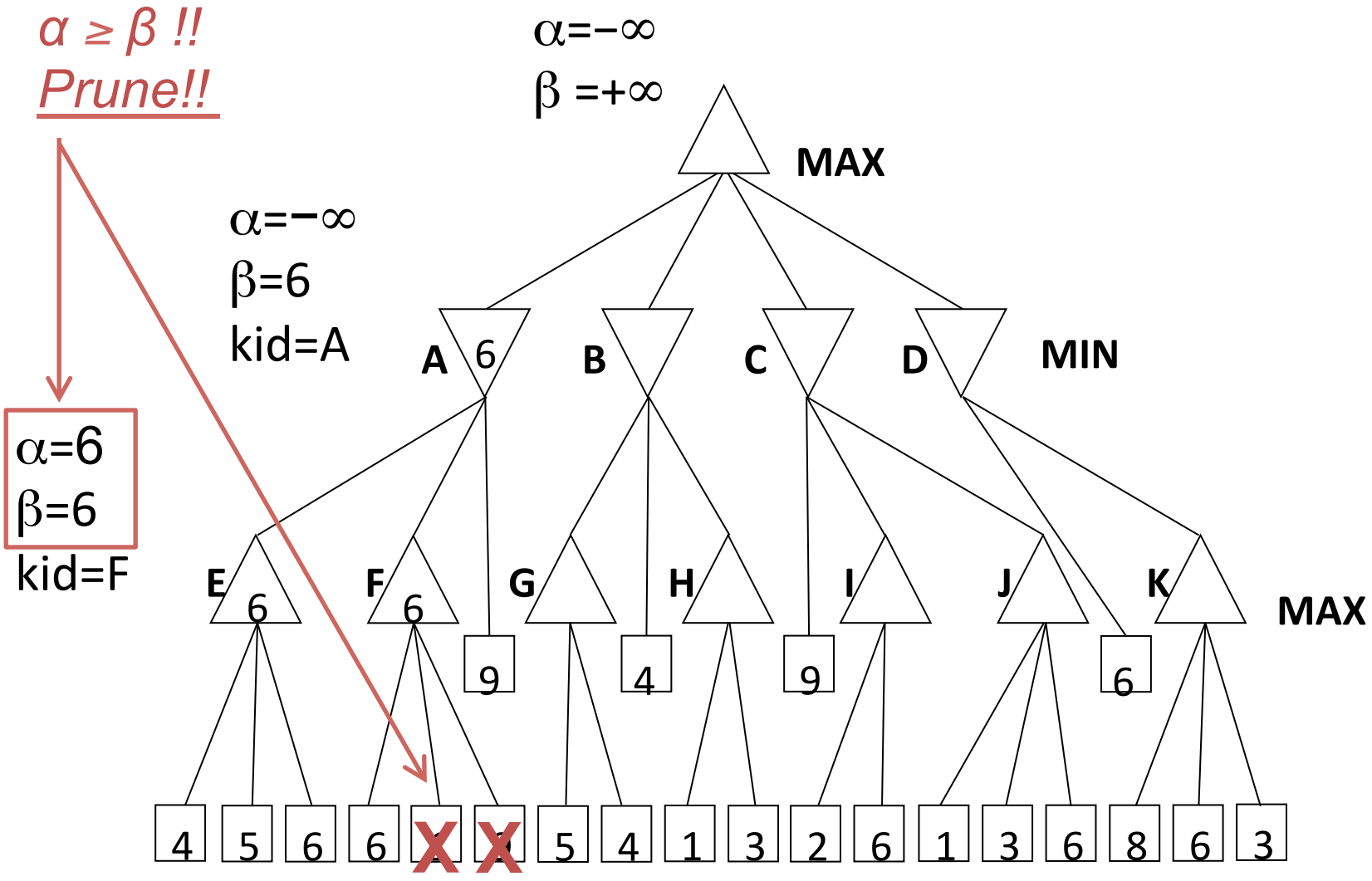

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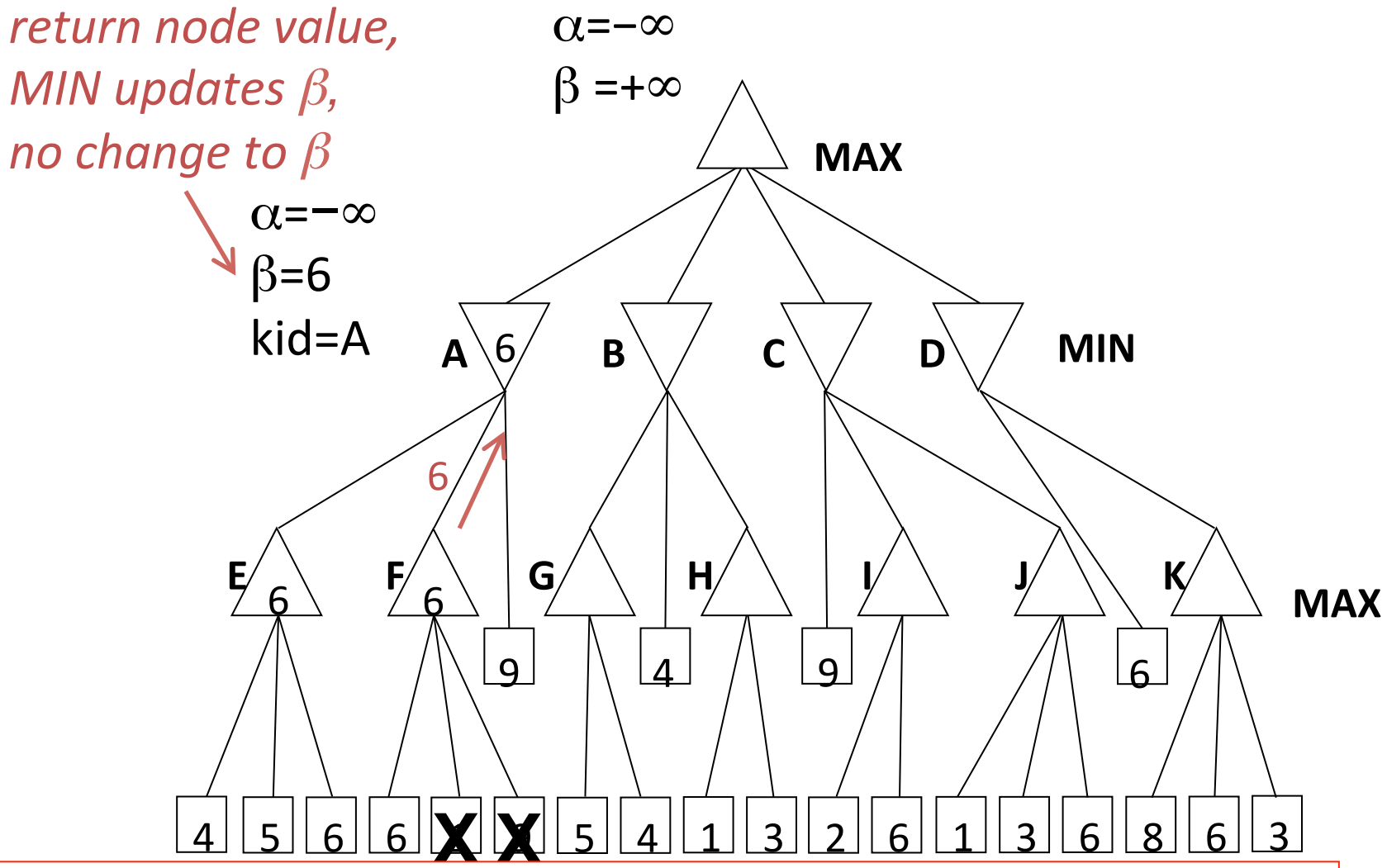
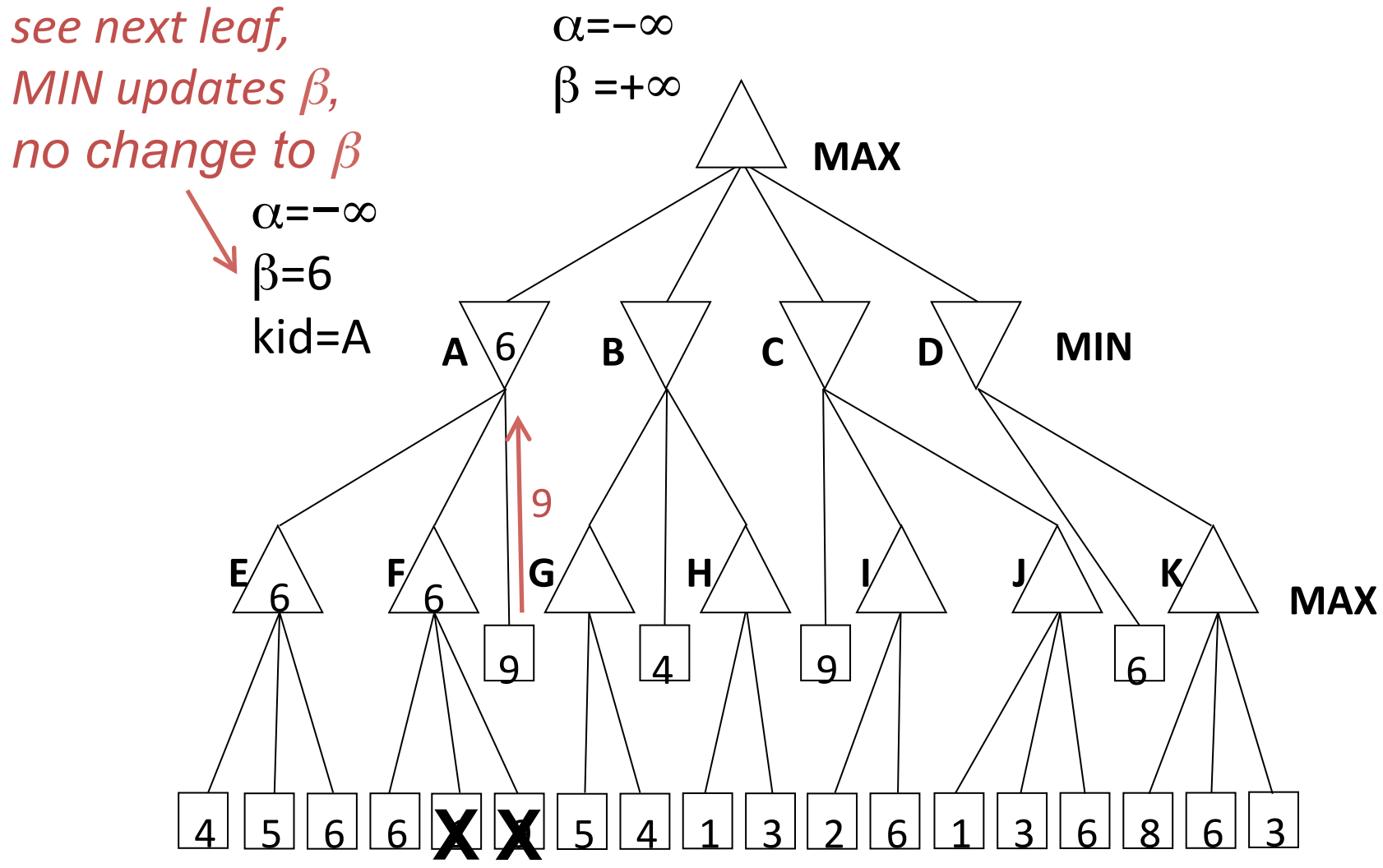


If we had continued searching at node F, we would see the 9 from its third leaf. Our returned value would be 9 instead of 6. But at A, MIN would choose E(=6) instead of F(=9). Internal values may change; root values do not.

Longer Alpha-Beta Example



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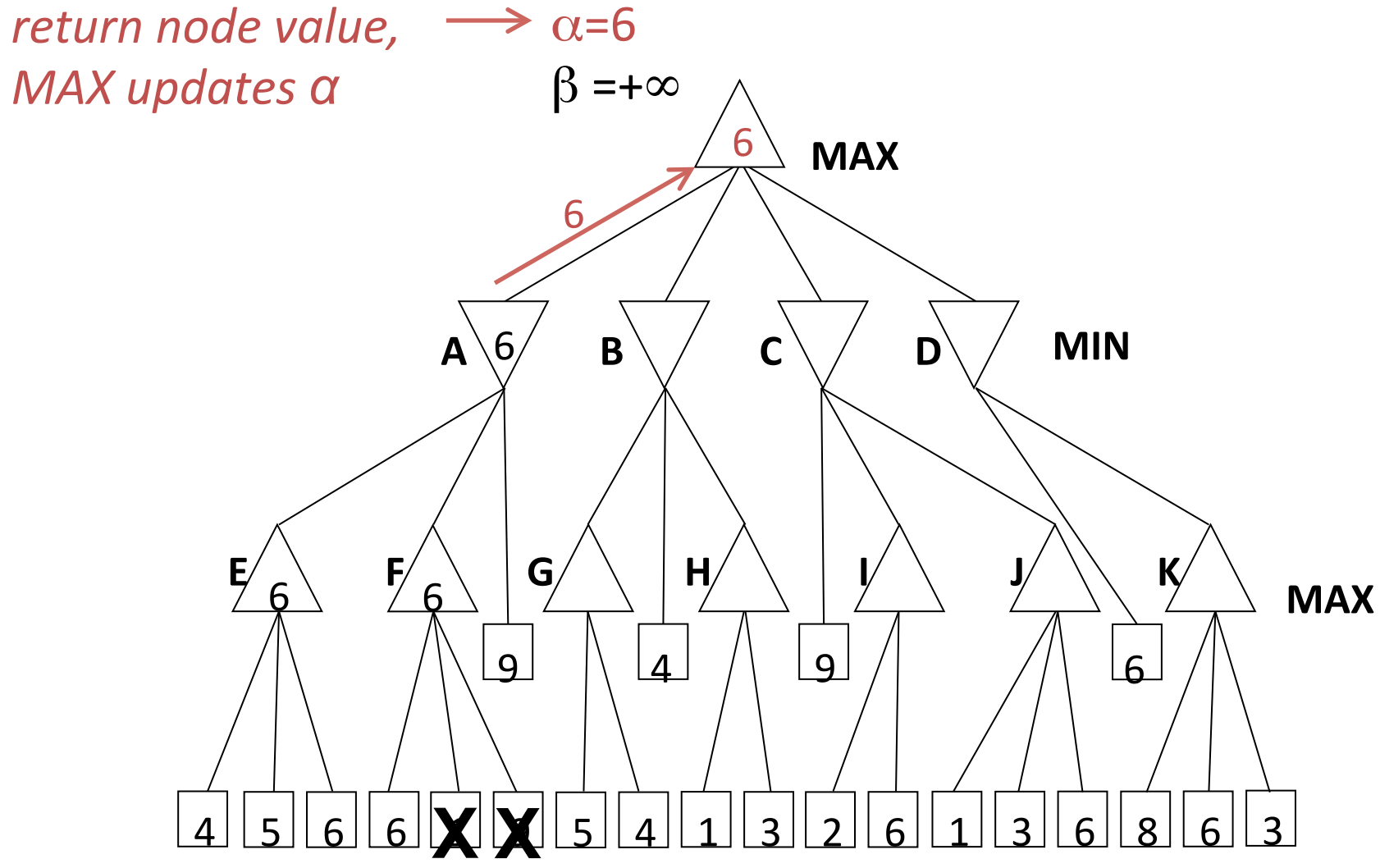
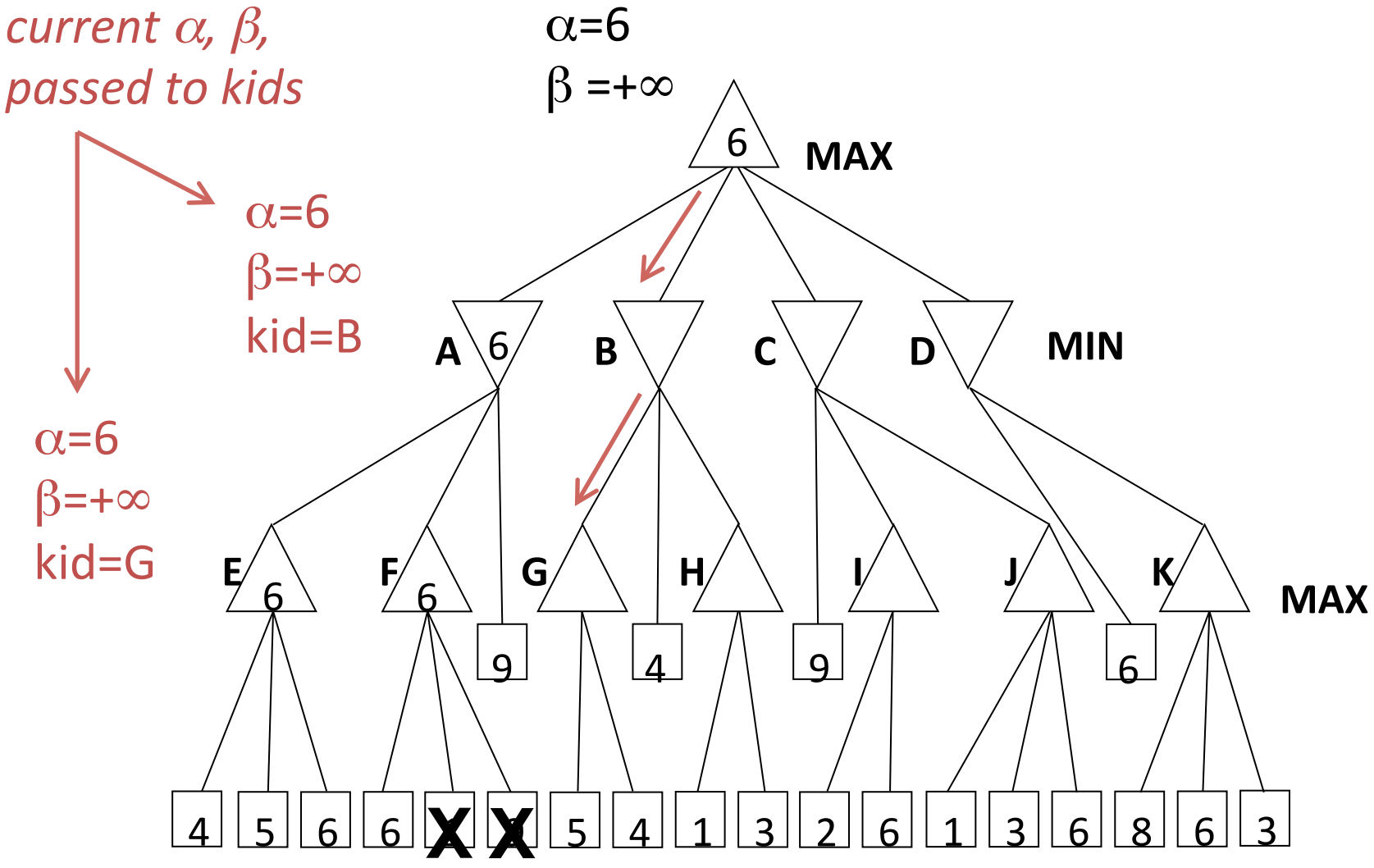
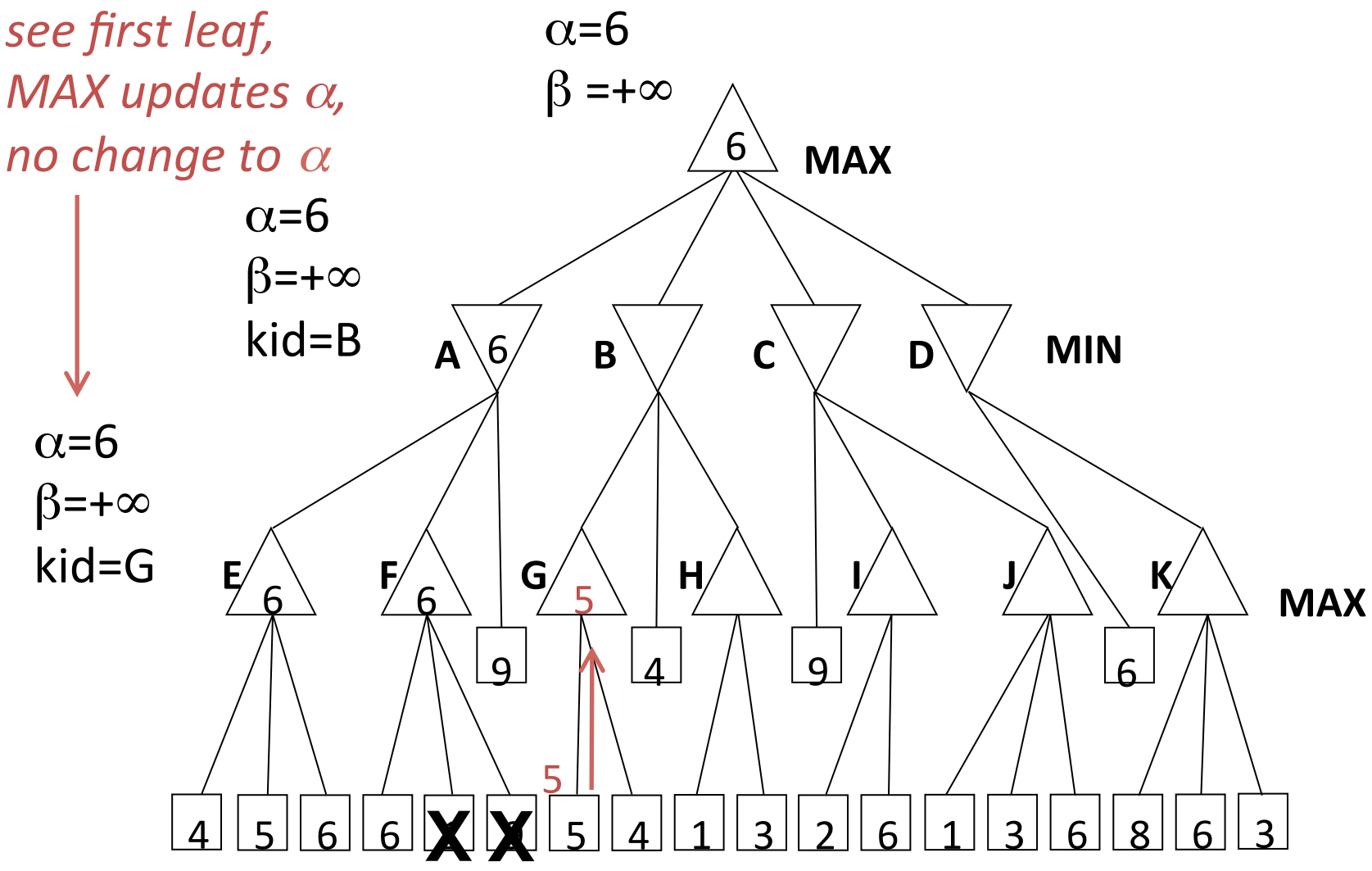


