Knowledge Representation using First-Order Logic (Part II)

Reading: Chapter 8, 9.1-9.2

First lecture slides read: 8.1-8.2

Second lecture slides read: 8.3-8.4

Third lecture slides read: Chapter 9.1-9.2

(lecture slides spread across two class sessions)

(Please read lecture topic material before and after each lecture on that topic)

Aside: More syntactic sugar --- uniqueness

- $\exists!$ x is “syntactic sugar” for “There exists a unique x”
  - “There exists one and only one x”
  - “There exists exactly one x”
- For example, E! x PresidentOfTheUSA(x)
- This is just syntactic sugar:
  - $\exists! \; x \; P(x)$ is the same as $\exists \; x \; P(x) \wedge (\forall \; y \; P(y) \Rightarrow (x = y))$

Outline

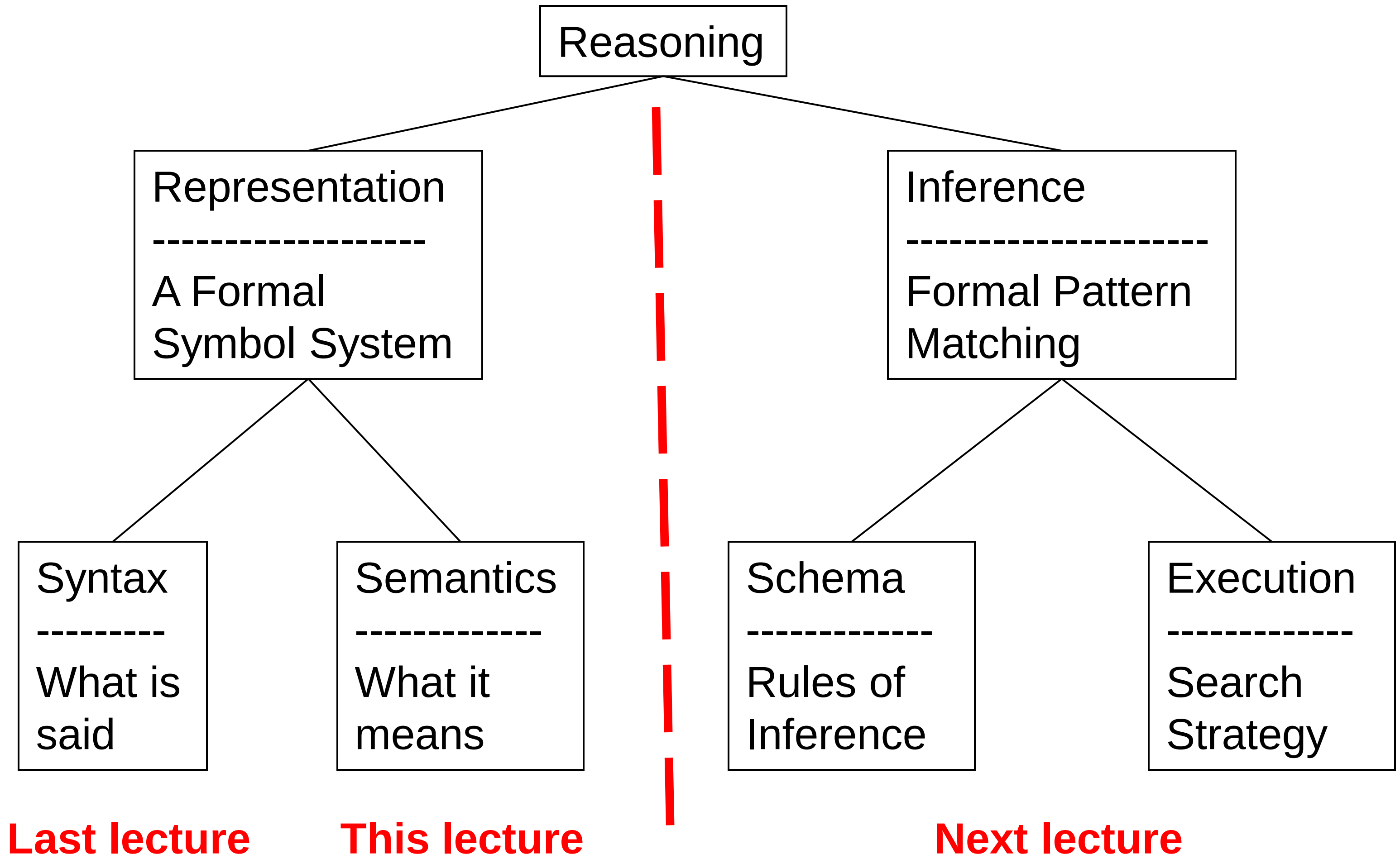
- Review: KB |= S is equivalent to |= (KB $\Rightarrow$ S)
  - So what does { } |= S mean?
- Review: Follows, Entails, Derives
  - Follows: "Is it the case?"
  - Entails: "Is it true?"
  - Derives: "Is it provable?"
- Semantics of FOL (FOPC)
- FOL can be TOO expressive, can offer TOO MANY choices
  - Likely confusion, especially for **teams** of Knowledge Engineers
  - Different team members can make different representation choices
  - E.g., represent "Ball43 is Red." as:
    - a predicate (= verb)? E.g., "Red(Ball43)" ?
    - an object (= noun)? E.g., "Red = Color(Ball43))" ?
    - a property (= adjective)? E.g., "HasProperty(Ball43, Red)" ?
  - SOLUTION: An upon-agreed **ontology** that settles these questions
    - Ontology = what exists in the world & how it is represented
    - The Knowledge Engineering teams agrees upon an ontology BEFORE they begin encoding knowledge

