First-Order Logic
Knowledge Representation

Reading: Chapter 8, ~~9.1-9.2, 9.5.1-9.5.5~~

FOL Syntax and Semantics read: 8.1-8.2

FOL Knowledge Engineering read: 8.3-8.5

~~FOL Inference read: Chapter 9.1-9.2, 9.5.1-9.5.5~~

(Please read lecture topic material before and after each lecture on that topic)

Outline

- Review --- Syntactic Ambiguity
- Using FOL
  - Tell, Ask
- Example: Wumpus world
- Deducing Hidden Properties
  - Keeping track of change
  - Describing the results of Actions
- Set Theory in First-Order Logic
- Knowledge engineering in FOL
- The electronic circuits domain

You will be expected to know

- Seven steps of Knowledge Engineering (R&N section 8.4.1)
- Given a simple Knowledge Engineering problem, produce a simple FOL Knowledge Base that solves the problem

Review --- Syntactic Ambiguity

- FOPC provides many ways to represent the same thing.
- E.g., “Ball-5 is red.”
  - HasColor(Ball-5, Red)
    - Ball-5 and Red are objects related by HasColor.
  - Red(Ball-5)
    - Red is a unary predicate applied to the Ball-5 object.
  - HasProperty(Ball-5, Color, Red)
    - Ball-5, Color, and Red are objects related by HasProperty.
  - ColorOf(Ball-5) = Red
    - Ball-5 and Red are objects, and ColorOf() is a function.
  - HasColor(Ball-5(), Red())
    - Ball-5() and Red() are functions of zero arguments that both return an object, which objects are related by HasColor.
  - …

- This can GREATLY confuse a pattern-matching reasoner.
  - Especially if multiple people collaborate to build the KB, and they all have different representational conventions.

Review --- Syntactic Ambiguity --- Partial Solution

- FOL can be TOO expressive, can offer TOO MANY choices

- Likely confusion, especially for **teams** of Knowledge Engineers

- Different team members can make different representation choices
  - E.g., represent "Ball43 is Red." as:
    - a predicate (= verb)? E.g., "Red(Ball43)" ?
    - an object (= noun)? E.g., "Red = Color(Ball43))" ?
    - a property (= adjective)? E.g., "HasProperty(Ball43, Red)" ?

- PARTIAL SOLUTION:
  - An upon-agreed **ontology** that settles these questions
  - Ontology = what exists in the world & how it is represented
  - The Knowledge Engineering teams agrees upon an ontology BEFORE they begin encoding knowledge

Using FOL

- We want to TELL things to the KB, e.g.

TELL(KB, $\forall x, King(x) \Rightarrow Person(x)$)

TELL(KB, *King(John)*)

These sentences are assertions

- We also want to ASK things to the KB,

ASK(KB, $\exists x, Person(x)$)

these are queries or goals

The KB should return the list of x's for which Person(x) is true: {x/John,x/Richard,...}

Knowledge engineering in FOL

1. Identify the task
2. Assemble the relevant knowledge
3. Decide on a vocabulary of predicates, functions, and constants
4. Encode general knowledge about the domain
5. Encode a description of the specific problem instance
6. Pose queries to the inference procedure and get answers
7. Debug the knowledge base

FOL Version of Wumpus World

- Typical percept sentence:
  Percept([Stench,Breeze,Glitter,None,None],5)

- Actions:
  Turn(Right), Turn(Left), Forward, Shoot, Grab, Release, Climb

- To determine best action, construct query:
  $\forall$ a BestAction(a,5)

- ASK solves this and returns {a/Grab}
  - And TELL about the action.

Knowledge Base for Wumpus World

- Perception
  - $\forall$s,b,g,x,y,t Percept([s,Breeze,g,x,y],t) $\Rightarrow$ Breeze(t)
  - $\forall$s,b,x,y,t Percept([s,b,Glitter,x,y],t) $\Rightarrow$ Glitter(t)

- Reflex action
  - $\forall$t Glitter(t) $\Rightarrow$ BestAction(Grab,t)

- Reflex action with internal state
  - $\forall$t Glitter(t) $\wedge\neg$Holding(Gold,t) $\Rightarrow$ BestAction(Grab,t)

Holding(Gold,t) can not be observed: keep track of change.