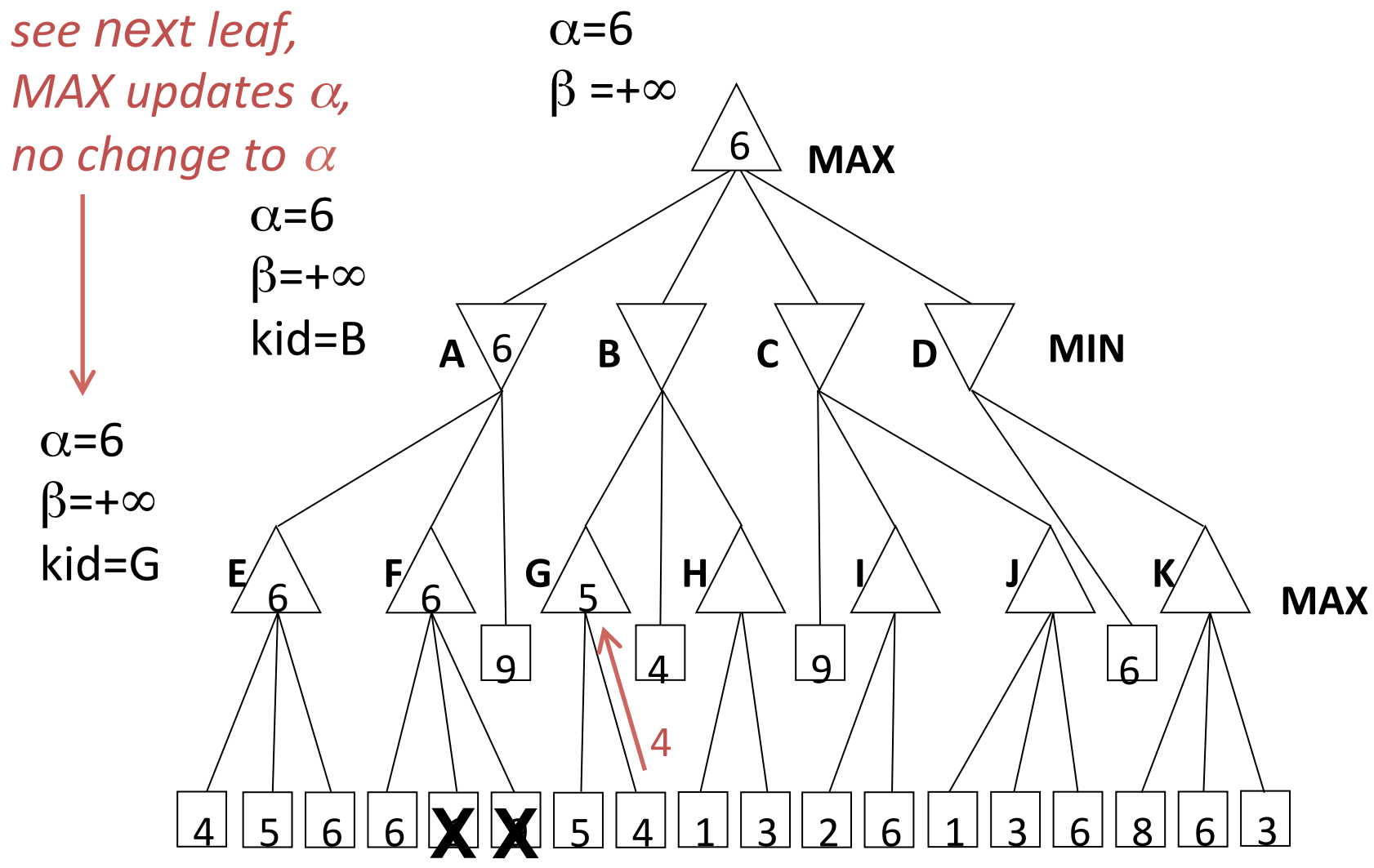
Longer Alpha-Beta Example



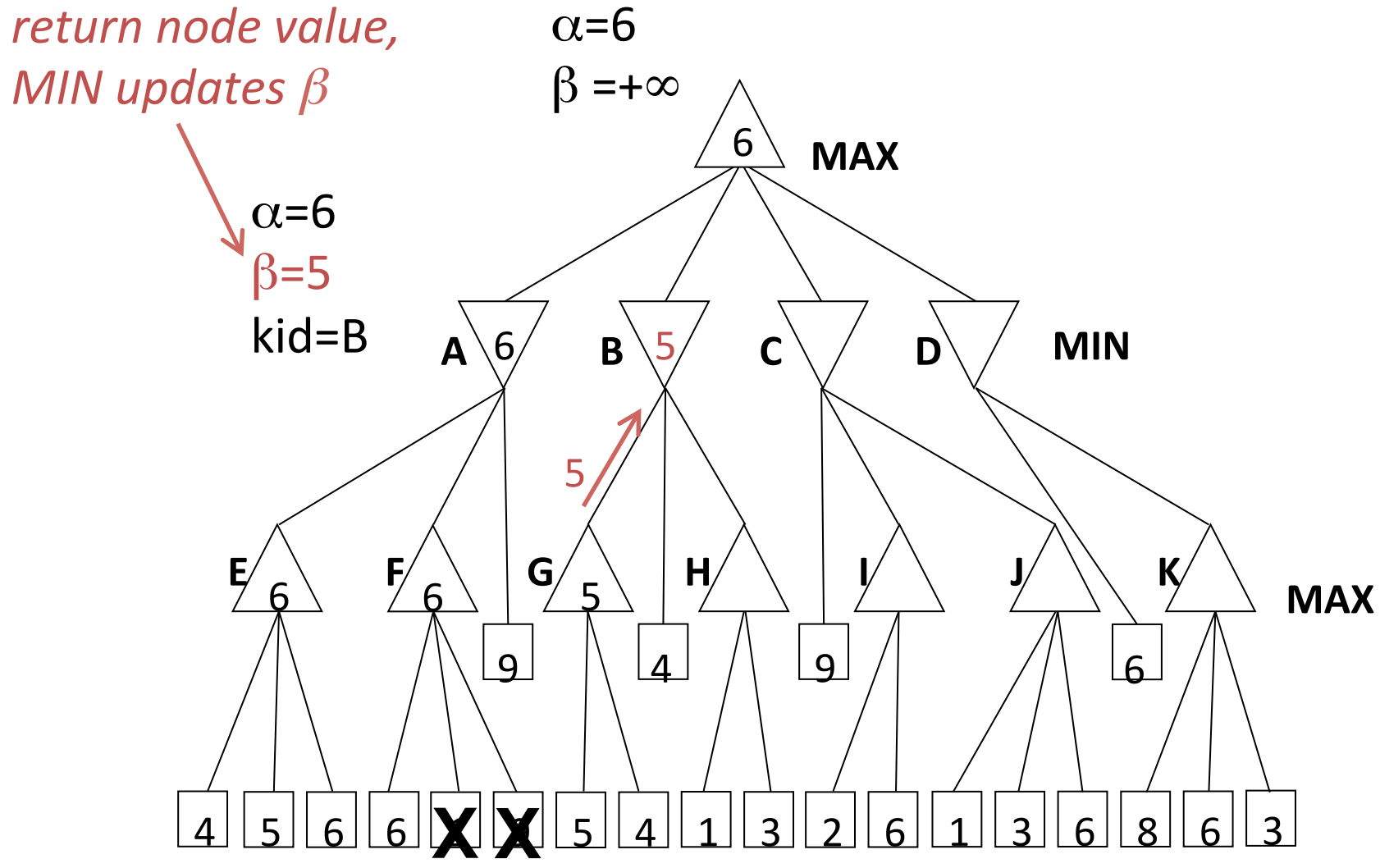
Longer Alpha-Beta Example



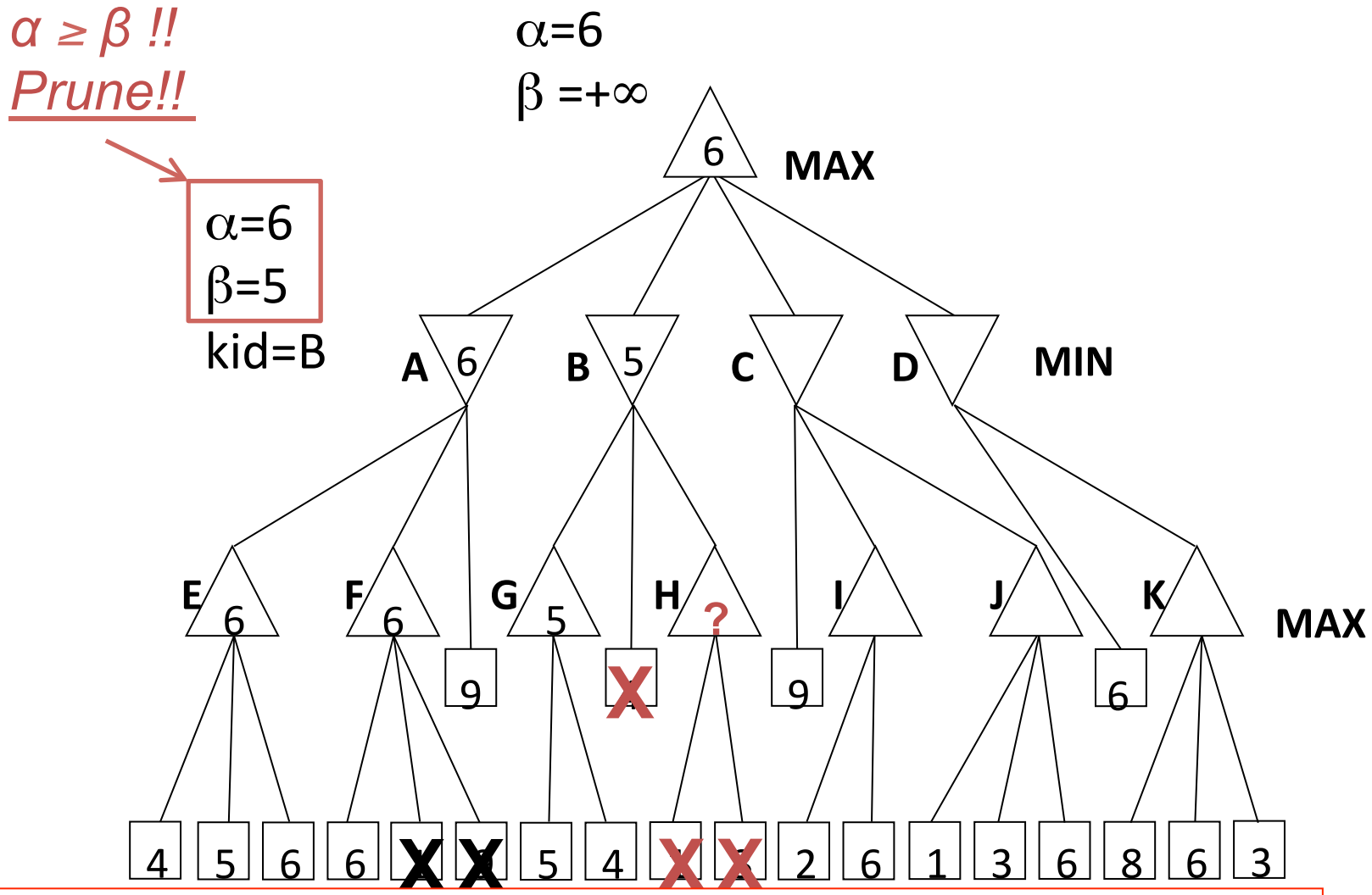
Longer Alpha-Beta Example



Longer Alpha-Beta Example



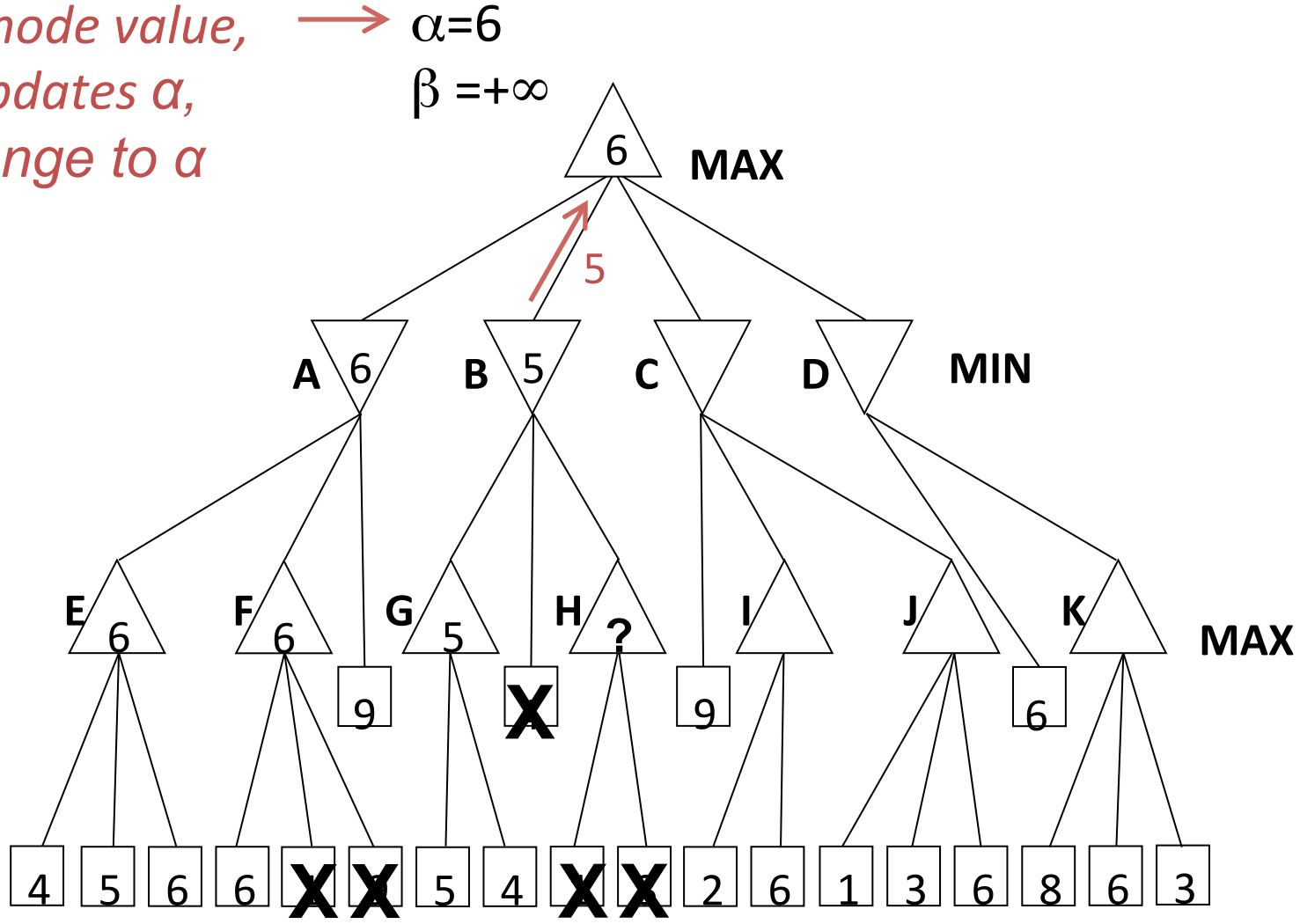
Longer Alpha-Beta Example



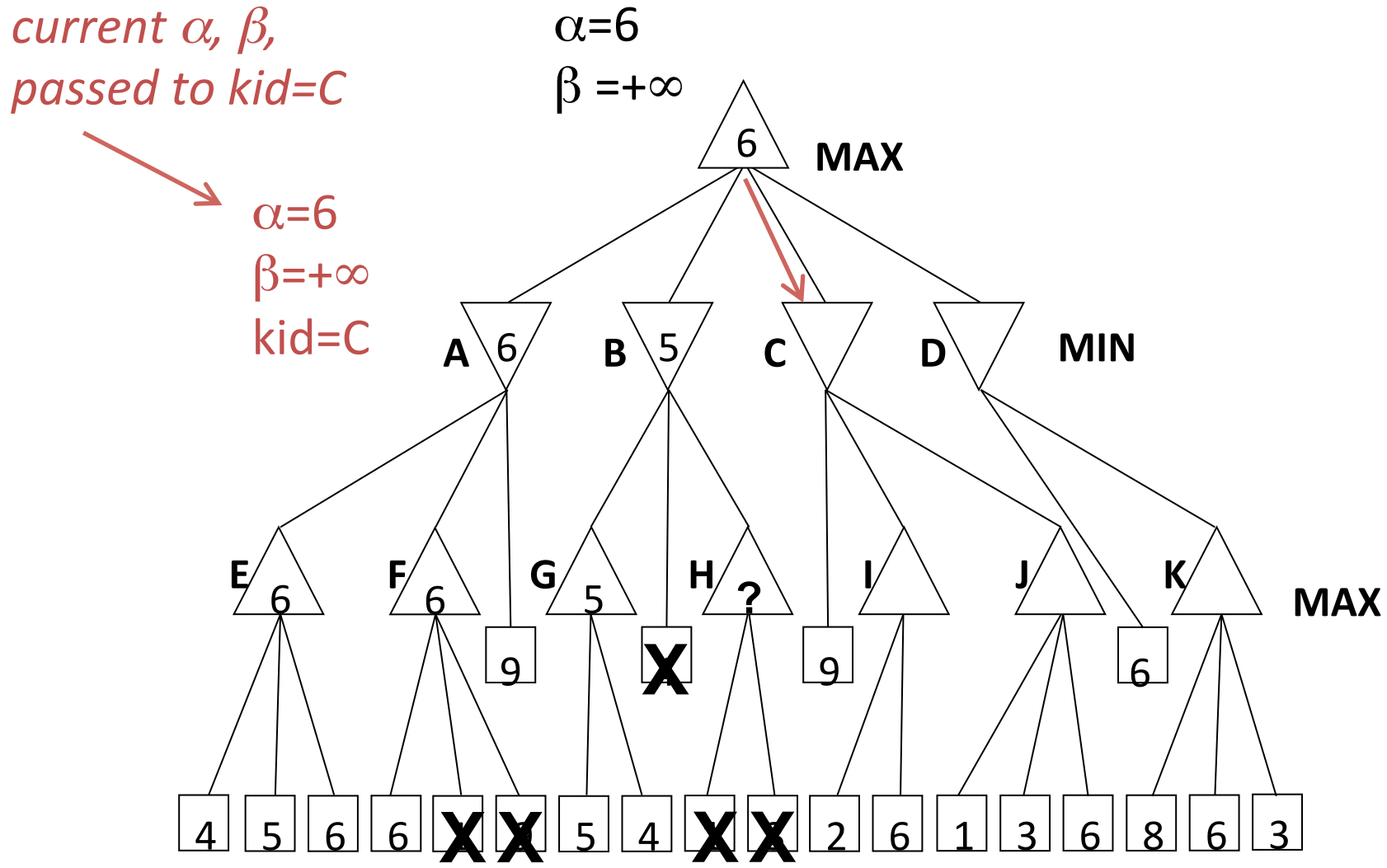
Note that we never find out, what is the node value of H? But we have proven it doesn't matter, so we don't care.

Longer Alpha-Beta Example

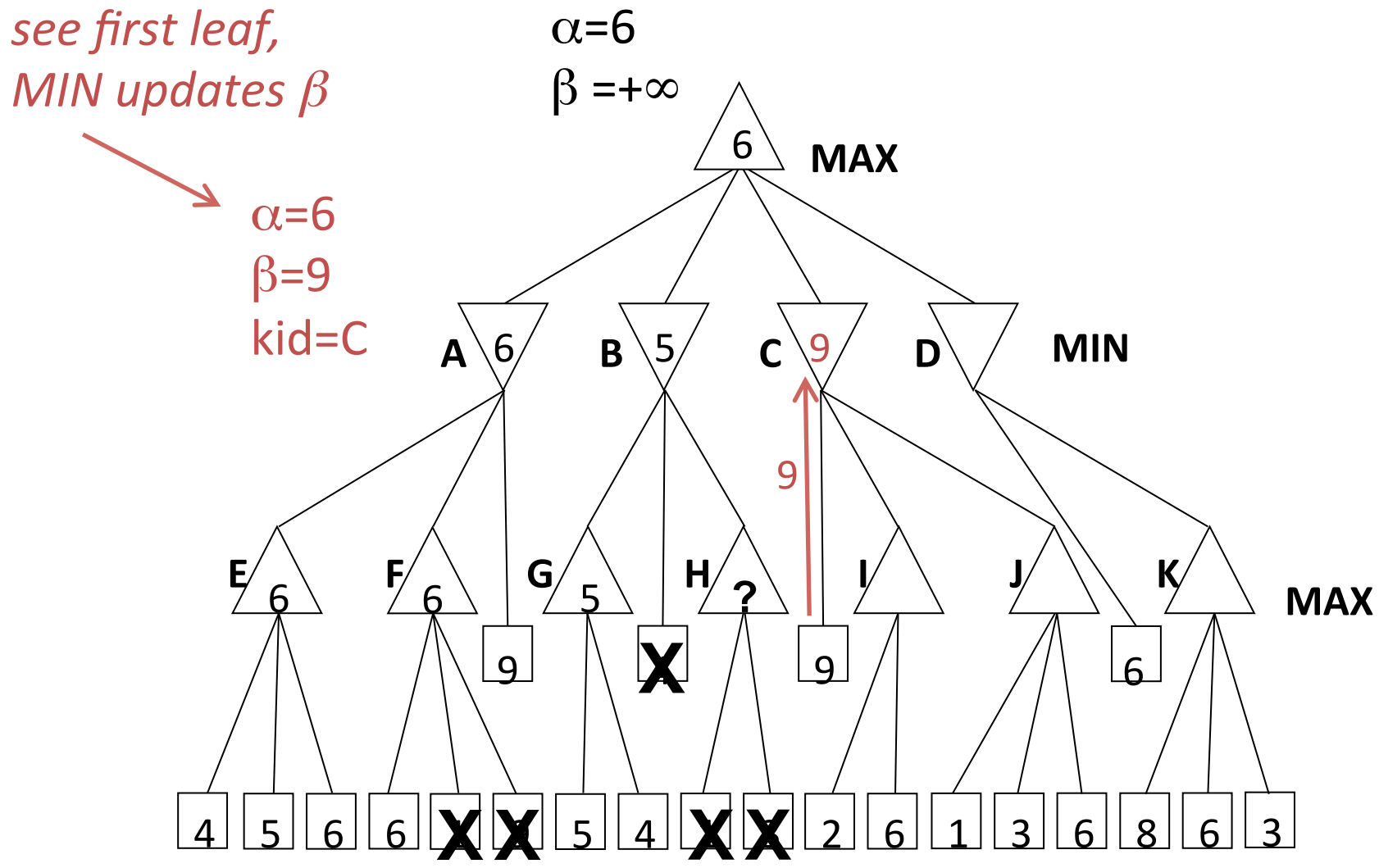
return node value,
MAX updates α,
no change to α