FOL (or FOPC) Ontology:

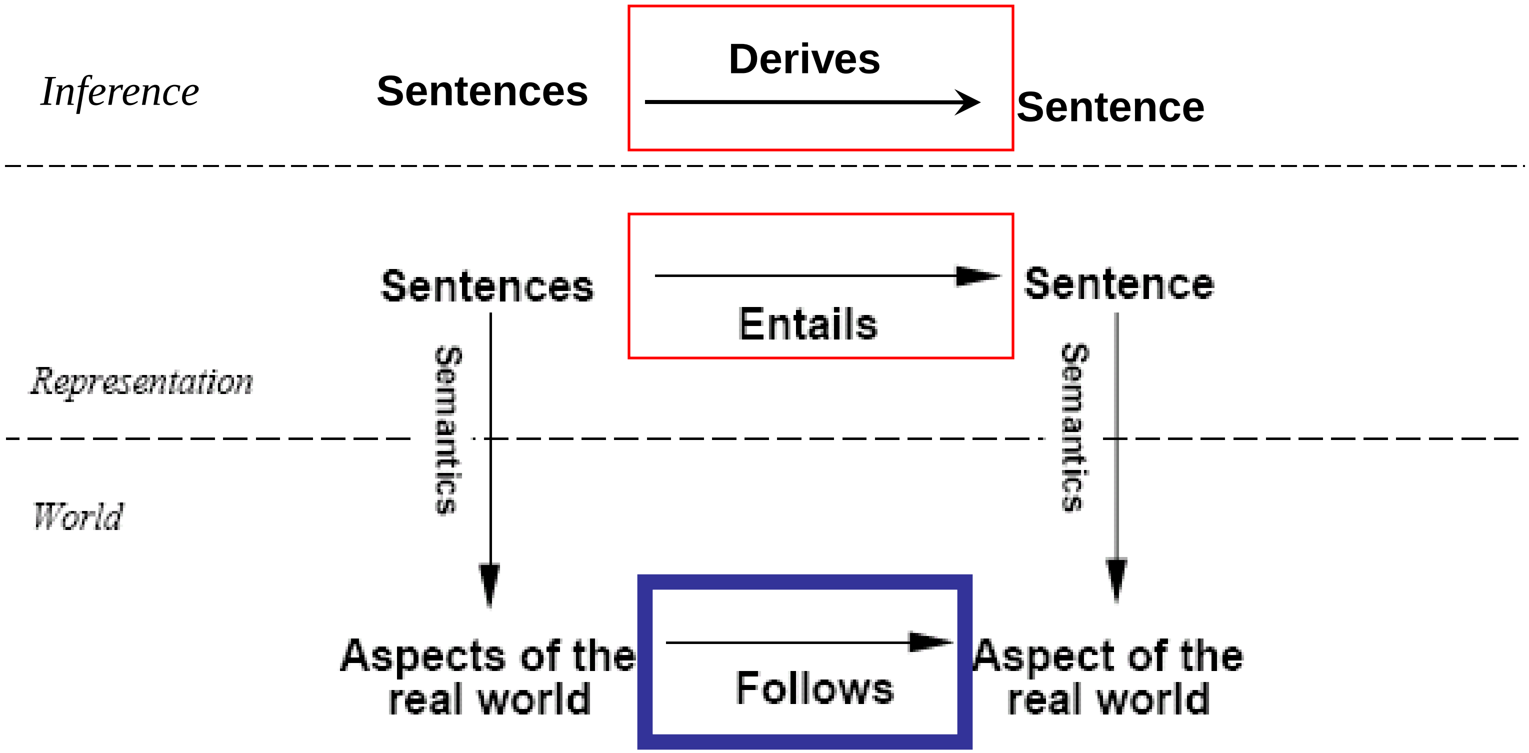
What kind of things exist in the world?