Deducing hidden properties

Environment definition:

$\forall$x,y,a,b *Adjacent*([x,y],[a,b]) $\Leftrightarrow$

[a,b] $\in$ {[x+1,y], [x-1,y],[x,y+1],[x,y-1]}

Properties of locations:

$\forall$s,t *At*(Agent,s,t) $\wedge$ Breeze(t) $\Rightarrow$ Breezy(s)

Squares are breezy near a pit:

– Diagnostic rule---infer cause from effect

$\forall$s Breezy(s) $\Leftrightarrow$ $\exists$ r Adjacent(r,s) $\wedge$ Pit(r)

– Causal rule---infer effect from cause (model based reasoning)

$\forall$r Pit(r) $\Rightarrow$ [$\forall$s Adjacent(r,s) $\Rightarrow$ Breezy(s)]

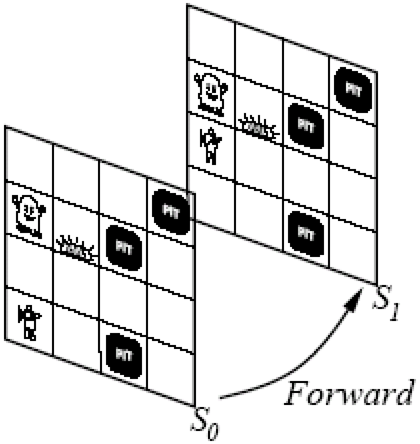
Keeping track of change

Facts hold in situations, rather than eternally
E.g., $Holding(Gold, Now)$ rather than just $Holding(Gold)$

Situation calculus is one way to represent change in FOL:

Adds a situation argument to each non-eternal predicate
E.g., Now in $Holding(Gold, Now)$ denotes a situation

Situations are connected by the $Result$ function
$Result(a, s)$ is the situation that results from doing a in s



Describing actions I

"Effect" axiom—describe changes due to action
$\forall s \;\; AtGold(s) \;\Rightarrow\; Holding(Gold, Result(Grab, s))$

"Frame" axiom—describe non-changes due to action
$\forall s \;\; HaveArrow(s) \;\Rightarrow\; HaveArrow(Result(Grab, s))$

Frame problem: find an elegant way to handle non-change
(a) representation—avoid frame axioms
(b) inference—avoid repeated "copy-overs" to keep track of state

Qualification problem: true descriptions of real actions require endless caveats—what if gold is slippery or nailed down or . . .

Ramification problem: real actions have many secondary consequences—what about the dust on the gold, wear and tear on gloves, . . .

Describing actions II

Successor-state axioms solve the representational frame problem

Each axiom is "about" a predicate (not an action per se):

P true afterwards $\Leftrightarrow$ [an action made P true
$\vee$ P true already and no action made P false]

For holding the gold:

$$\forall\, a, s \;\; Holding(Gold, Result(a, s)) \;\Leftrightarrow$$
$$[(a = Grab \wedge AtGold(s))$$
$$\vee\, (Holding(Gold, s) \wedge a \neq Release)]$$

Set Theory in First-Order Logic

Can we define set theory using FOL?
- individual sets, union, intersection, etc

Answer is yes.

Basics:
- empty set = constant = { }

- unary predicate Set( ), true for sets

- binary predicates:
  - $x \in s$ (true if x is a member of the set s)
  - $s_1 \subseteq s_2$ (true if s1 is a subset of s2)

- binary functions:
  - intersection $s_1 \cap s_2$, union $s_1 \cup s_2$, adjoining $\{x|s\}$

A Possible Set of FOL Axioms for Set Theory

The only sets are the empty set and sets made by adjoining an element to a set

$\forall s\ Set(s) \Leftrightarrow (s = \{\}\ ) \vee (\exists x,s_2\ Set(s_2) \wedge s = \{x|s_2\})$

The empty set has no elements adjoined to it

$\neg\exists x,s\ \{x|s\} = \{\}$

Adjoining an element already in the set has no effect

$\forall x,s\ x \in s \Leftrightarrow s = \{x|s\}$

The only elements of a set are those that were adjoined into it. Expressed recursively:

$\forall x,s\quad x \in s \Leftrightarrow [\ \exists y,s_2\ \ (s = \{y|s_2\} \wedge (x = y \vee x \in s_2))]$

A Possible Set of FOL Axioms for Set Theory

A set is a subset of another set iff all the first set's members are members of the 2nd set

$\forall s_1, s_2 \; s_1 \subseteq s_2 \Leftrightarrow (\forall x \; x \in s_1 \Rightarrow x \in s_2)$

Two sets are equal iff each is a subset of the other

$\forall s_1, s_2 \; (s_1 = s_2) \Leftrightarrow (s_1 \subseteq s_2 \wedge s_2 \subseteq s_1)$