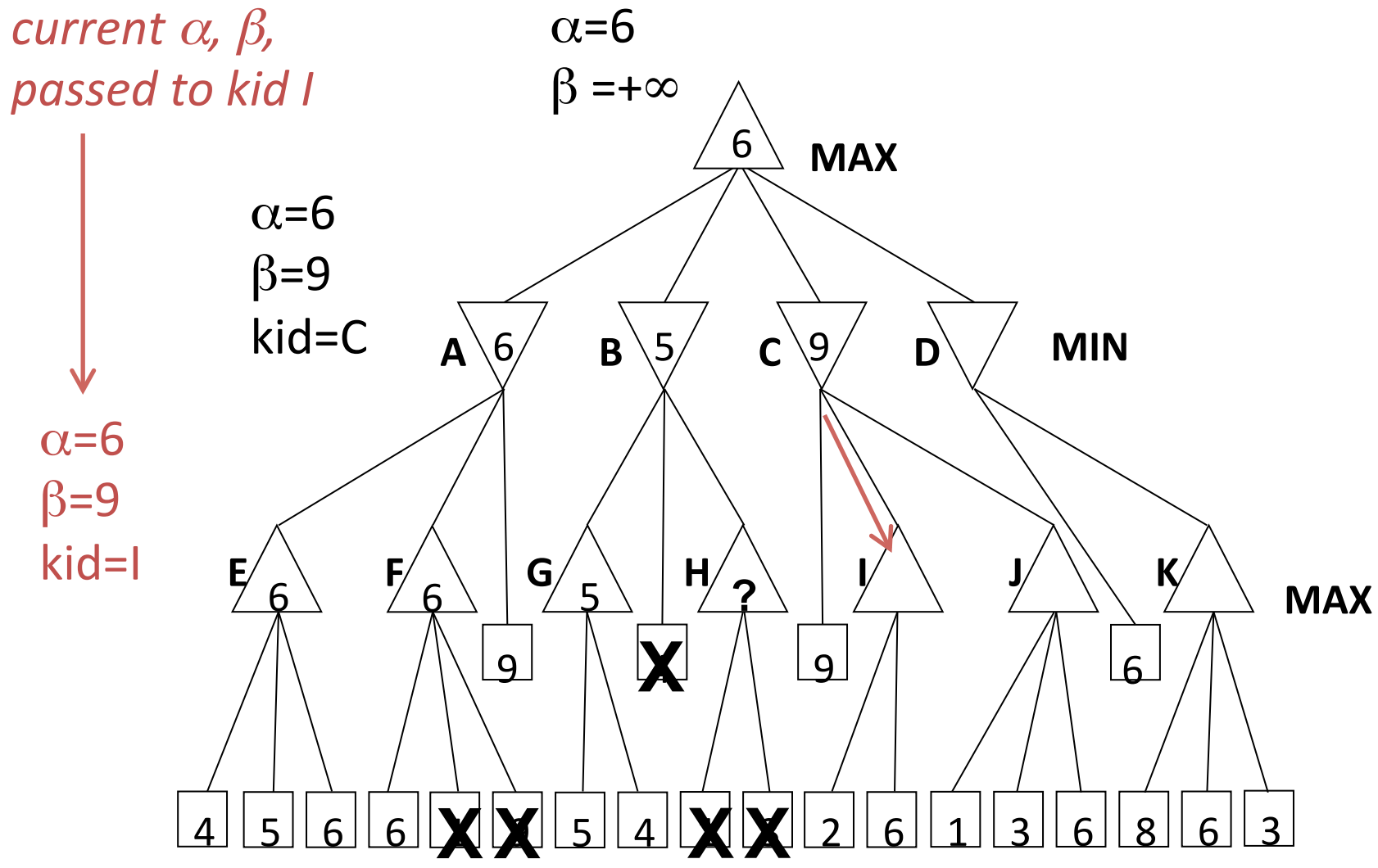
Longer Alpha-Beta Example



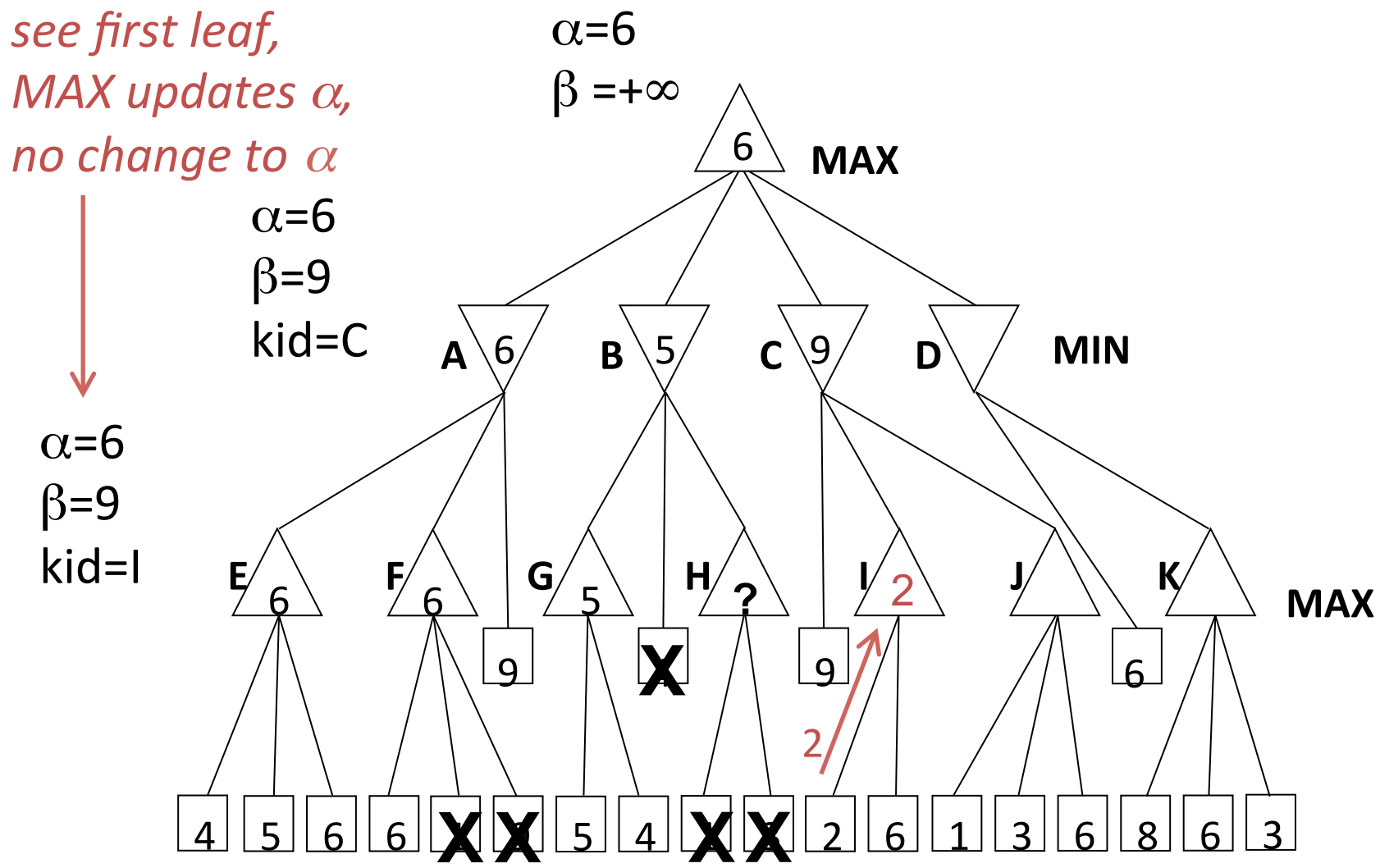
Longer Alpha-Beta Example



Longer Alpha-Beta Example



Longer Alpha-Beta Example



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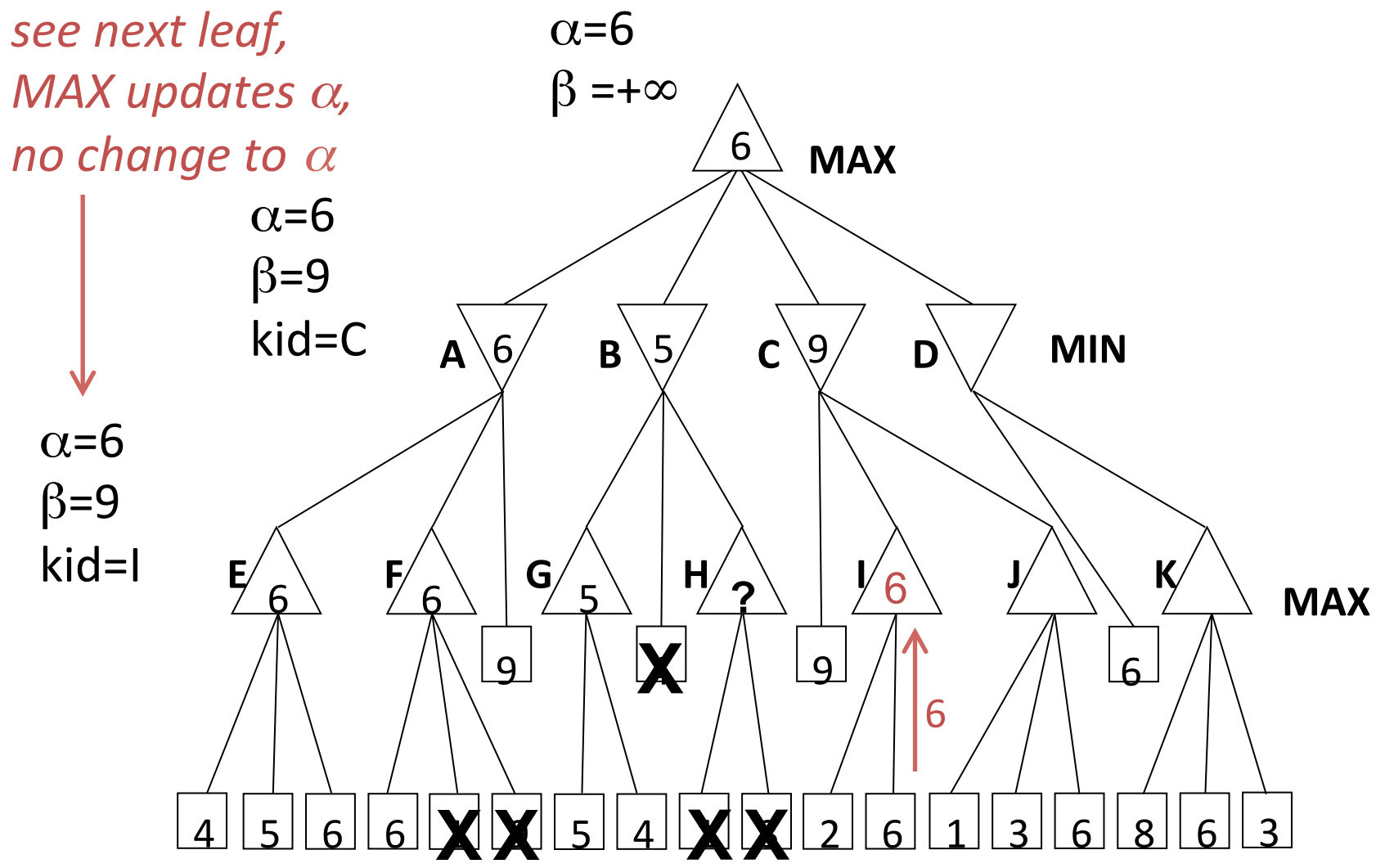
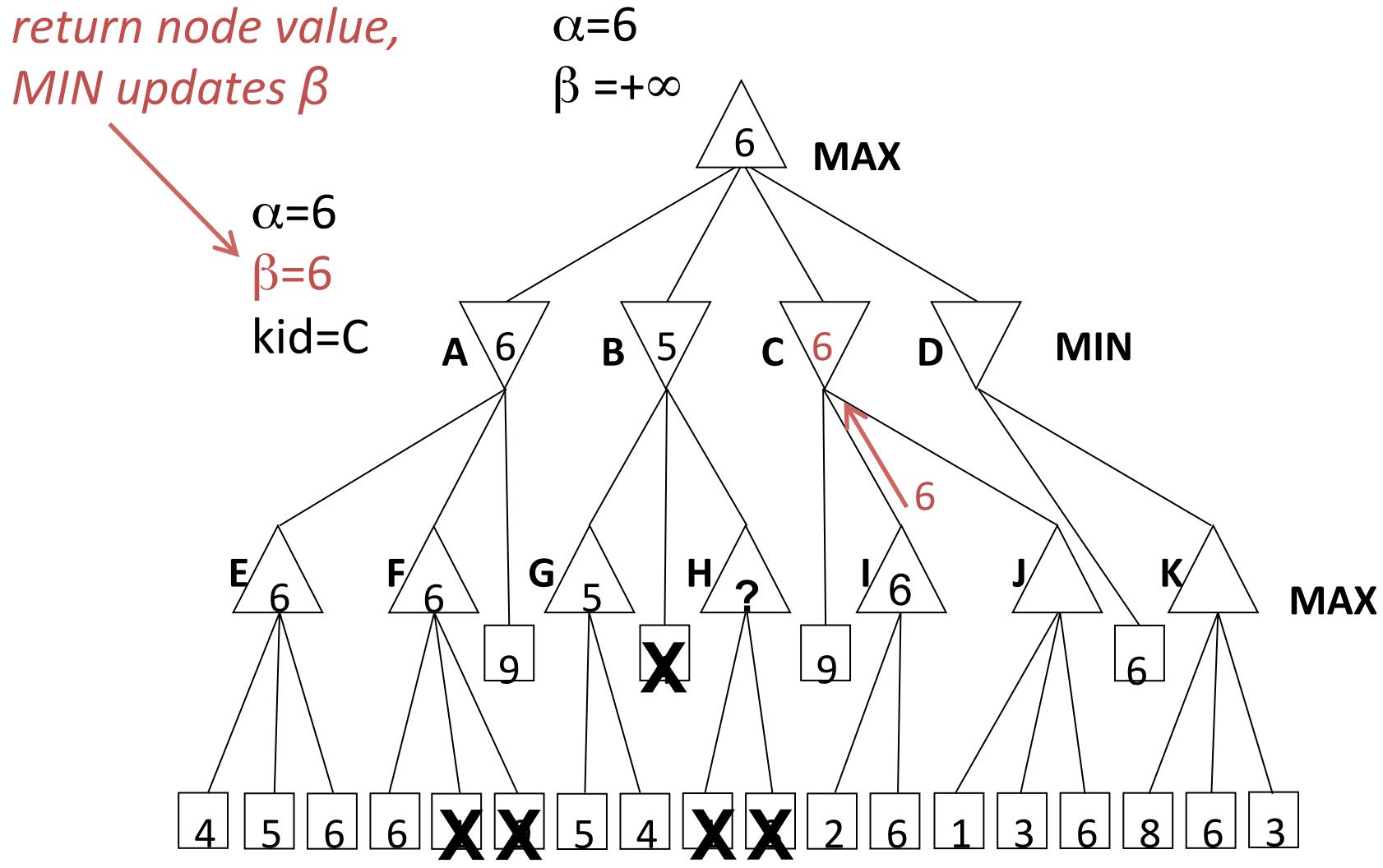