What do we need to describe and reason about?

Objects --- with their relations, functions, predicates, properties, and general rules.



Review: Schematic for Follows, Entails, and Derives



If KB is true in the real world,

*then any sentence α **entailed** by KB*

*and any sentence α **derived** from KB*

by a sound inference procedure

is also true in the real world.

Schematic Example: Follows, Entails, and Derives

Inference

"Mary is Sue's sister and
Amy is Sue's daughter."

"An aunt is a sister
of a parent."

Derives →

Is it provable?

"Mary is
Amy's aunt."

Representation

"Mary is Sue's sister and
Amy is Sue's daughter."

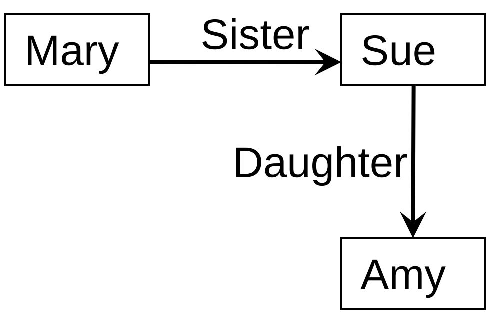
"An aunt is a sister
of a parent."

Entails →

Is it true?

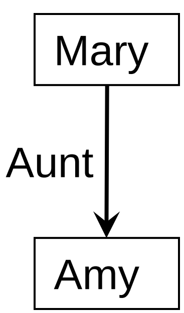
"Mary is
Amy's aunt."