An object is in the intersection of 2 sets only if a member of both

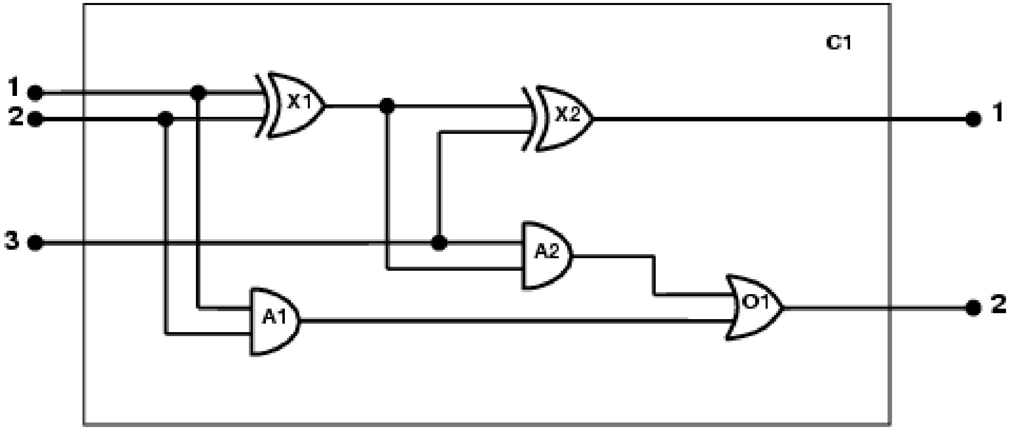
$\forall x, s_1, s_2 \; x \in (s_1 \cap s_2) \Leftrightarrow (x \in s_1 \wedge x \in s_2)$

An object is in the union of 2 sets only if a member of either

$\forall x, s_1, s_2 \; x \in (s_1 \cup s_2) \Leftrightarrow (x \in s_1 \vee x \in s_2)$

The electronic circuits domain

One-bit full adder



Possible queries:

- does the circuit function properly?
- what gates are connected to the first input terminal?
- what would happen if one of the gates is broken?

and so on

The electronic circuits domain

1. Identify the task
   - Does the circuit actually add properly?

2. Assemble the relevant knowledge
   - Composed of wires and gates; Types of gates (AND, OR, XOR, NOT)
   -
   - Irrelevant: size, shape, color, cost of gates
   -

3. Decide on a vocabulary
   - Alternatives:
   -

     Type(X_1) = XOR (function)
     Type(X_1, XOR) (binary predicate)
     XOR(X_1)
     (unary predicate)

The electronic circuits domain

4. Encode general knowledge of the domain
   - $\forall t_1,t_2\ Connected(t_1, t_2) \Rightarrow Signal(t_1) = Signal(t_2)$
   - $\forall t\ Signal(t) = 1 \vee Signal(t) = 0$
   - $1 \neq 0$
   - $\forall t_1,t_2\ Connected(t_1, t_2) \Rightarrow Connected(t_2, t_1)$
   - $\forall g\ Type(g) = OR \Rightarrow Signal(Out(1,g)) = 1 \Leftrightarrow \exists n\ Signal(In(n,g)) = 1$
   - $\forall g\ Type(g) = AND \Rightarrow Signal(Out(1,g)) = 0 \Leftrightarrow \exists n\ Signal(In(n,g)) = 0$
   - $\forall g\ Type(g) = XOR \Rightarrow Signal(Out(1,g)) = 1 \Leftrightarrow Signal(In(1,g)) \neq Signal(In(2,g))$
   - $\forall g\ Type(g) = NOT \Rightarrow Signal(Out(1,g)) \neq Signal(In(1,g))$

The electronic circuits domain

5. Encode the specific problem instance

$Type(X_1) = XOR$ $Type(X_2) = XOR$
$Type(A_1) = AND$ $Type(A_2) = AND$
$Type(O_1) = OR$

$Connected(Out(1,X_1),In(1,X_2))$
$Connected(Out(1,X_1),In(2,A_2))$
$Connected(Out(1,A_2),In(1,O_1))$
$Connected(Out(1,A_1),In(2,O_1))$
$Connected(Out(1,X_2),Out(1,C_1))$
$Connected(Out(1,O_1),Out(2,C_1))$

$Connected(In(1,C_1),In(1,X_1))$
$Connected(In(1,C_1),In(1,A_1))$
$Connected(In(2,C_1),In(2,X_1))$
$Connected(In(2,C_1),In(2,A_1))$
$Connected(In(3,C_1),In(2,X_2))$
$Connected(In(3,C_1),In(1,A_2))$

The electronic circuits domain

6. Pose queries to the inference procedure

   What are the possible sets of values of all the terminals for the adder circuit?

   $\exists i_1, i_2, i_3, o_1, o_2$ Signal(In(1,C_1)) = $i_1 \wedge$ Signal(In(2,C_1)) = $i_2 \wedge$ Signal(In(3,C_1)) = $i_3 \wedge$ Signal(Out(1,C_1)) = $o_1 \wedge$ Signal(Out(2,C_1)) = o_2

7. Debug the knowledge base

   May have omitted assertions like $1 \neq 0$

Review --- Knowledge engineering in FOL

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Summary

- First-order logic:
  - Much more expressive than propositional logic
  - Allows objects and relations as semantic primitives
  - Universal and existential quantifiers
  - syntax: constants, functions, predicates, equality, quantifiers
  -

- Knowledge engineering using FOL
  - Capturing domain knowledge in logical form

- Inference and reasoning in FOL
  - Next lecture

- Required Reading:
  - All of Chapter 8
  - Next lecture: Chapter 9