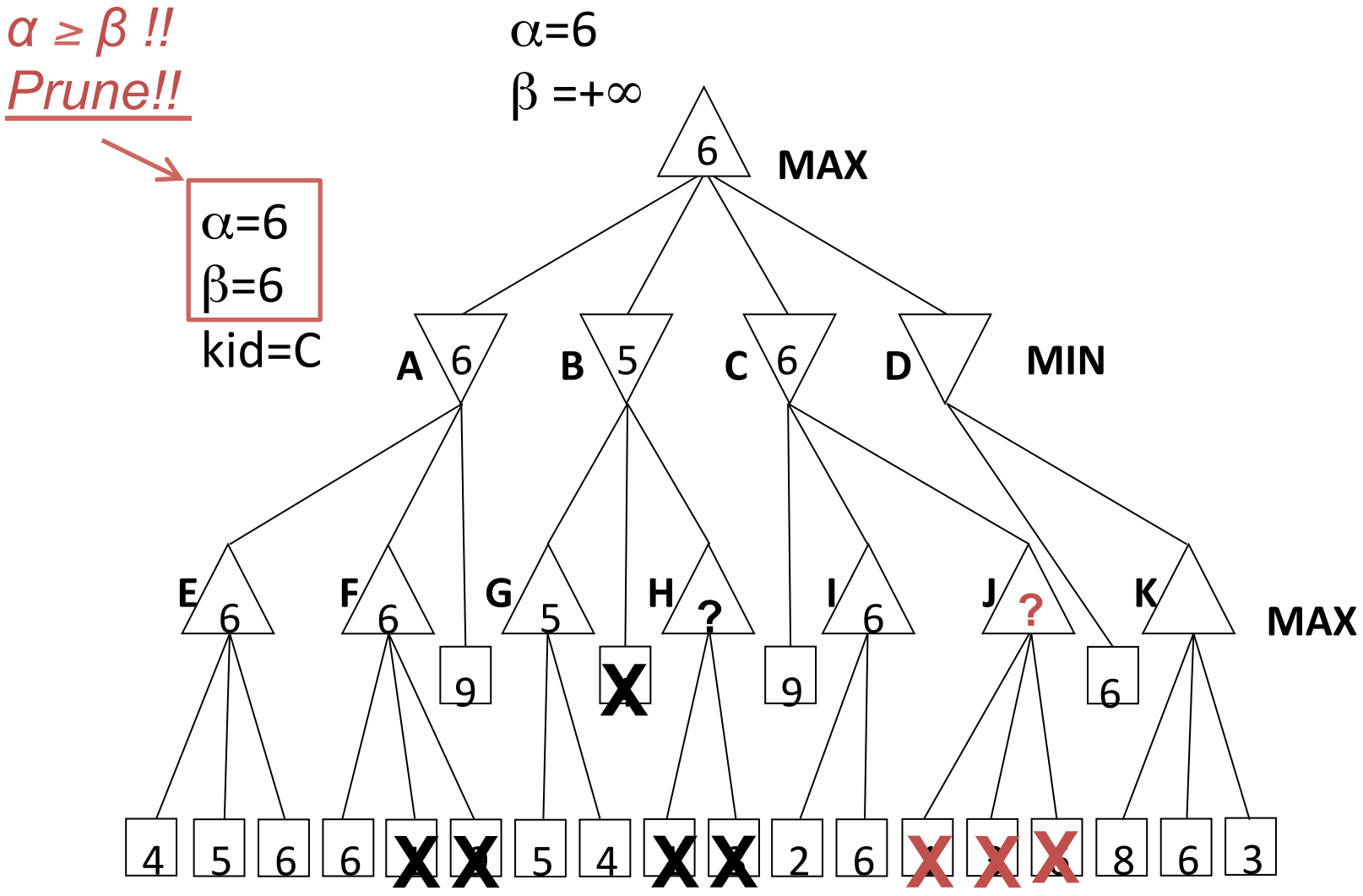


Longer Alpha-Beta Example

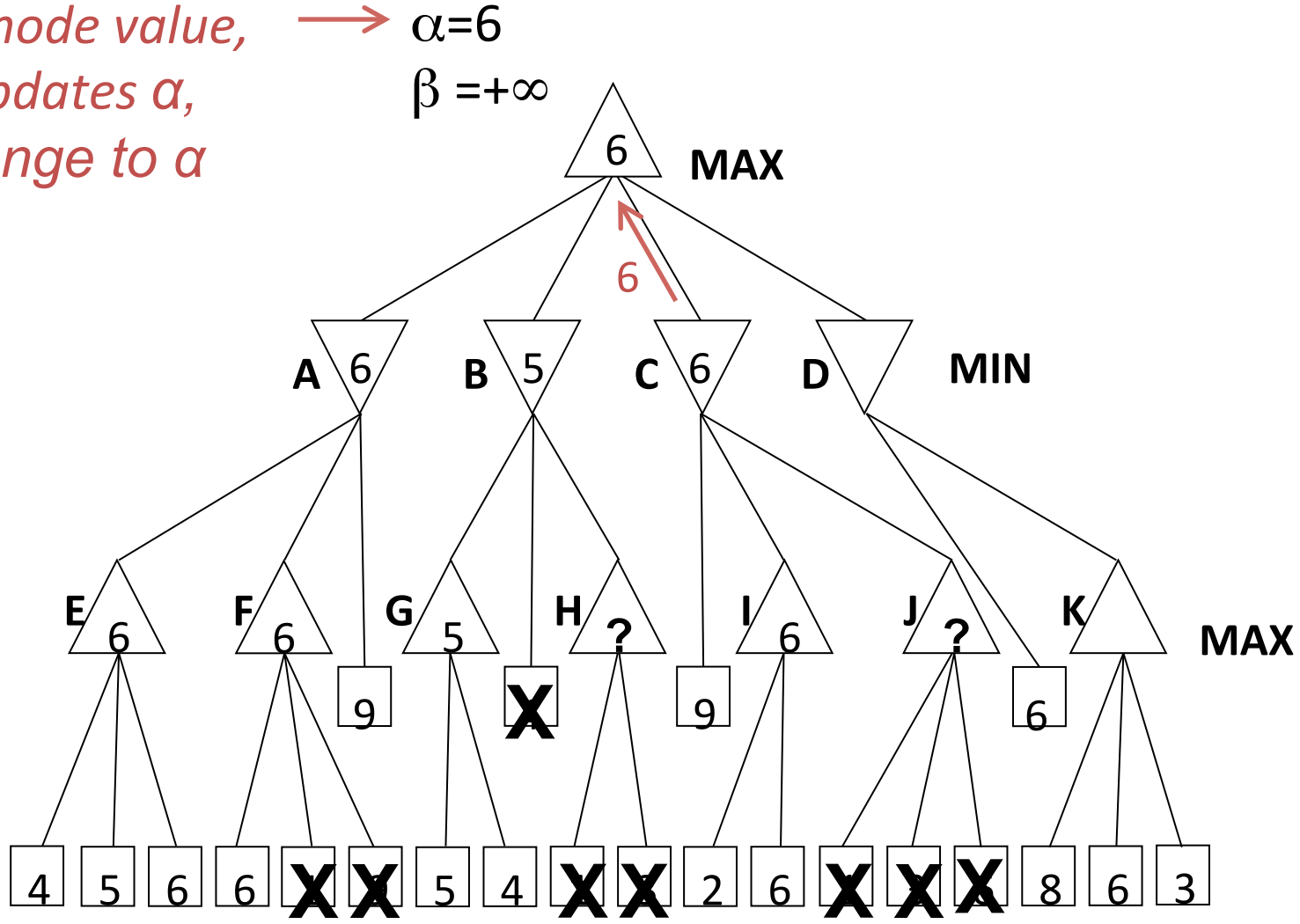


Longer Alpha-Beta Example

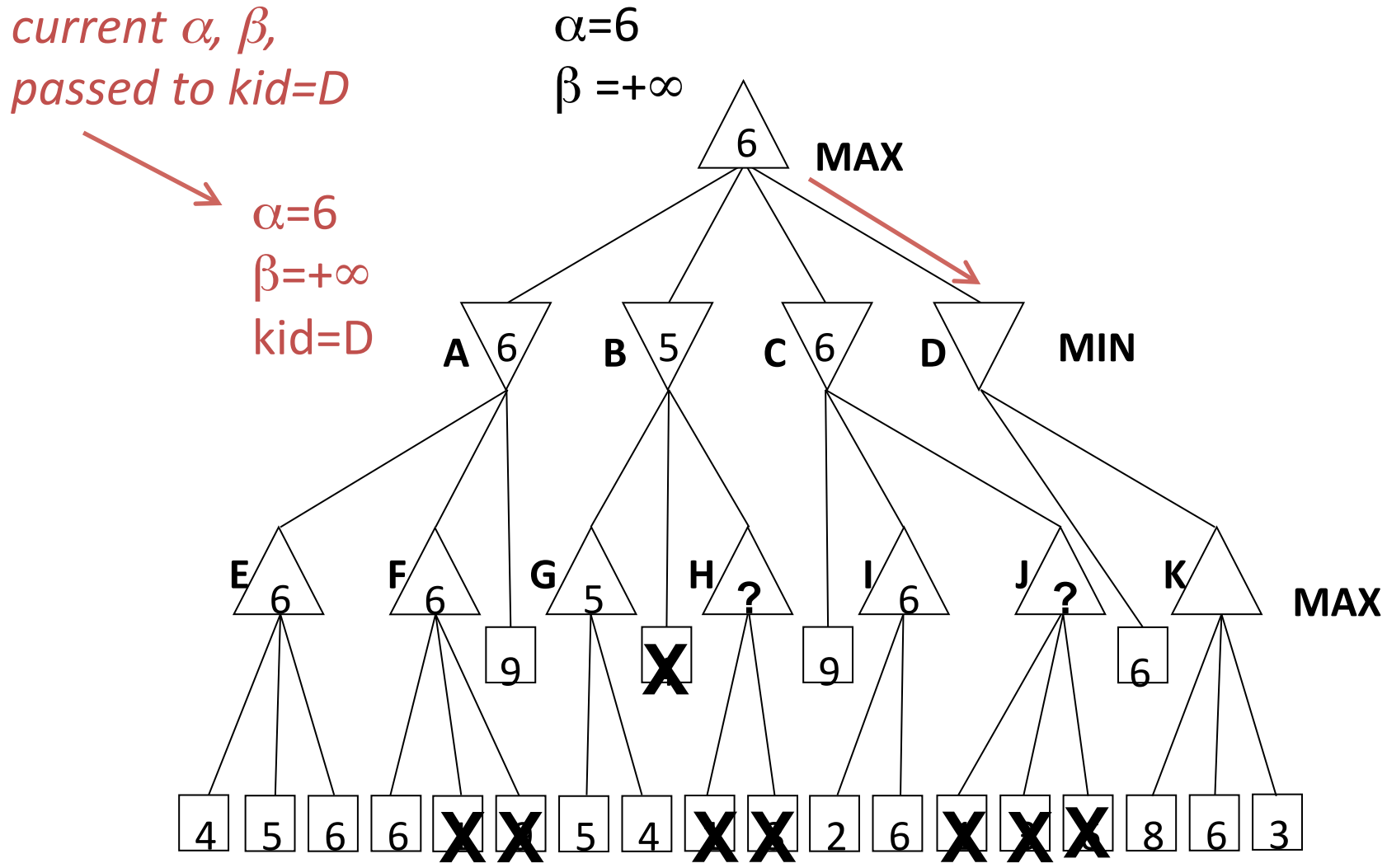


Longer Alpha-Beta Example

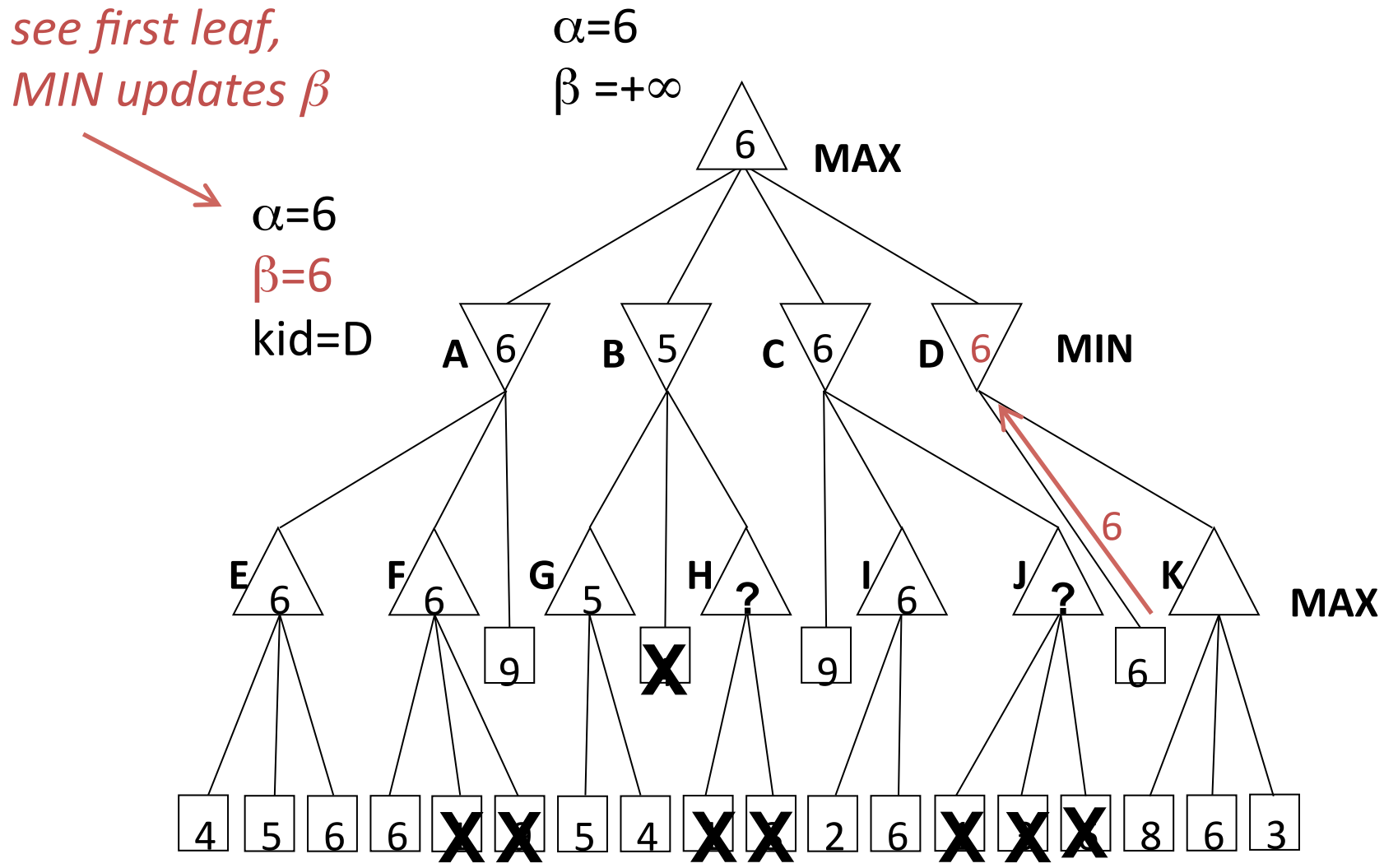
return node value,
MAX updates α,
no change to α