World



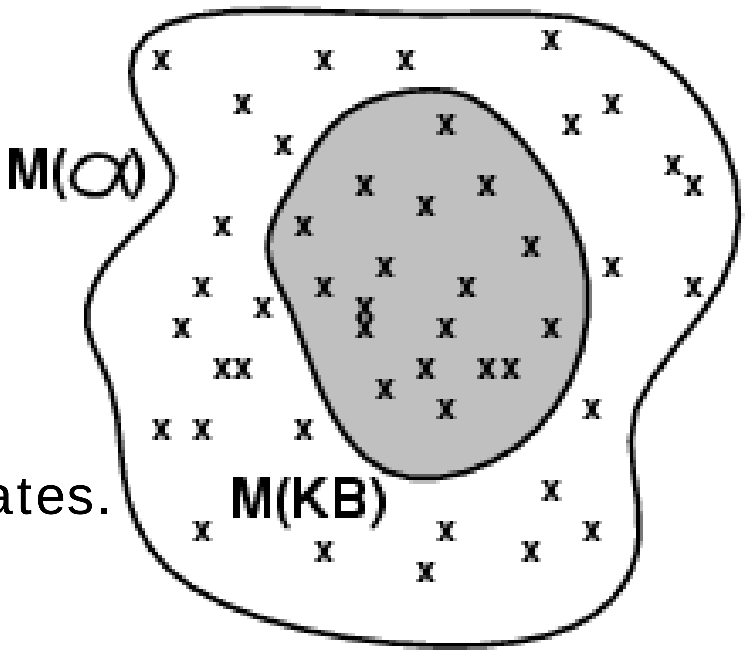
Follows →

Is it the case?

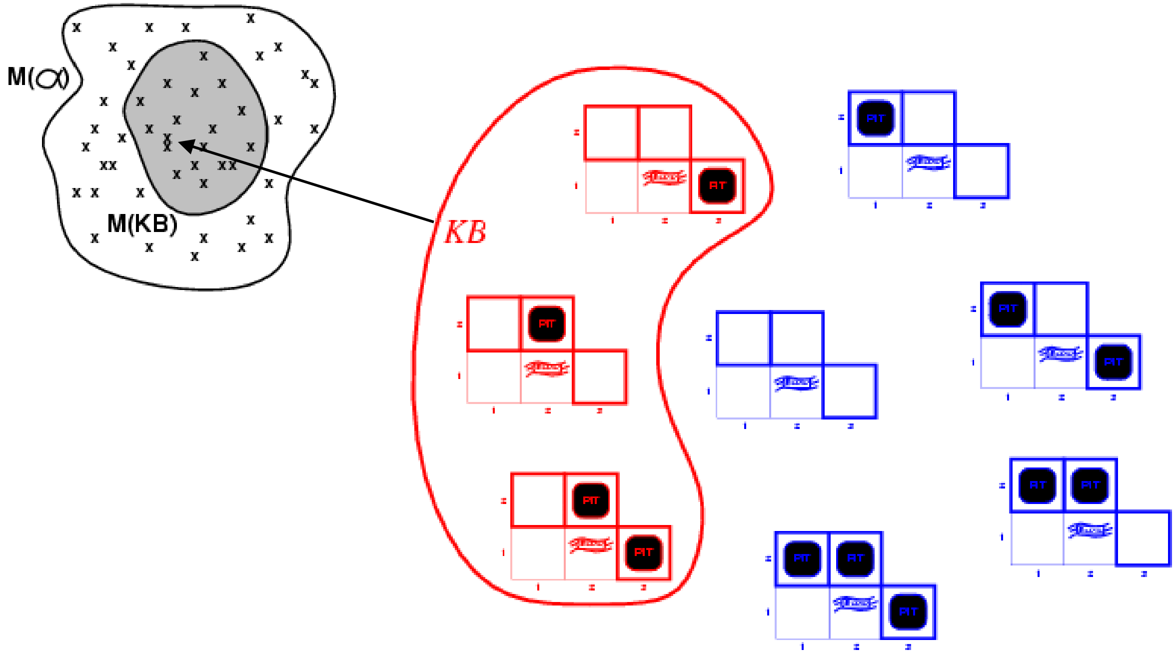


Review: Models (and in FOL, Interpretations)

- Models are formal worlds in which truth can be evaluated
- We say *m* is a model of a sentence α if α is true in *m*
- *M(α)* is the set of all models of α
- Then KB $\models$ α iff $M(KB) \subseteq M(\alpha)$
  - E.g. *KB,* = "Mary is Sue's sister and Amy is Sue's daughter."
  - α = "Mary is Amy's aunt."
- Think of KB and α as constraints, and of models m as possible states.
- M(KB) are the solutions to KB and M(α) the solutions to α.
- Then, KB $\models$ α, i.e., $\models$ (KB $\Rightarrow$ a) , when all solutions to KB are also solutions to α.

