

Knowledge Representation using First-Order Logic (Part III)

This lecture: R&N Chapters 8, 9

Next lecture: Chapter 13; Chapter 14.1-14.2

(Please read lecture topic material before and after
each lecture on that topic)

Outline

- Review: $KB \models S$ is equivalent to $\models (KB \Rightarrow S)$
 - So what does $\{ \} \models S$ mean?
- Review: Follows, Entails, Derives
 - Follows: “Is it the case?”
 - Entails: “Is it true?”
 - Derives: “Is it provable?”
- Review: FOL syntax

- Finish FOL Semantics, FOL examples

- Inference in FOL

Using FOL

- We want to TELL things to the KB, e.g.
TELL(KB, $\forall x, King(x) \Rightarrow Person(x)$)
TELL(KB, $King(John)$)

These sentences are assertions

- We also want to ASK things to the KB,
ASK(KB, $\exists x, Person(x)$)

these are queries or goals

The KB should return the list of x's for which $Person(x)$ is true:
{x/John,x/Richard,...}

FOL Version of Wumpus World

- Typical percept sentence:
Percept([Stench,Breeze,Glitter,None,None],5)
- Actions:
Turn(Right), Turn(Left), Forward, Shoot, Grab, Release, Climb
- To determine best action, construct query:
 $\forall a \text{ BestAction}(a,5)$
- ASK solves this and returns {a/Grab}
 - And TELL about the action.

Knowledge Base for Wumpus World

- Perception
 - $\forall s, b, g, x, y, t \text{ Percept}([s, \text{Breeze}, g, x, y], t) \Rightarrow \text{Breeze}(t)$
 - $\forall s, b, x, y, t \text{ Percept}([s, b, \text{Glitter}, x, y], t) \Rightarrow \text{Glitter}(t)$
- Reflex action
 - $\forall t \text{ Glitter}(t) \Rightarrow \text{BestAction}(\text{Grab}, t)$
- Reflex action with internal state
 - $\forall t \text{ Glitter}(t) \wedge \neg \text{Holding}(\text{Gold}, t) \Rightarrow \text{BestAction}(\text{Grab}, t)$

$\text{Holding}(\text{Gold}, t)$ can not be observed: keep track of change.

Deducing hidden properties

Environment definition:

$$\forall x,y,a,b \text{ Adjacent}([x,y],[a,b]) \Leftrightarrow \\ [a,b] \in \{[x+1,y], [x-1,y],[x,y+1],[x,y-1]\}$$

Properties of locations:

$$\forall s,t \text{ At}(\text{Agent},s,t) \wedge \text{Breeze}(t) \Rightarrow \text{Breezy}(s)$$

Squares are breezy near a pit:

- **Diagnostic** rule---infer cause from effect

$$\forall s \text{ Breezy}(s) \Leftrightarrow \exists r \text{ Adjacent}(r,s) \wedge \text{Pit}(r)$$

- **Causal** rule---infer effect from cause (model based reasoning)

$$\forall r \text{ Pit}(r) \Rightarrow [\forall s \text{ Adjacent}(r,s) \Rightarrow \text{Breezy}(s)]$$

Keeping track of change

Facts hold in **situations**, rather than eternally

E.g., $Holding(Gold, Now)$ rather than just $Holding(Gold)$

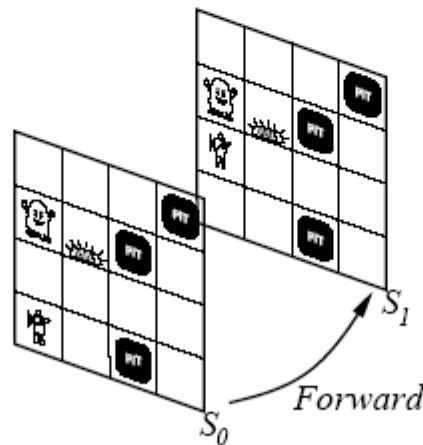
Situation calculus is one way to represent change in FOL:

Adds a situation argument to each non-eternal predicate

E.g., Now in $Holding(Gold, Now)$ denotes a situation

Situations are connected by the *Result* function

$Result(a, s)$ is the situation that results from doing a in s



Describing actions I

“Effect” axiom—describe changes due to action

$$\forall s \text{ AtGold}(s) \Rightarrow \text{Holding}(\text{Gold}, \text{Result}(\text{Grab}, s))$$

“Frame” axiom—describe **non-changes** due to action

$$\forall s \text{ HaveArrow}(s) \Rightarrow \text{HaveArrow}(\text{Result}(\text{Grab}, s))$$

Frame problem: find an elegant way to handle non-change

- (a) representation—avoid frame axioms
- (b) inference—avoid repeated “copy-overs” to keep track of state

Qualification problem: true descriptions of real actions require endless caveats—what if gold is slippery or nailed down or . . .