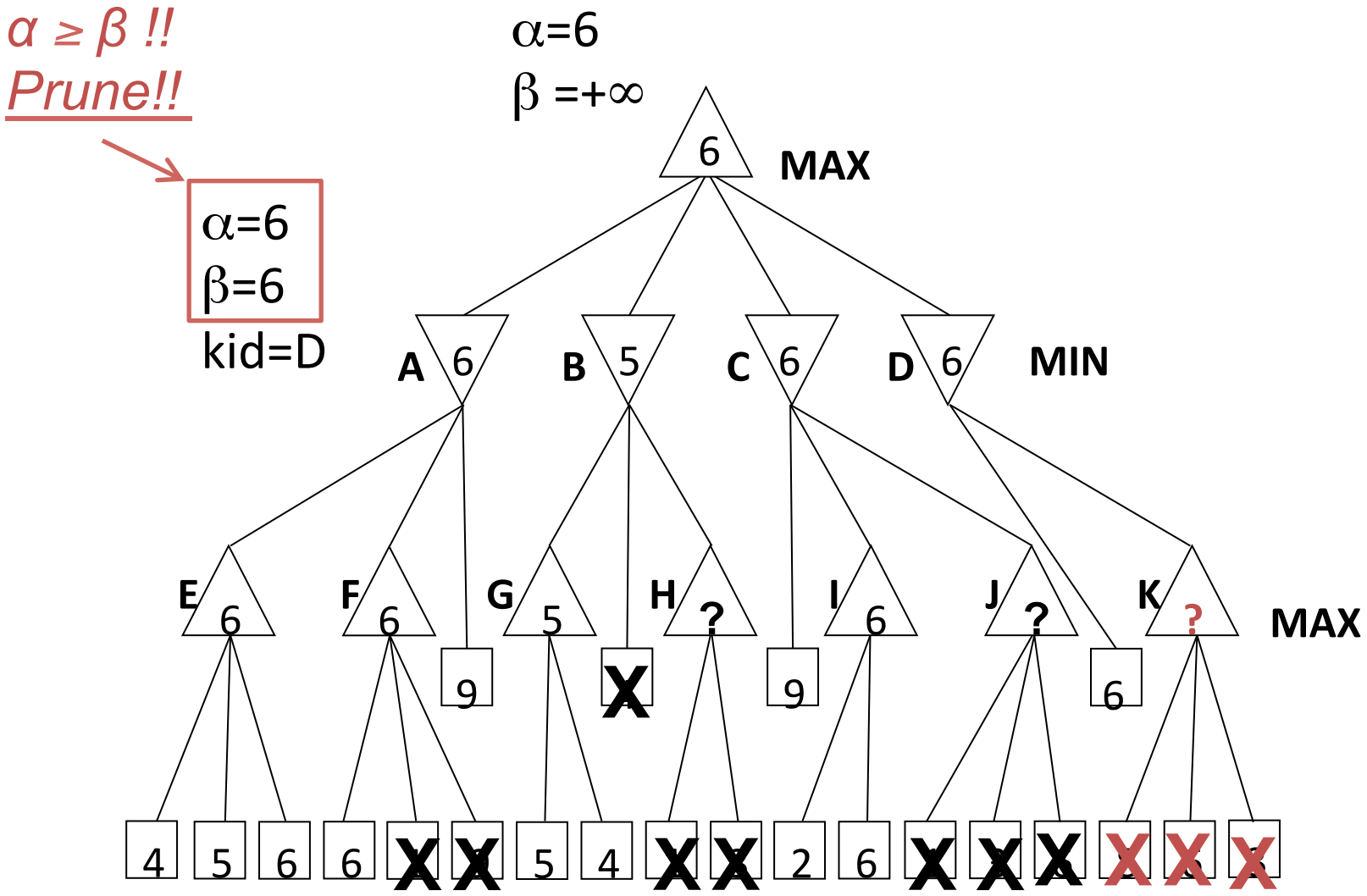
Longer Alpha-Beta Example



Longer Alpha-Beta Example



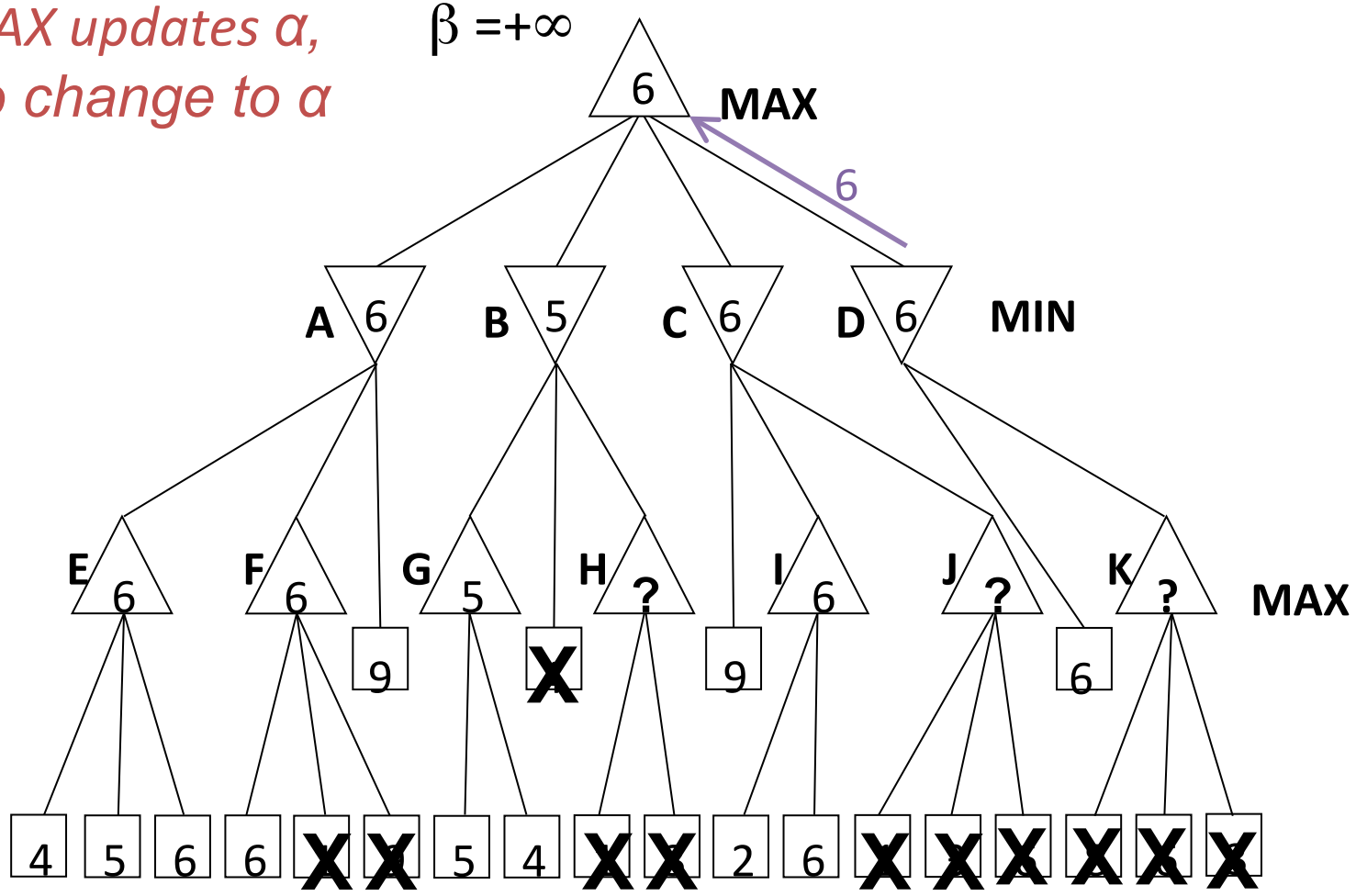
Longer Alpha-Beta Example



Alpha-Beta Example #2

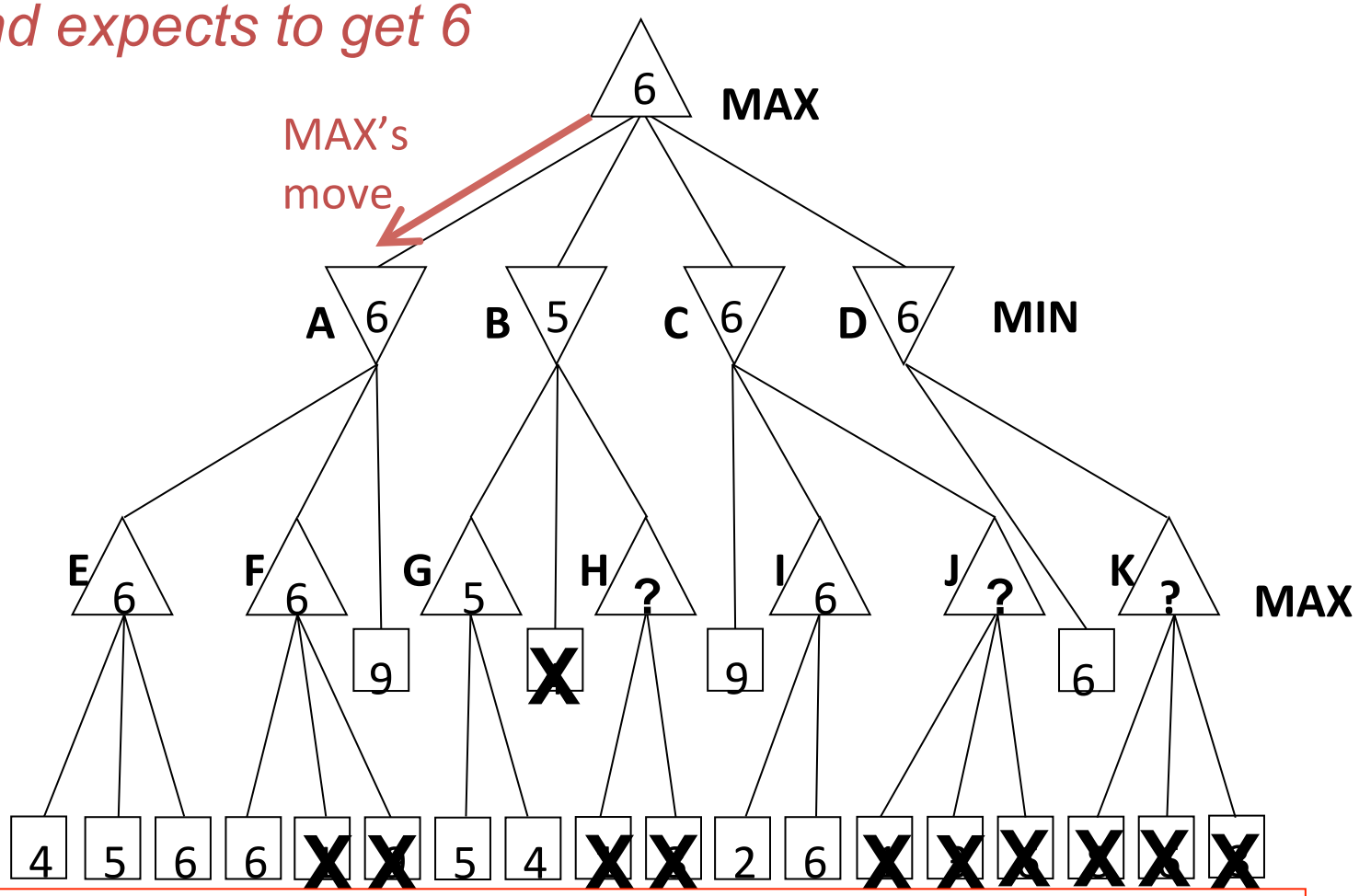
return node value,
MAX updates α,
no change to α

α=6
β =+∞



Alpha-Beta Example #2

MAX moves to A,
and expects to get 6



Although we may have changed some internal branch node return values, the final root action and expected outcome are identical to if we had not done alpha-beta pruning. Internal values may change; root values do not.

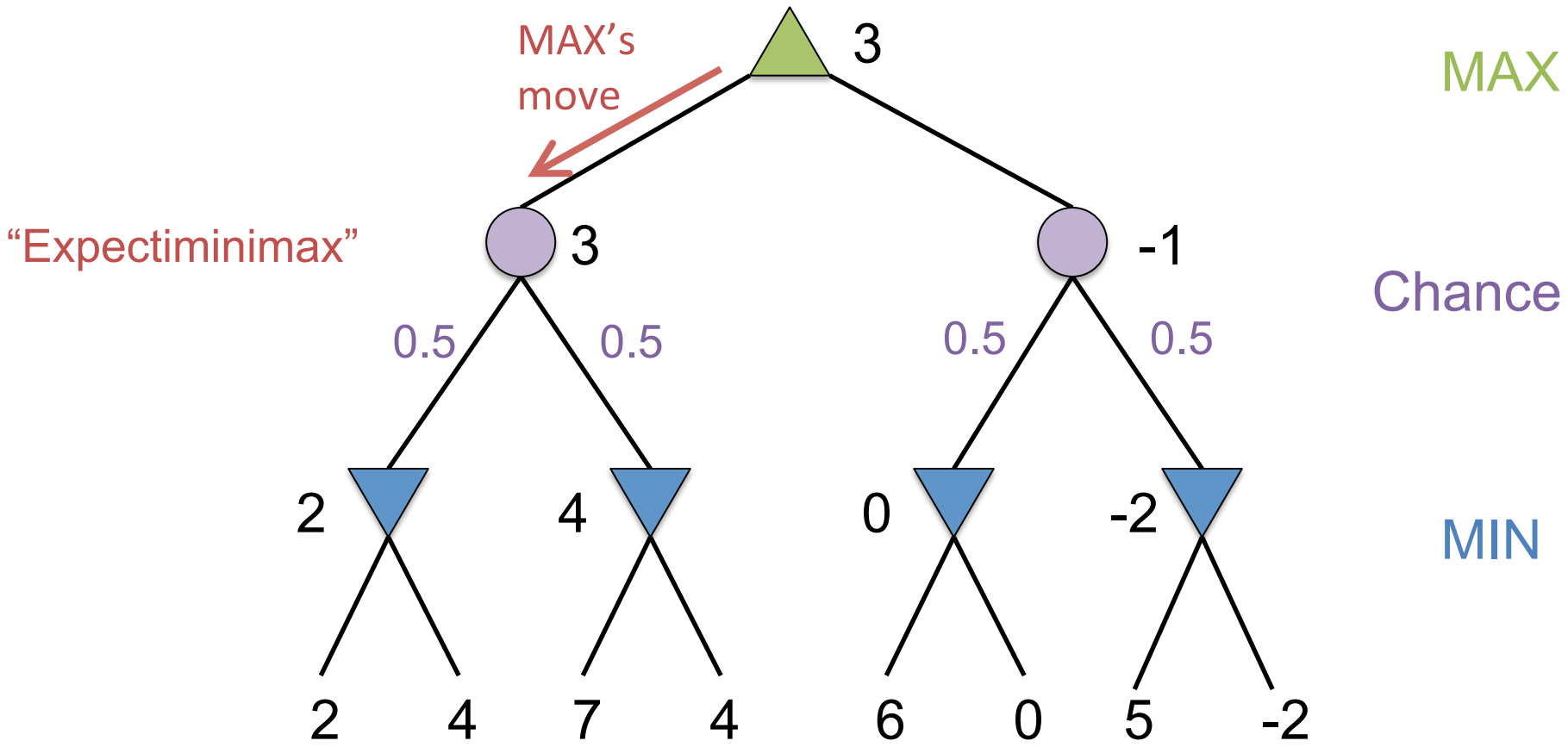
Nondeterministic games

- Ex: Backgammon
  - Roll dice to determine how far to move (random)
  - Player selects which checkers to move (strategy)

https://commons.wikimedia.org/wiki/File:Backgammon_lg.jpg

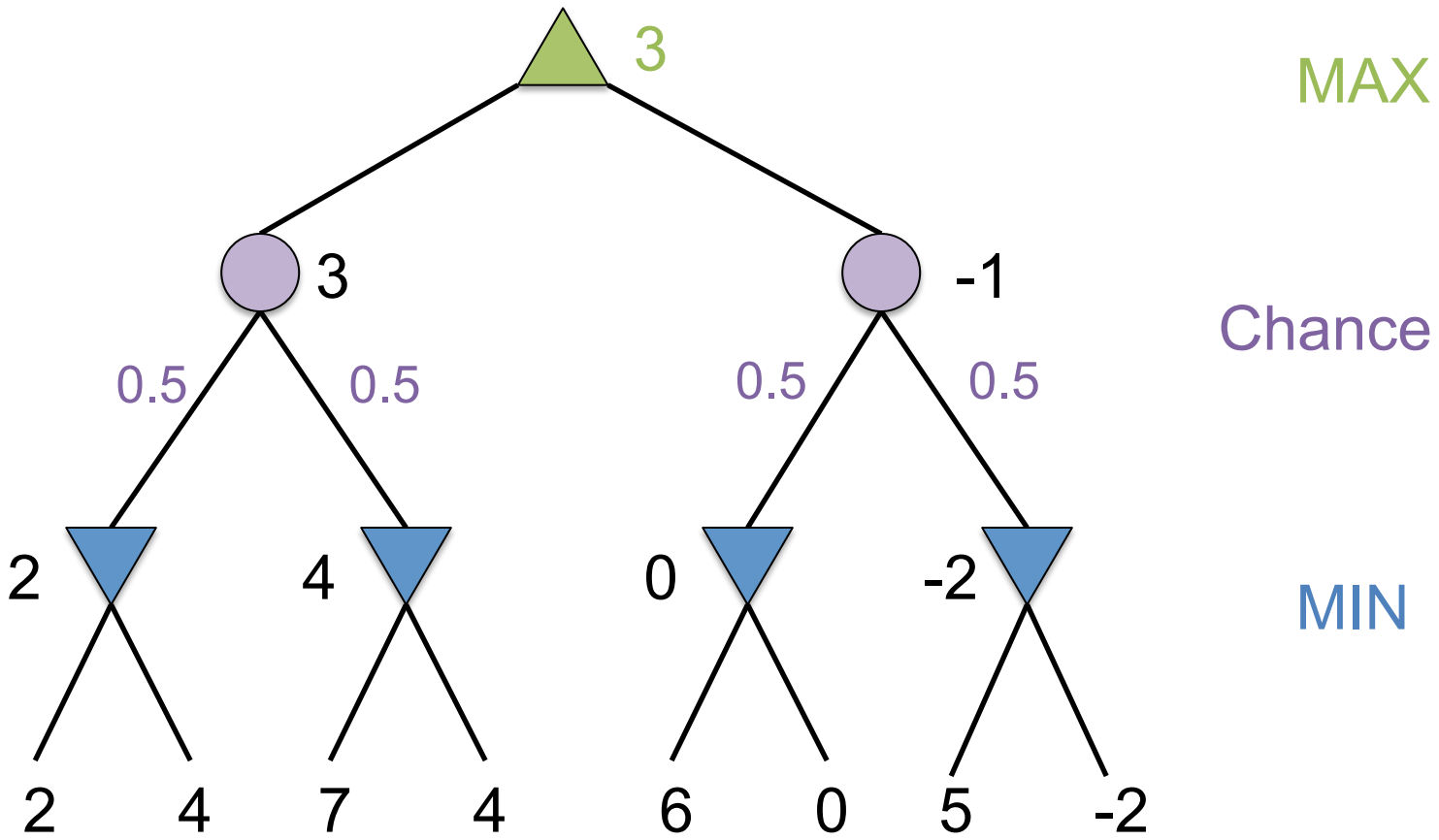
Nondeterministic games

- Chance (random effects) due to dice, card shuffle, …
- Chance nodes: expectation (weighted average) of successors
- Simplified example: coin flips