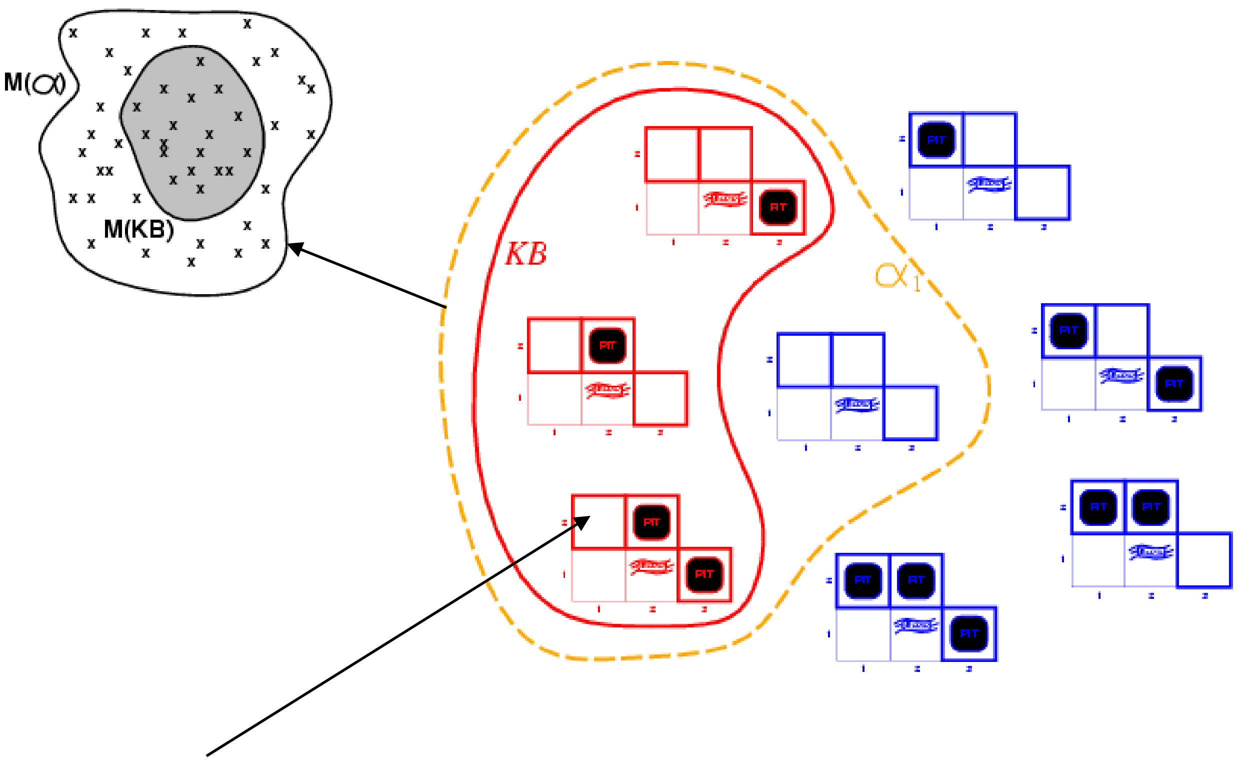


Review: Wumpus models



- *KB* = all possible wumpus-worlds consistent with the observations and the "physics" of the Wumpus world.

Review: Wumpus models



α_1 = "[1,2] is safe", $KB \models \alpha_1$, proved by model checking.

Every model that makes KB true also makes α_1 true.