Ramification problem: real actions have many secondary consequences—what about the dust on the gold, wear and tear on gloves, . . .

Describing actions II

Successor-state axioms solve the representational frame problem

Each axiom is “about” a **predicate** (not an action per se):

$$\begin{aligned} P \text{ true afterwards} &\Leftrightarrow [\text{an action made } P \text{ true} \\ &\vee P \text{ true already and no action made } P \text{ false}] \end{aligned}$$

For holding the gold:

$$\begin{aligned} \forall a, s \text{ Holding}(\text{Gold}, \text{Result}(a, s)) &\Leftrightarrow \\ &[(a = \text{Grab} \wedge \text{AtGold}(s)) \\ &\vee (\text{Holding}(\text{Gold}, s) \wedge a \neq \text{Release})] \end{aligned}$$

Set Theory in First-Order Logic

Can we define set theory using FOL?

- individual sets, union, intersection, etc

Answer is yes.

Basics:

- empty set = constant = $\{ \}$

- unary predicate $\text{Set}()$, true for sets

- binary predicates:

$x \in s$ (true if x is a member of the set s)

$s_1 \subseteq s_2$ (true if s_1 is a subset of s_2)

- binary functions:

intersection $s_1 \cap s_2$, union $s_1 \cup s_2$, adjoining $\{x|s\}$

A Possible Set of FOL Axioms for Set Theory

The only sets are the empty set and sets made by adjoining an element to a set

$$\forall s \text{ Set}(s) \Leftrightarrow (s = \{\}) \vee (\exists x, s_2 \text{ Set}(s_2) \wedge s = \{x|s_2\})$$

The empty set has no elements adjoined to it

$$\neg \exists x, s \{x|s\} = \{\}$$

Adjoining an element already in the set has no effect

$$\forall x, s \ x \in s \Leftrightarrow s = \{x|s\}$$

The only elements of a set are those that were adjoined into it.
Expressed recursively:

$$\forall x, s \quad x \in s \Leftrightarrow [\exists y, s_2 (s = \{y|s_2\} \wedge (x = y \vee x \in s_2))]$$

A Possible Set of FOL Axioms for Set Theory

A set is a subset of another set iff all the first set's members are members of the 2nd set

$$\forall S_1, S_2 \ S_1 \subseteq S_2 \Leftrightarrow (\forall x \ x \in S_1 \Rightarrow x \in S_2)$$

Two sets are equal iff each is a subset of the other

$$\forall S_1, S_2 \ (S_1 = S_2) \Leftrightarrow (S_1 \subseteq S_2 \wedge S_2 \subseteq S_1)$$

An object is in the intersection of 2 sets only if a member of both

$$\forall x, S_1, S_2 \ x \in (S_1 \cap S_2) \Leftrightarrow (x \in S_1 \wedge x \in S_2)$$

An object is in the union of 2 sets only if a member of either

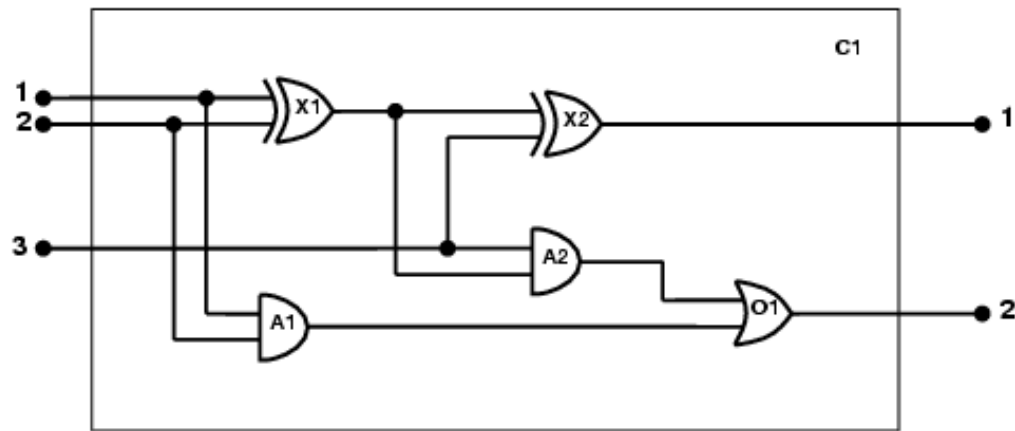
$$\forall x, S_1, S_2 \ x \in (S_1 \cup S_2) \Leftrightarrow (x \in S_1 \vee x \in S_2)$$