Pruning in nondeterministic games

- Can still apply a form of alpha-beta pruning



Pruning in nondeterministic games

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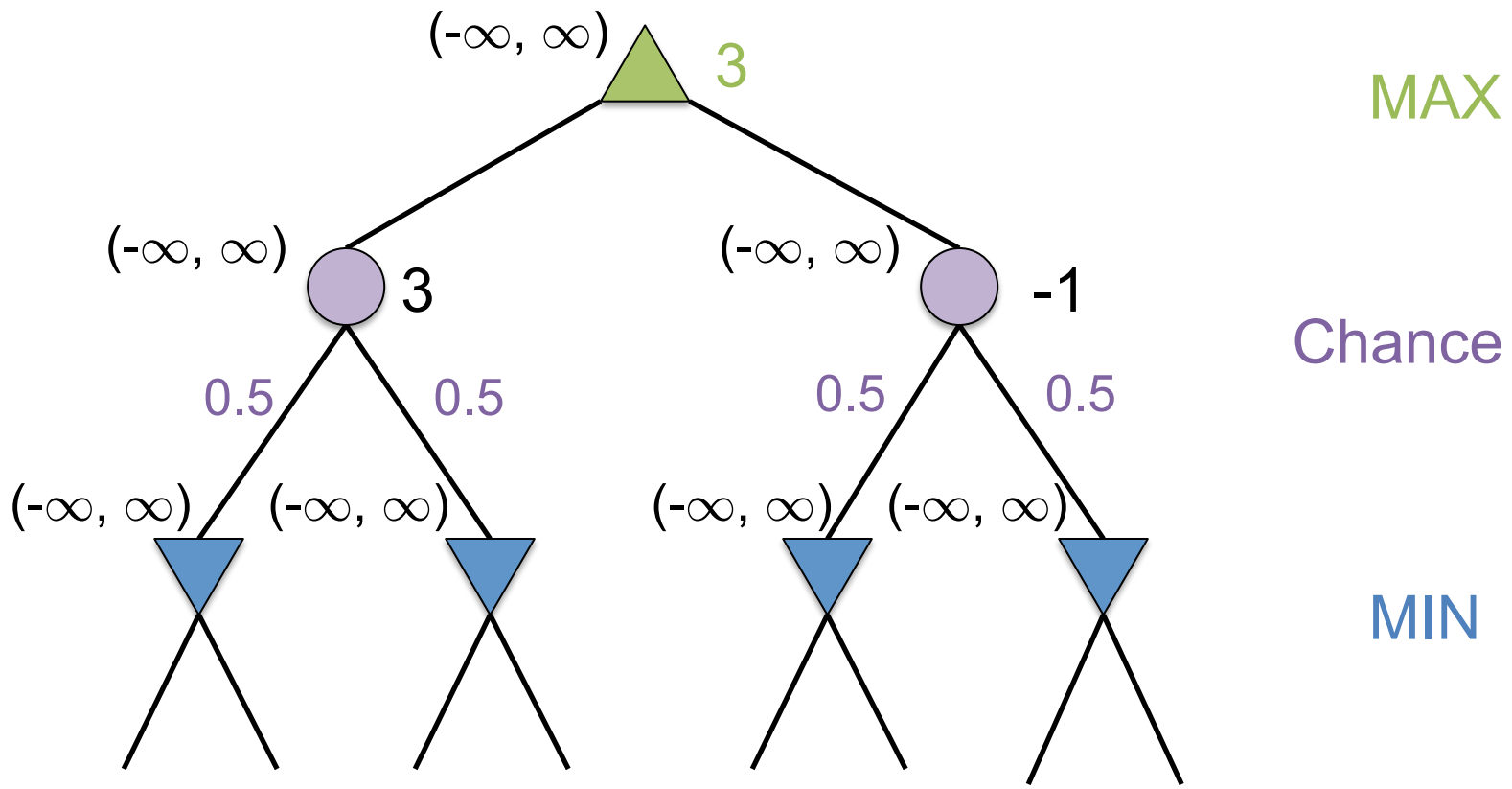
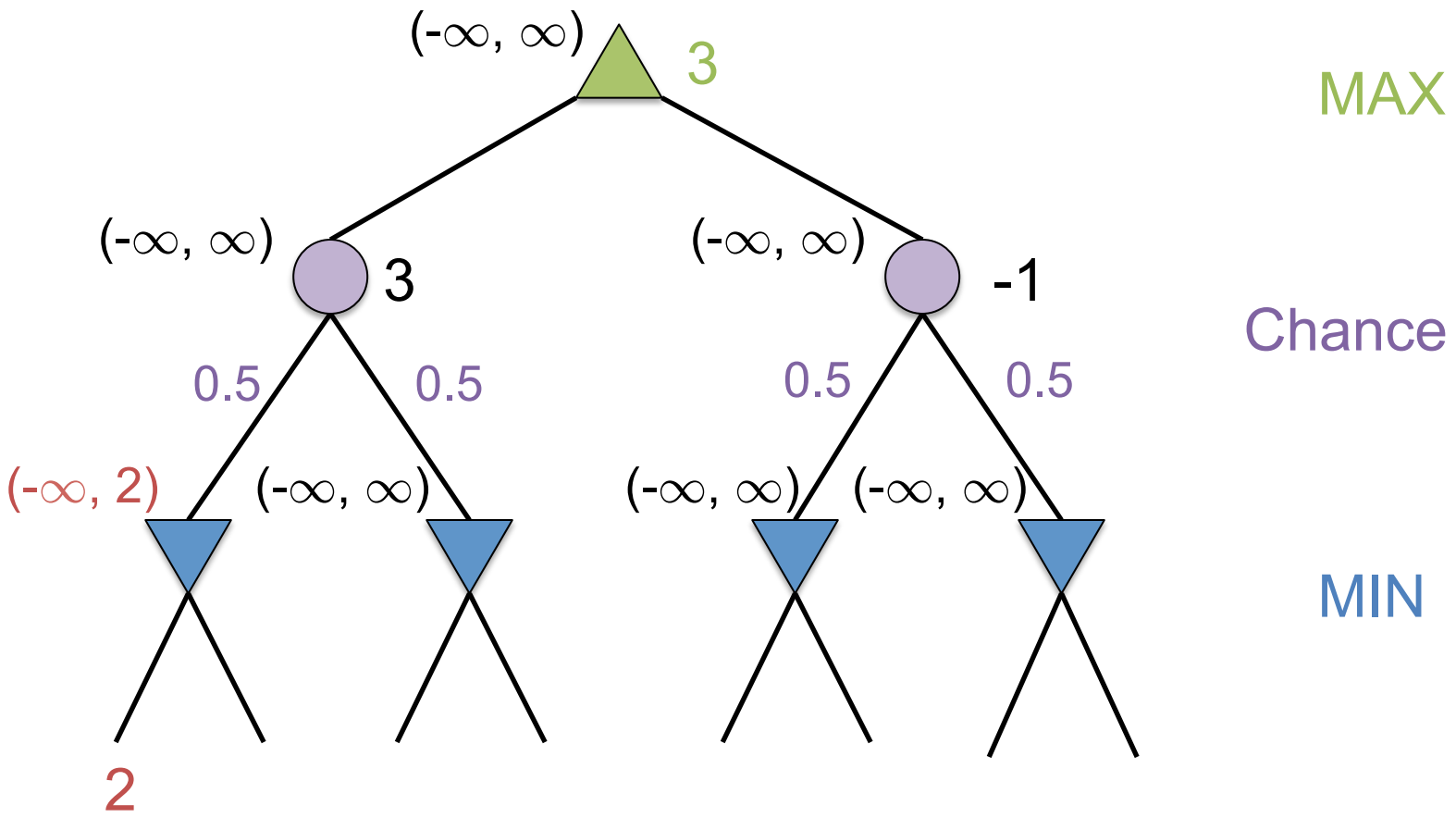