Semantics: Worlds

- **The** world **consists of** objects **that have** properties**.**
  - **There are** relations **and** functions **between these objects**
  - **Objects in the world, individuals:** people, houses, numbers, colors, baseball games, wars, centuries
    - Clock A, John, 7, the-house in the corner, Tel-Aviv, Ball43
  - **Functions** on individuals:
    - father-of, best friend, third inning of, one more than
  - **Relations**:
    - brother-of, bigger than, inside, part-of, has color, occurred after
  - **Properties (a relation of arity 1)**:
    - red, round, bogus, prime, multistoried, beautiful

Semantics: Interpretation

- An **interpretation** of a sentence (wff) is an assignment that maps
  - Object constant symbols to objects in the world,
  - n-ary function symbols to n-ary functions in the world,
  - n-ary relation symbols to n-ary relations in the world
- Given an interpretation, an atomic sentence has the value "true" if it denotes a relation that holds for those individuals denoted in the terms. Otherwise it has the value "false."
  - Example: Kinship world:
    - Symbols = Ann, Bill, Sue, Married, Parent, Child, Sibling, …
  - World consists of individuals in relations:
    - Married(Ann,Bill) is false, Parent(Bill,Sue) is true, …

Truth in first-order logic

- Sentences are true with respect to a model and an interpretation
- Model contains objects (domain elements) and relations among them
- Interpretation specifies referents for

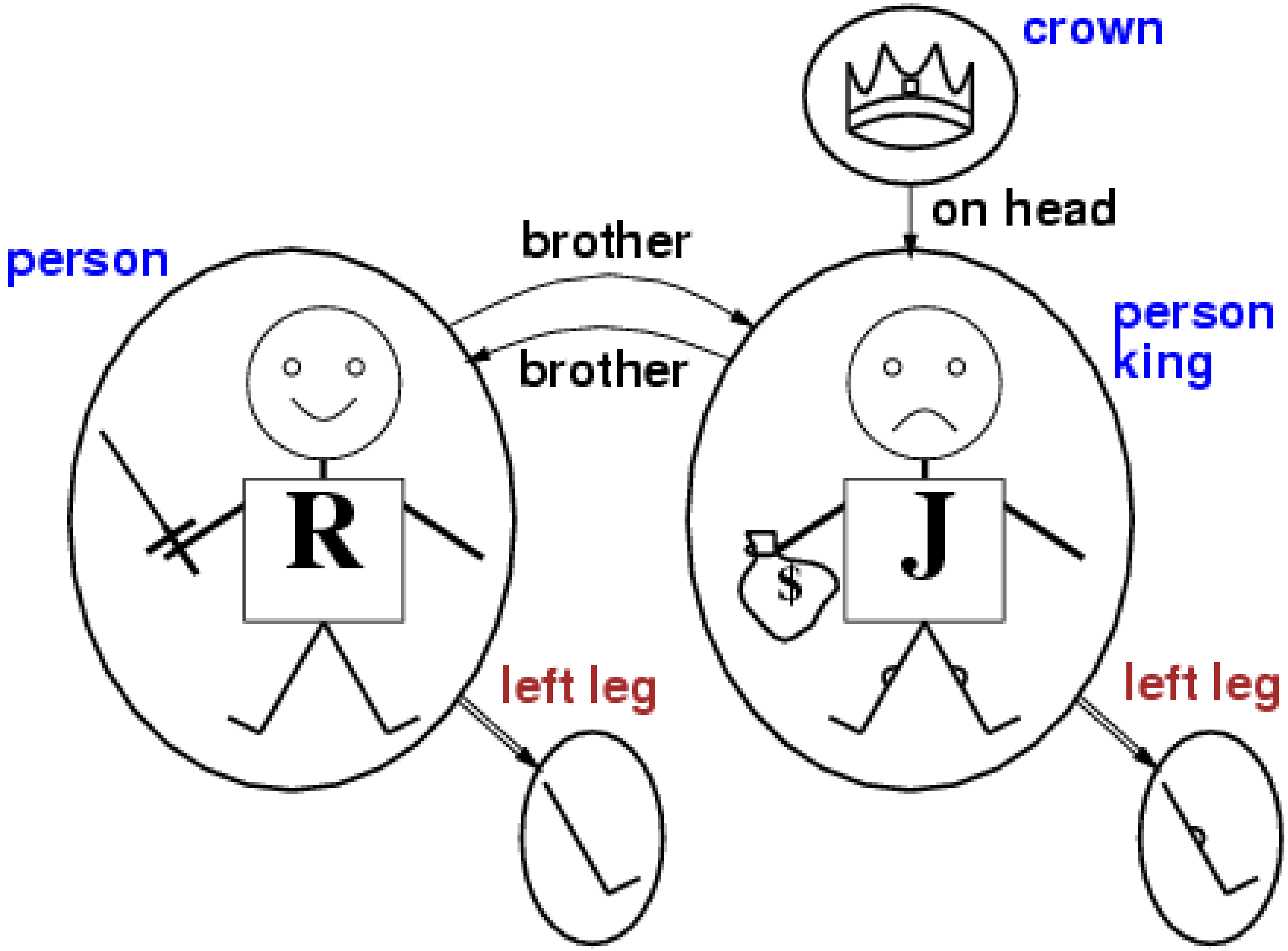
constant symbols	→	objects
predicate symbols	→	relations
function symbols	→	functional relations

- An atomic sentence $predicate(term_1, \ldots, term_n)$ is true iff the objects referred to by $term_1, \ldots, term_n$ are in the relation referred to by *predicate*

Semantics: Models

- **An interpretation** satisfies **a wff (sentence) if the wff has the value "true" under the interpretation.**
- **Model: A domain and an interpretation that satisfies a wff is a** model **of that wff**
- **Validity: Any wff that has the value "true" under all interpretations is** valid
- **Any wff that does not have a model is inconsistent or unsatisfiable**
- **If a wff w has a value true under all the models of a set of sentences KB then KB logically entails w**

Models for FOL: Example



Fun with sentences

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$\forall\, x, y \;\; Sibling(x, y) \;\Leftrightarrow\; Sibling(y, x).$

One's mother is one's female parent

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$\forall\, x, y\;\; Sibling(x, y) \;\Leftrightarrow\; Sibling(y, x).$

One's mother is one's female parent

$\forall\, x, y\;\; Mother(x, y) \;\Leftrightarrow\; (Female(x) \land Parent(x, y)).$

A first cousin is a child of a parent's sibling

Fun with sentences

Brothers are siblings

$\forall\, x, y \;\; Brother(x, y) \;\Rightarrow\; Sibling(x, y).$

"Sibling" is symmetric

$\forall\, x, y \;\; Sibling(x, y) \;\Leftrightarrow\; Sibling(y, x).$

One's mother is one's female parent

$\forall\, x, y \;\; Mother(x, y) \;\Leftrightarrow\; (Female(x) \wedge Parent(x, y)).$

A first cousin is a child of a parent's sibling

$\forall\, x, y \;\; FirstCousin(x, y) \;\Leftrightarrow\; \exists\, p, ps \;\; Parent(p, x) \wedge Sibling(ps, p) \wedge Parent(ps, y)$

Syntactic Ambiguity

- FOPC provides many ways to represent the same thing.
- E.g., “Ball-5 is red.”
  - HasColor(Ball-5, Red)
    - Ball-5 and Red are objects related by HasColor.
  - Red(Ball-5)
    - Red is a unary predicate applied to the Ball-5 object.
  - HasProperty(Ball-5, Color, Red)
    - Ball-5, Color, and Red are objects related by HasProperty.
  - ColorOf(Ball-5) = Red
    - Ball-5 and Red are objects, and ColorOf() is a function.
  - HasColor(Ball-5(), Red())
    - Ball-5() and Red() are functions of zero arguments that both return an object, which objects are related by HasColor.
  - ...
- This can GREATLY confuse a pattern-matching reasoner.
  - Especially if multiple people collaborate to build the KB, and they all have different representational conventions.