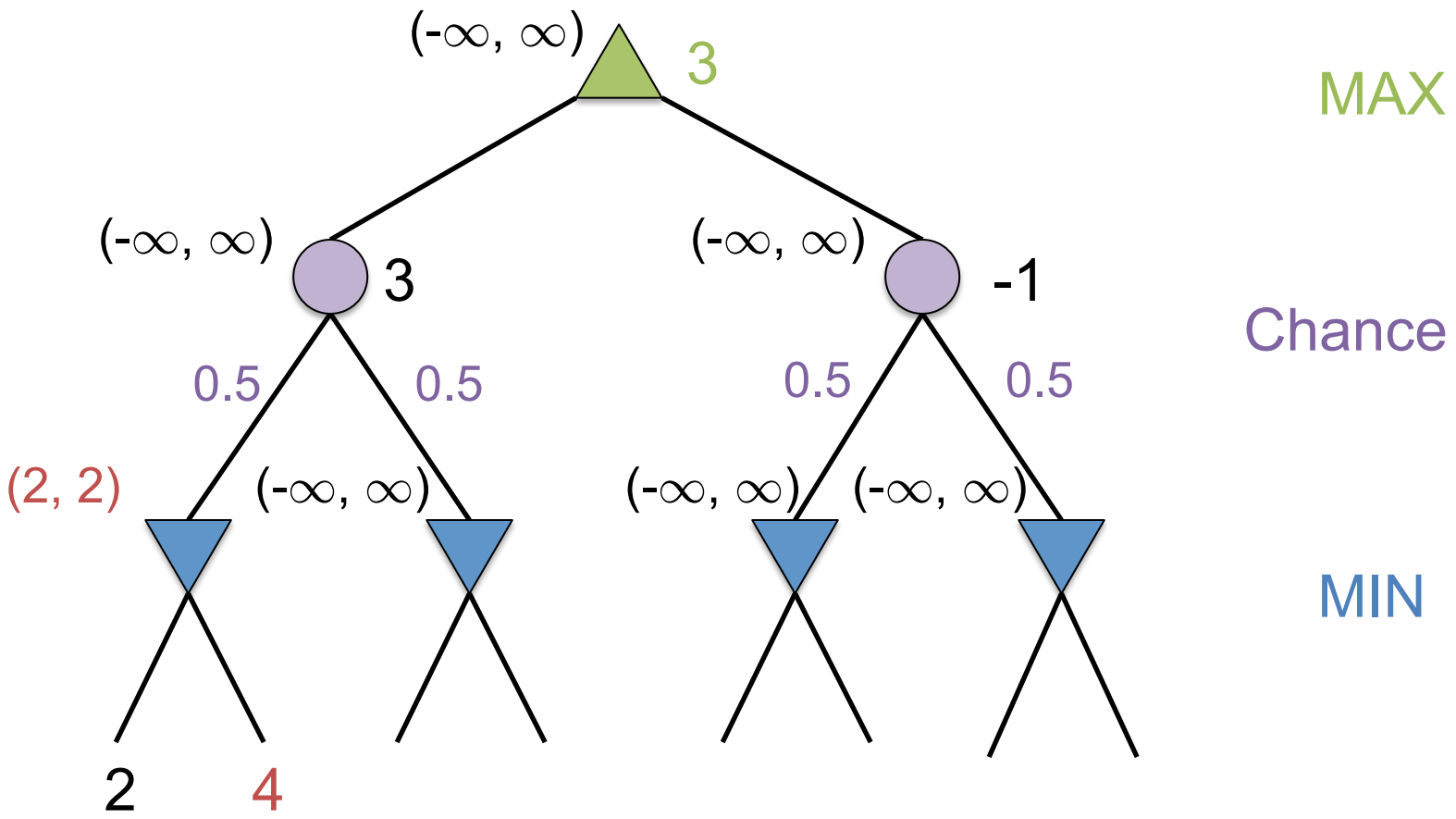
Pruning in nondeterministic games

- Can still apply a form of alpha-beta pruning



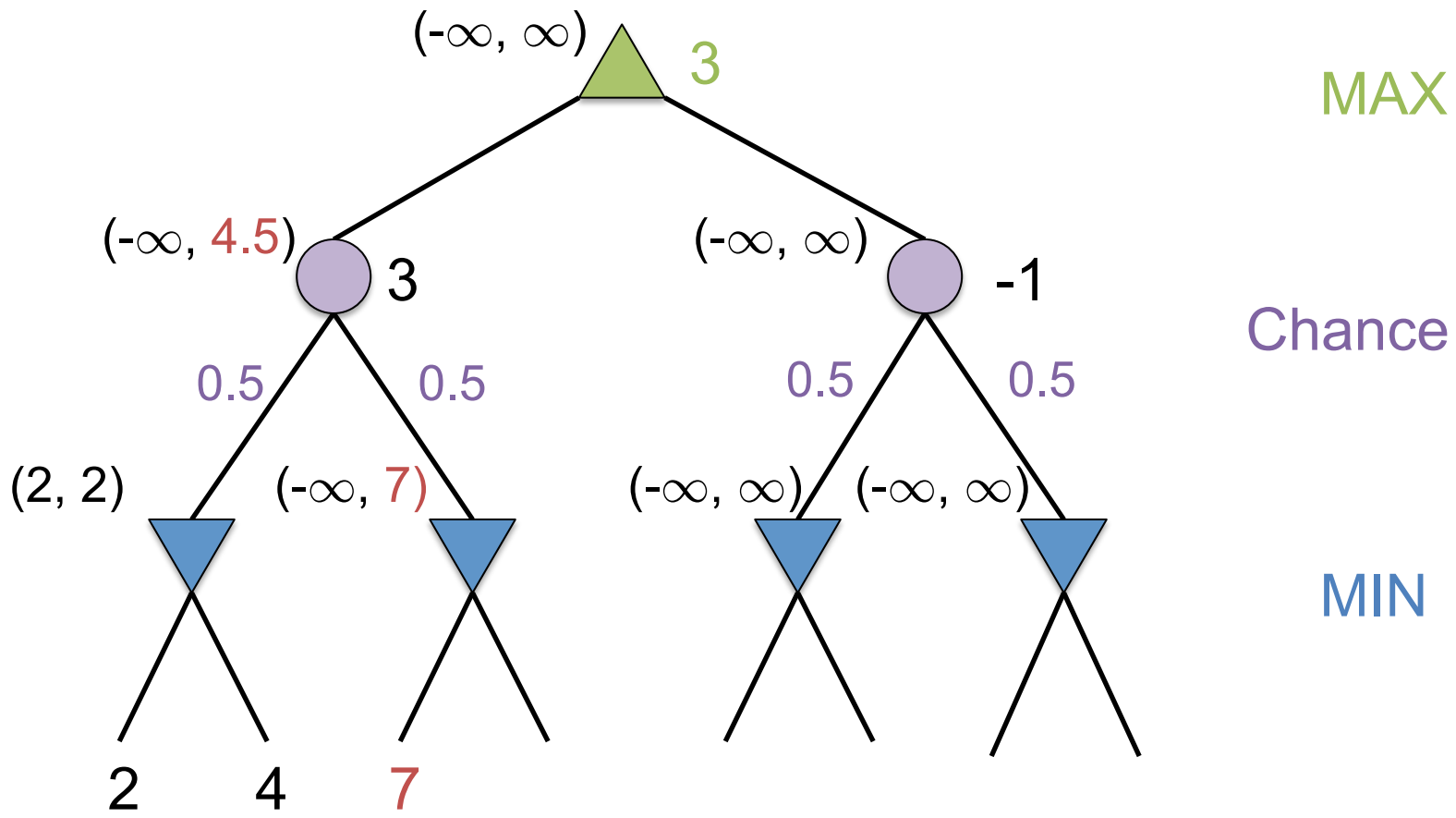
Pruning in nondeterministic games

- Can still apply a form of alpha-beta pruning



Pruning in nondeterministic games

- Can still apply a form of alpha-beta pruning



Pruning in nondeterministic games

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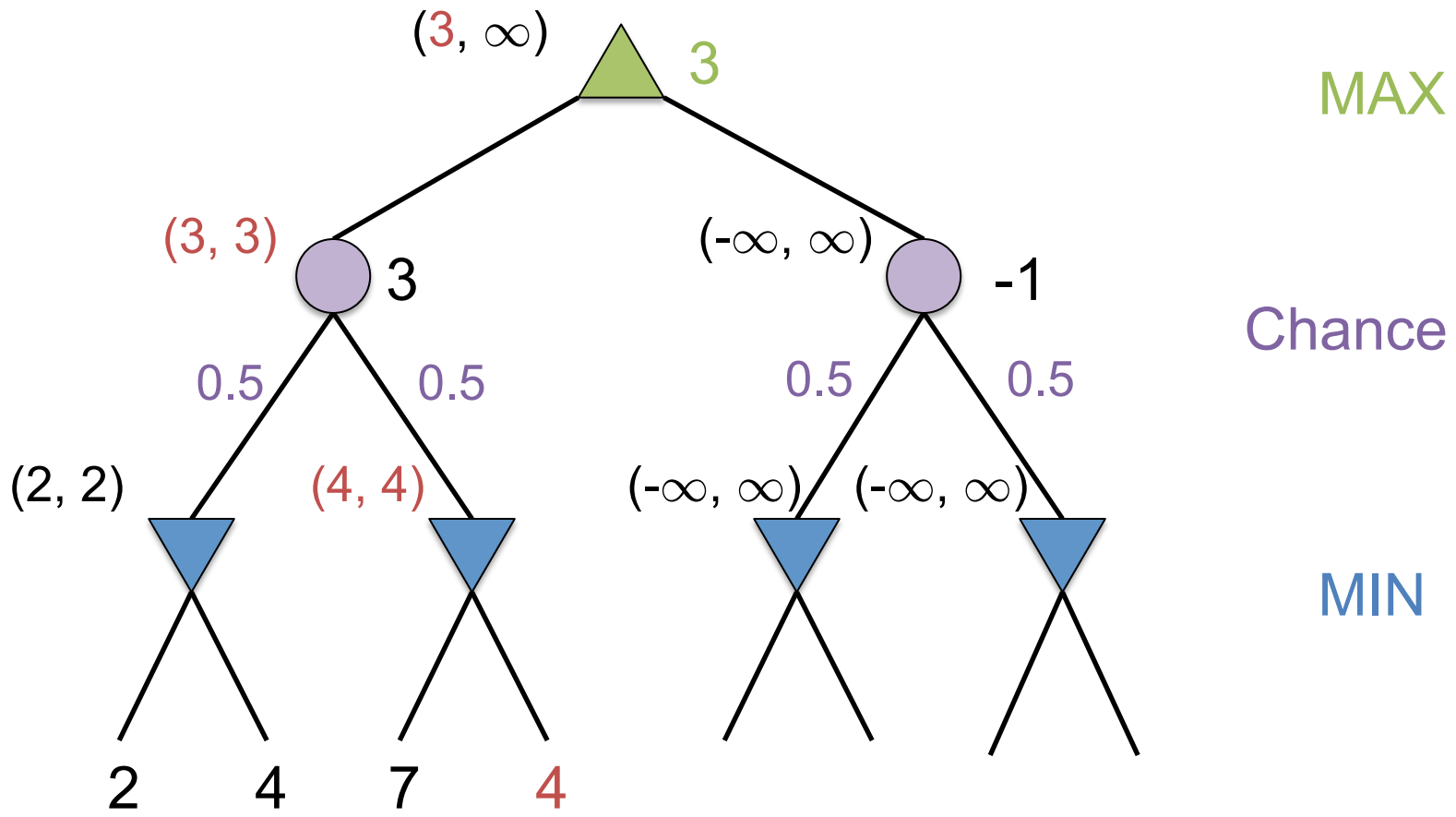
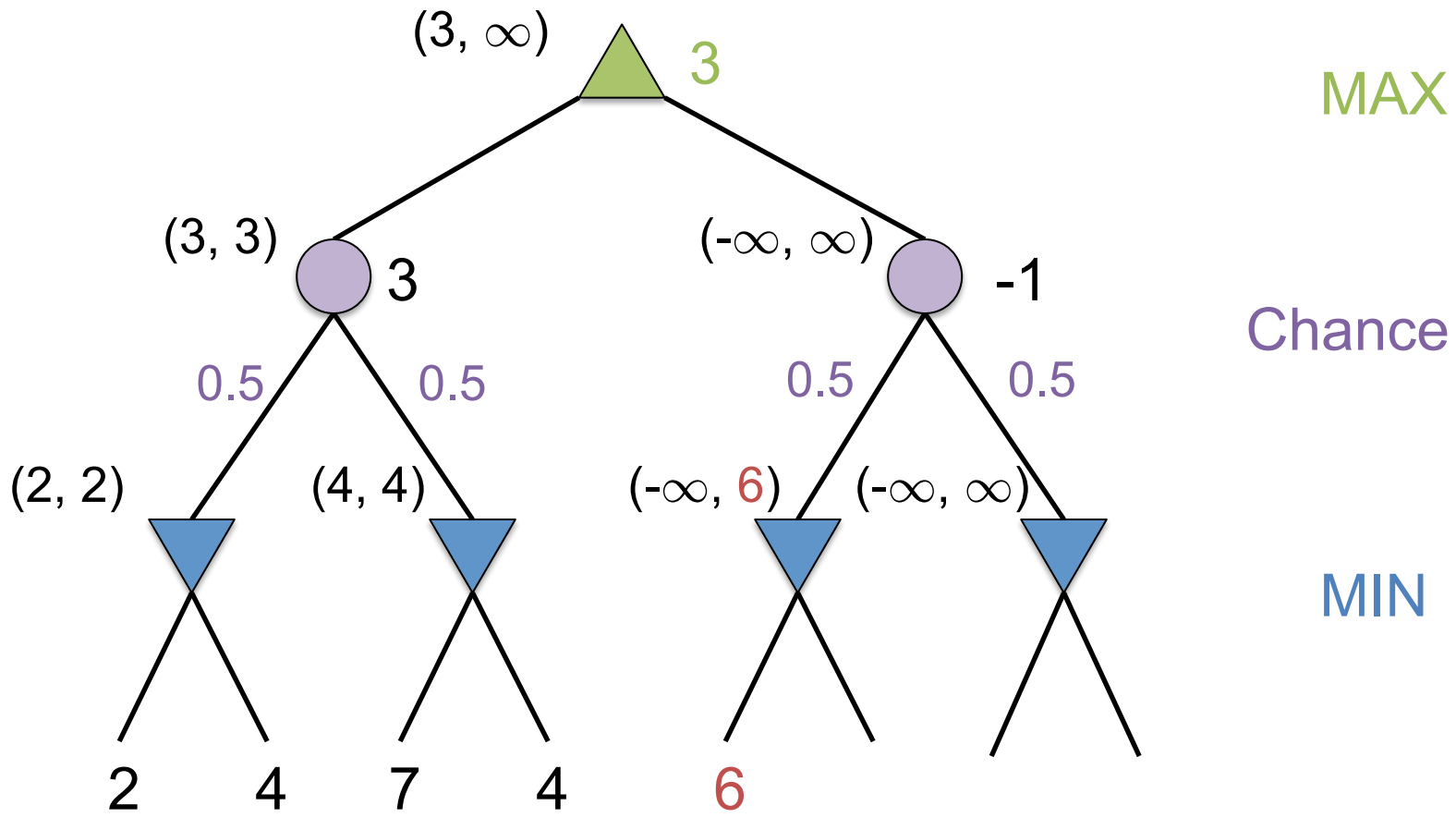


Pruning in nondeterministic games

- Can still apply a form of alpha-beta pruning



Pruning in nondeterministic games

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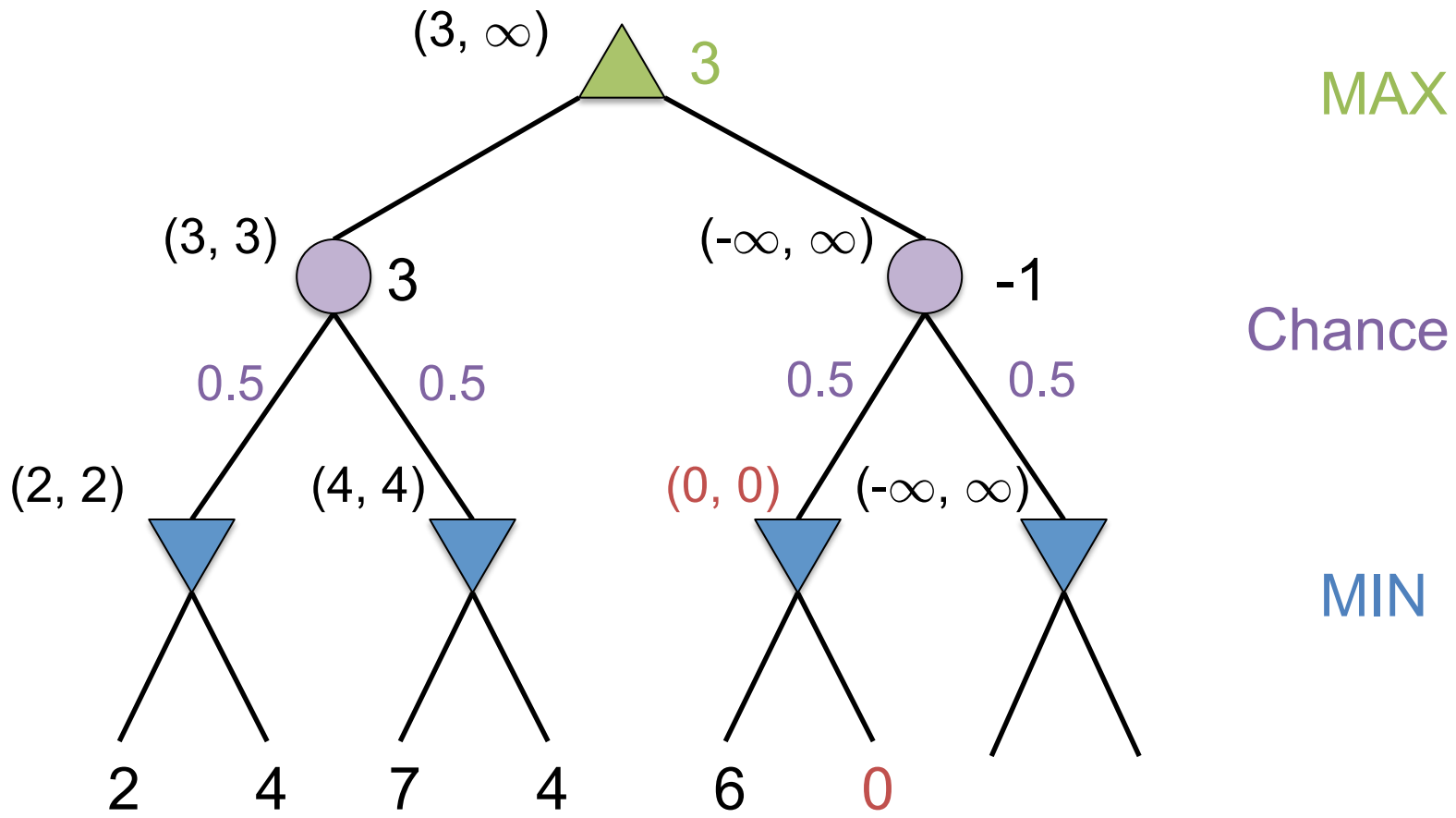
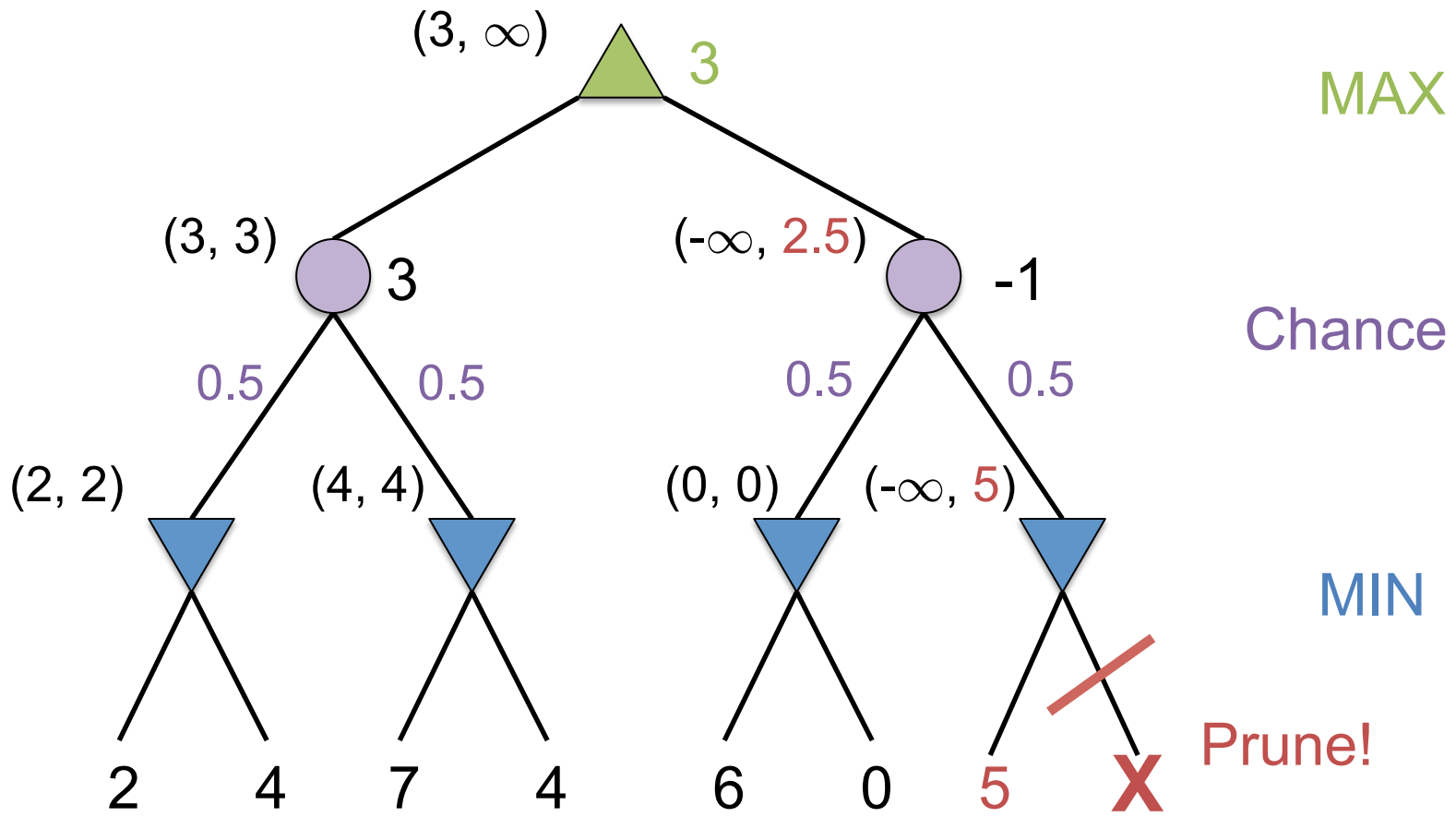


Pruning in nondeterministic games

- Can still apply a form of alpha-beta pruning

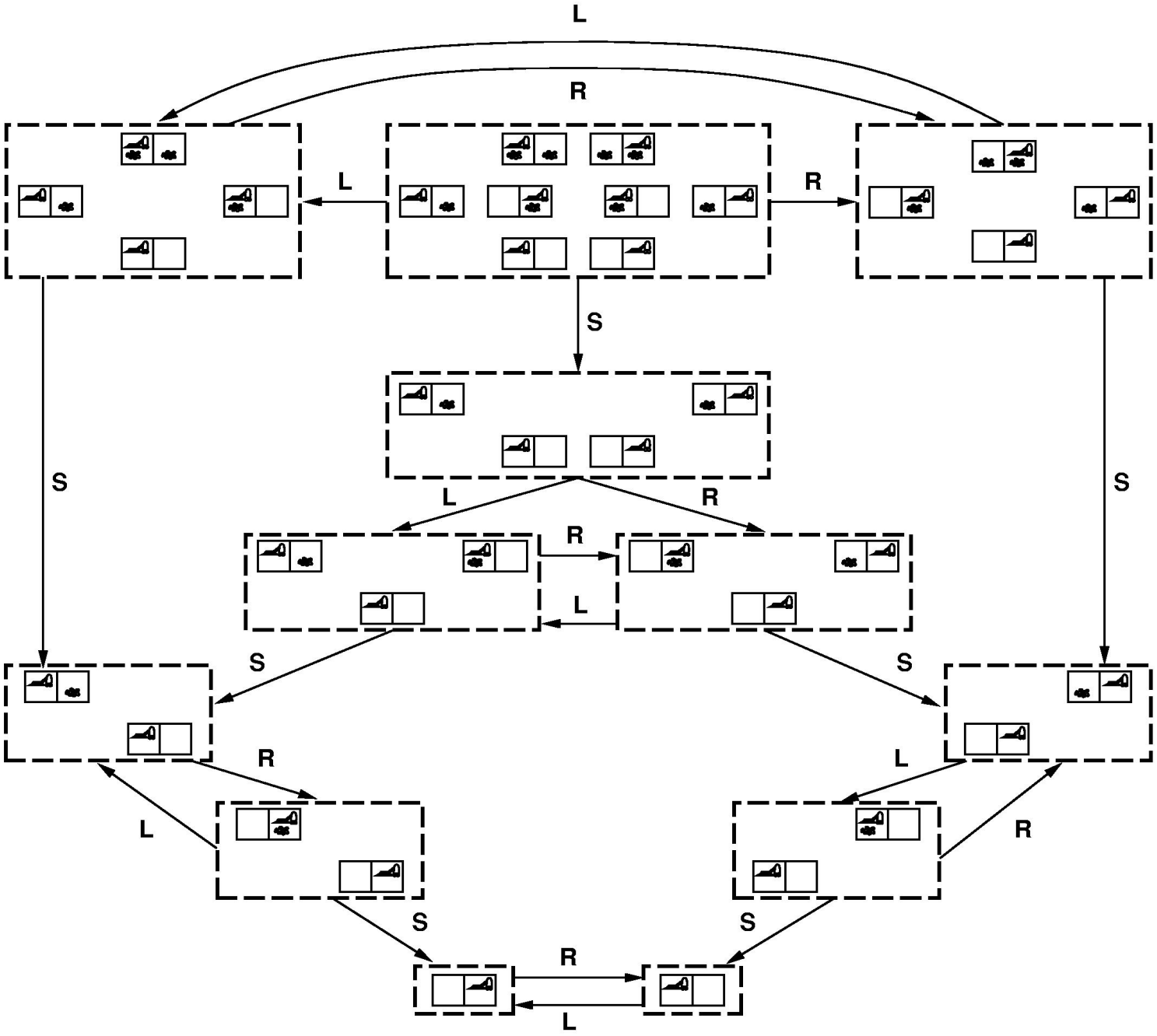


Partially observable games

- R&N Chapter 5.6 – "The fog of war"
- Background: R&N, Chapter 4.3-4
  - Searching with Nondeterministic Actions/Partial Observations

- Search through Belief States (see Fig. 4.14)
  - Agent's current belief about which states it might be in,
    given the sequence of actions & percepts to that point
- Actions(b) = ?? Union? Intersection?
  - Tricky: an action legal in one state may be illegal in another
  - Is an illegal action a NO-OP? or the end of the world?
- Transition Model:
  - Result(b,a) = { s' : s' = Result(s, a) and s is a state in b }
- Goaltest(b) = every state in b is a goal state

Belief States for Unobservable Vacuum World



Partially observable games

- R&N Chapter 5.6
- Player's current node is a belief state
- Player's move (action) generates child belief state
- Opponent's move is replaced by Percepts(s)
  - Each possible percept leads to the belief state that is consistent with that percept
- Strategy = a move for every possible percept sequence
- Minimax returns the worst state in the belief state

- Many more complications and possibilities!!
  - Opponent may select a move that is not optimal, but instead minimizes the information transmitted, or confuses the opponent
  - May not be reasonable to consider ALL moves; open P-QR3??
- **See R&N, Chapter 5.6, for more info**

The State of Play

- Checkers:
  - Chinook ended 40-year-reign of human world champion Marion Tinsley in 1994.
- Chess:
  - Deep Blue defeated human world champion Garry Kasparov in a six-game match in 1997.
- Othello:
  - human champions refuse to compete against computers:

    they are too good.
- Go:
  - AlphaGo recently (3/2016) beat 9th dan Lee Sedol
  - b > 300 (!); full game tree has > 10^760 leaf nodes (!!)
- See (e.g.) http://www.cs.ualberta.ca/~games/ for more info

High branching factors

- What can we do when the search tree is too large?
  - Ex: Go (b = 50-200+ moves per state)
  - Heuristic state evaluation (score a partial game)
- Where does this heuristic come from?
  - Hand designed
  - Machine learning on historical game patterns
  - Monte Carlo methods – play random games

Monte Carlo heuristic scoring

- Idea: play out the game randomly, and use the results as a score
  - Easy to generate & score lots of random games
  - May use 1000s of games for a node
- The basis of Monte Carlo tree search algorithms…