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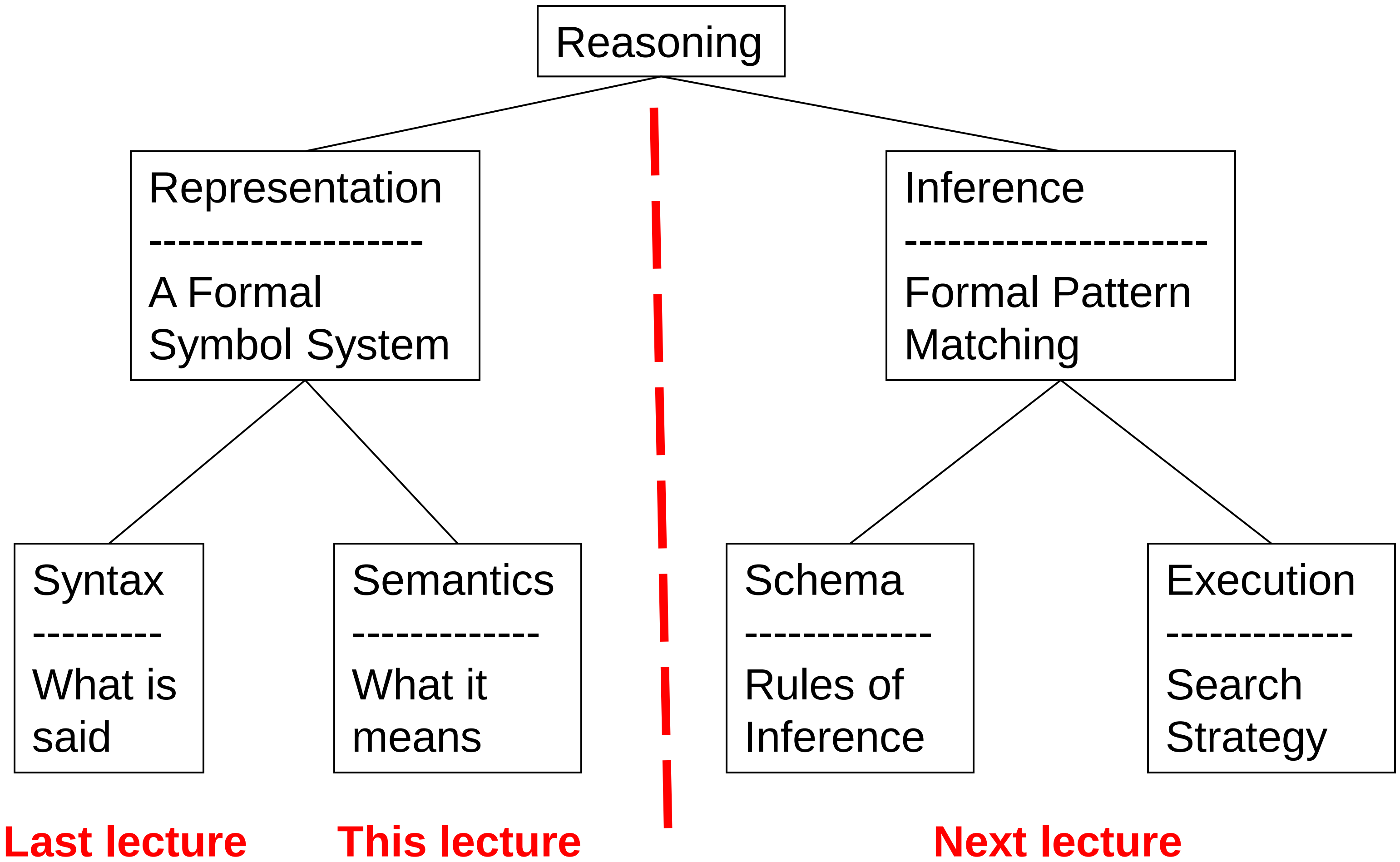

Summary

- First-order logic:
  - Much more expressive than propositional logic
  - Allows objects and relations as semantic primitives
  - Universal and existential quantifiers
  - syntax: constants, functions, predicates, equality, quantifiers
  -

- Knowledge engineering using FOL
  - Capturing domain knowledge in logical form

- Inference and reasoning in FOL
  - Next lecture

- Required Reading:
  - Chapter 8.1-8.4
  - Next lecture: 8.3-8.4
  - Next lecture: Chapter 9.1-9.2