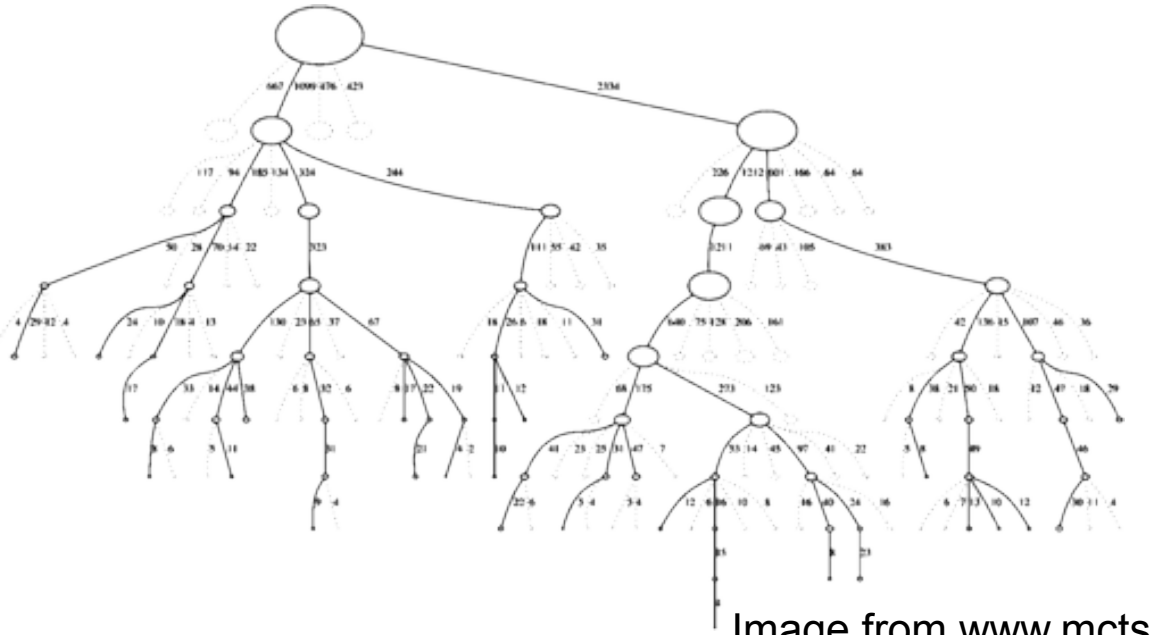


Image from www.mcts.ai

Monte Carlo Tree Search

- Should we explore the whole (top of) the tree?
  - Some moves are obviously not good…
  - Should spend time exploring / scoring promising ones
- This is a *multi-armed bandit* problem:
- Want to spend our time on good moves
- Which moves have high payout?
  - Hard to tell – random…
- *Explore* vs. *exploit* tradeoff

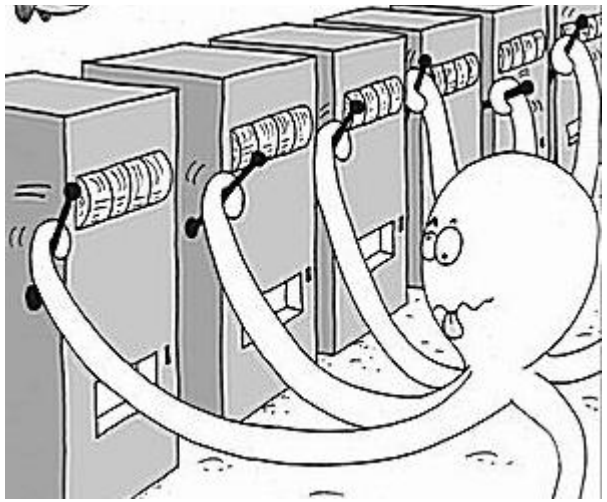
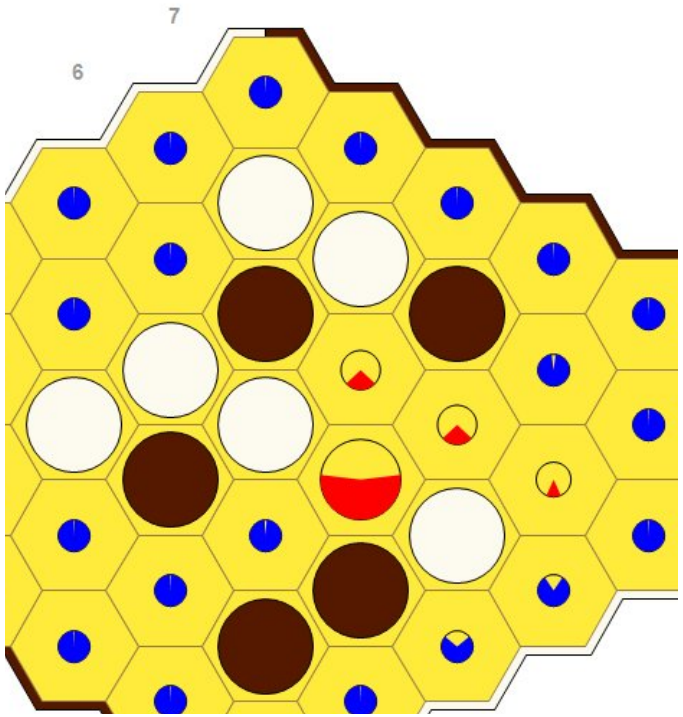
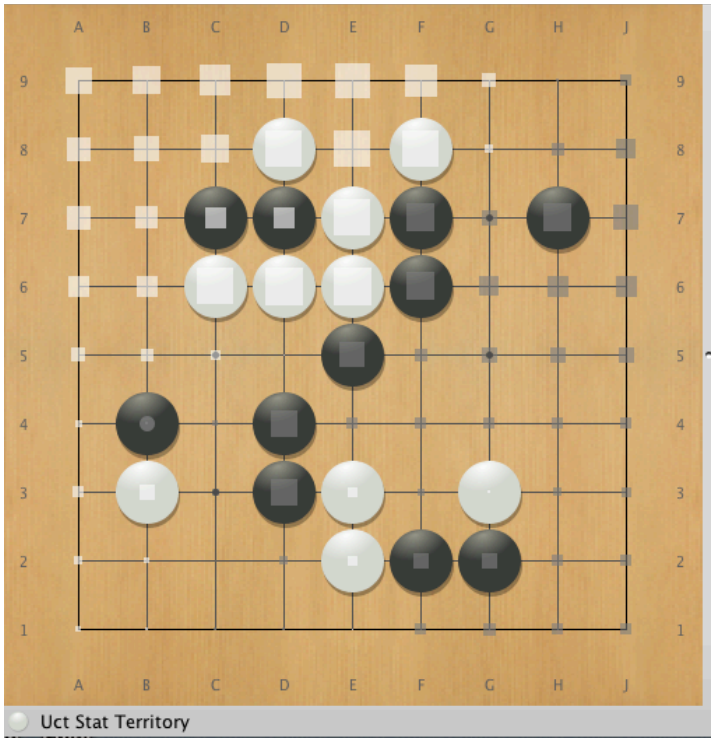


Image from Microsoft Research

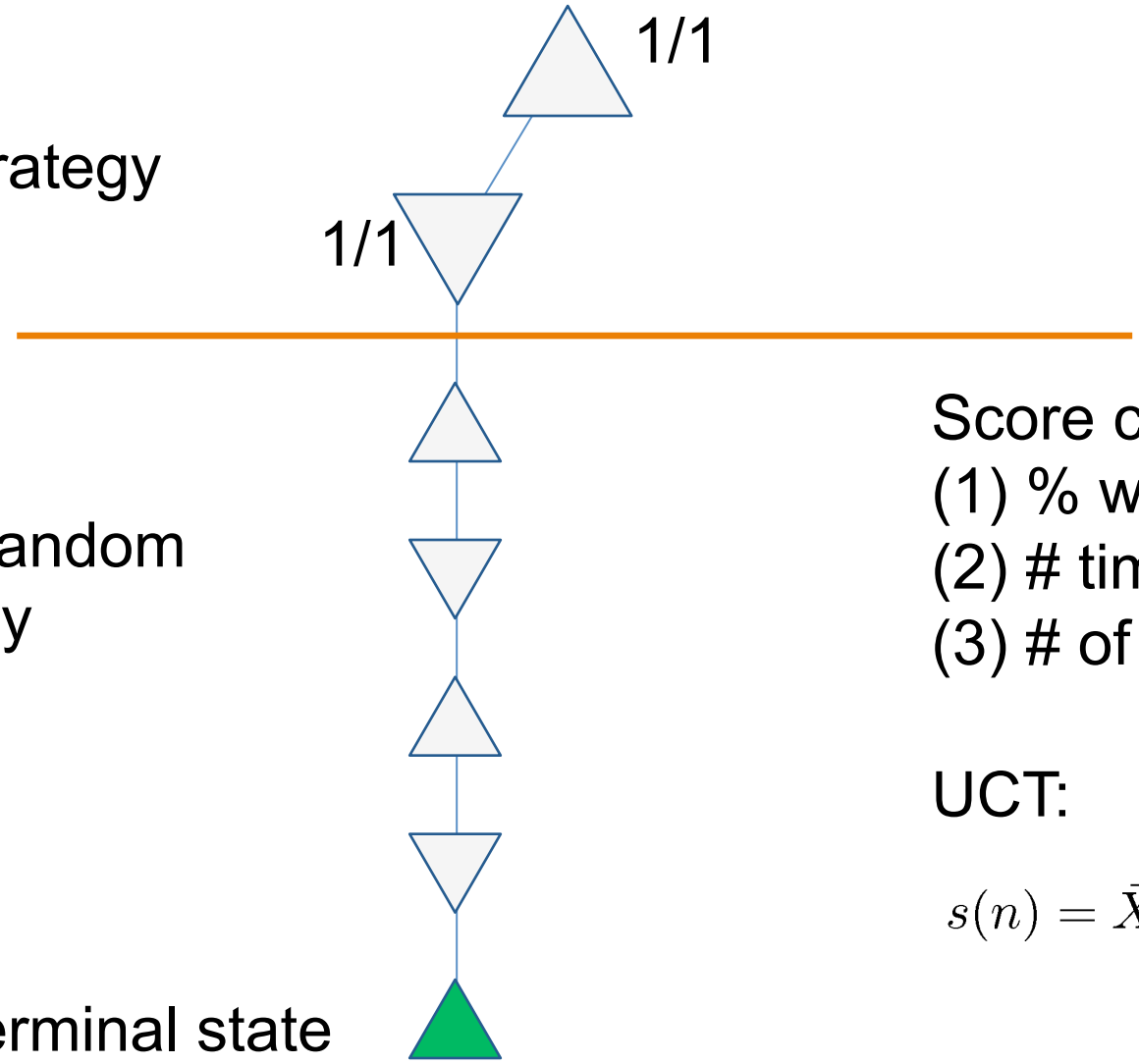
Visualizing MCTS

- At each level of the tree, keep track of
  - Number of times we've explored a path
  - Number of times we won
- Follow winning (from max/min perspective) strategies more often, but also explore others





MCTS



MAB strategy

Default / random strategy

Terminal state

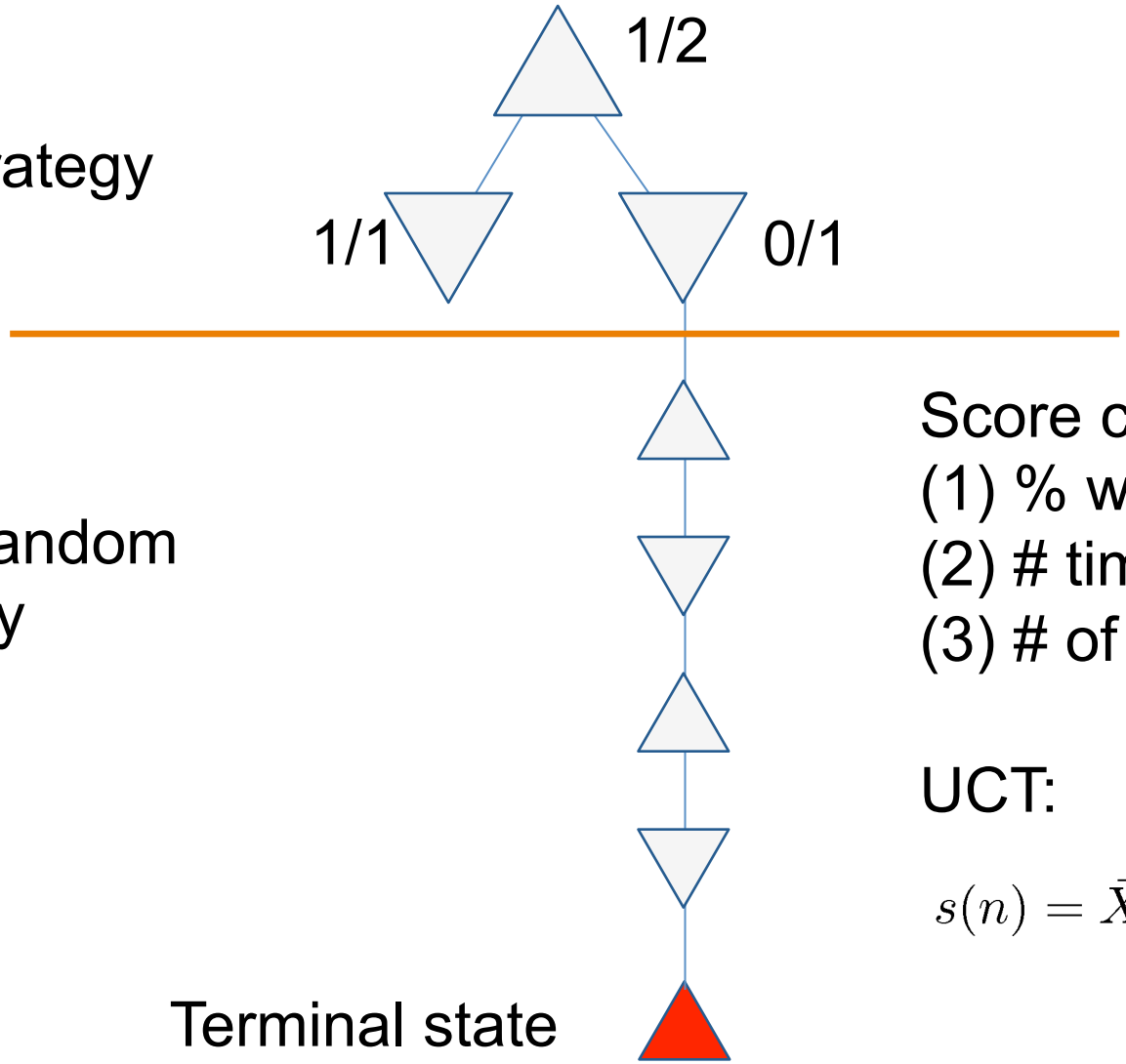
Score consists of
(1) % wins
(2) # times tried
(3) # of steps total

UCT:

$$s(n) = \bar{X}_n \pm \sqrt{\frac{\log n}{t(n)}}$$

MCTS

MAB strategy



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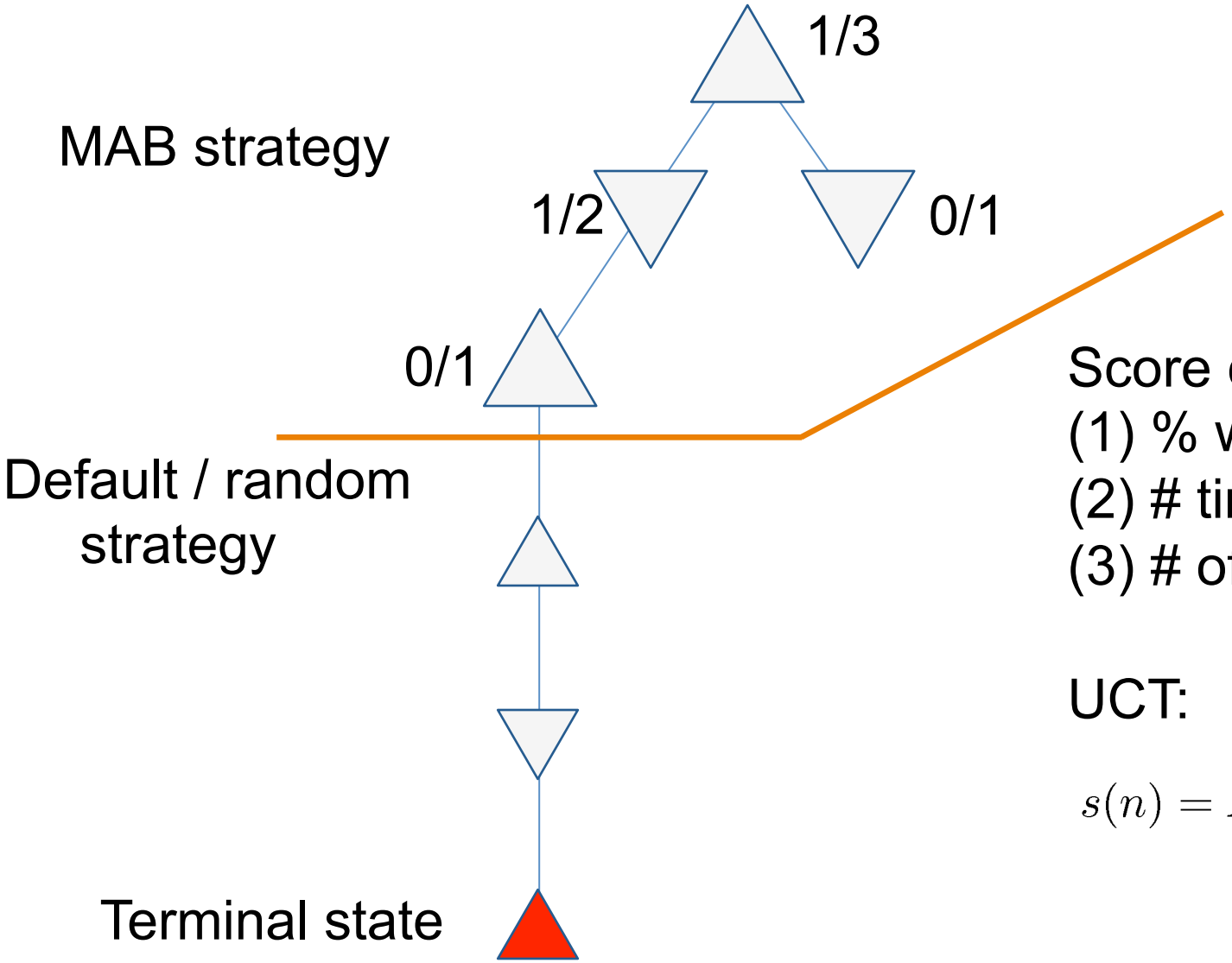
MCTS



Score consists of
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(3) # of steps total

UCT:

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Summary

- Game playing is best modeled as a search problem
- Game trees represent alternate computer/opponent moves
- Evaluation functions estimate the quality of a given board configuration for the Max player.
- Minimax is a procedure which chooses moves by assuming that the opponent will always choose the move which is best for them
- Alpha-Beta is a procedure which can prune large parts of the search tree and allow search to go deeper
- For many well-known games, computer algorithms based on heuristic search match or out-perform human world experts.