Knowledge engineering in FOL

1. Identify the task
2. Assemble the relevant knowledge
3. Decide on a vocabulary of predicates, functions, and constants
4. Encode general knowledge about the domain
5. Encode a description of the specific problem instance
6. Pose queries to the inference procedure and get answers
7. Debug the knowledge base

The electronic circuits domain

One-bit full adder



Possible queries:

- does the circuit function properly?
 - what gates are connected to the first input terminal?
 - what would happen if one of the gates is broken?
- and so on

The electronic circuits domain

1. Identify the task
 - Does the circuit actually add properly?
2. Assemble the relevant knowledge
 - Composed of wires and gates; Types of gates (AND, OR, XOR, NOT)
 -
 - Irrelevant: size, shape, color, cost of gates
 -
3. Decide on a vocabulary
 - Alternatives:
 -
 - $\text{Type}(X_1) = \text{XOR}$ (function)
 - $\text{Type}(X_1, \text{XOR})$ (binary predicate)
 - $\text{XOR}(X_1)$
(unary predicate)

The electronic circuits domain

4. Encode general knowledge of the domain
- $\forall t_1, t_2 \text{ Connected}(t_1, t_2) \Rightarrow \text{Signal}(t_1) = \text{Signal}(t_2)$
 - $\forall t \text{ Signal}(t) = 1 \vee \text{Signal}(t) = 0$
 - $1 \neq 0$
 - $\forall t_1, t_2 \text{ Connected}(t_1, t_2) \Rightarrow \text{Connected}(t_2, t_1)$
 - $\forall g \text{ Type}(g) = \text{OR} \Rightarrow \text{Signal}(\text{Out}(1, g)) = 1 \Leftrightarrow \exists n \text{ Signal}(\text{In}(n, g)) = 1$
 - $\forall g \text{ Type}(g) = \text{AND} \Rightarrow \text{Signal}(\text{Out}(1, g)) = 0 \Leftrightarrow \exists n \text{ Signal}(\text{In}(n, g)) = 0$
 - $\forall g \text{ Type}(g) = \text{XOR} \Rightarrow \text{Signal}(\text{Out}(1, g)) = 1 \Leftrightarrow \text{Signal}(\text{In}(1, g)) \neq \text{Signal}(\text{In}(2, g))$
 - $\forall g \text{ Type}(g) = \text{NOT} \Rightarrow \text{Signal}(\text{Out}(1, g)) \neq \text{Signal}(\text{In}(1, g))$

The electronic circuits domain

5. Encode the specific problem instance

Type(X_1) = XOR

Type(A_1) = AND

Type(O_1) = OR

Type(X_2) = XOR

Type(A_2) = AND

Connected(Out(1, X_1),In(1, X_2))

Connected(Out(1, X_1),In(2, A_2))

Connected(Out(1, A_2),In(1, O_1))

Connected(Out(1, A_1),In(2, O_1))

Connected(Out(1, X_2),Out(1, C_1))

Connected(Out(1, O_1),Out(2, C_1))

Connected(In(1, C_1),In(1, X_1))

Connected(In(1, C_1),In(1, A_1))

Connected(In(2, C_1),In(2, X_1))

Connected(In(2, C_1),In(2, A_1))

Connected(In(3, C_1),In(2, X_2))

Connected(In(3, C_1),In(1, A_2))

The electronic circuits domain

6. Pose queries to the inference procedure

What are the possible sets of values of all the terminals for the adder circuit?

$$\begin{aligned} \exists i_1, i_2, i_3, o_1, o_2 \text{ Signal(In}(1, C_1)) = i_1 \wedge \text{Signal(In}(2, C_1)) = i_2 \wedge \text{Signal(In}(3, C_1)) \\ = i_3 \wedge \text{Signal(Out}(1, C_1)) = o_1 \wedge \text{Signal(Out}(2, C_1)) = o_2 \end{aligned}$$

7. Debug the knowledge base

May have omitted assertions like $1 \neq 0$

Syntactic Ambiguity

- FOPC provides many ways to represent the same thing.
- E.g., "Ball-5 is red."
 - HasColor(Ball-5, Red)
 - Ball-5 and Red are objects related by HasColor.
 - Red(Ball-5)
 - Red is a unary predicate applied to the Ball-5 object.
 - HasProperty(Ball-5, Color, Red)
 - Ball-5, Color, and Red are objects related by HasProperty.
 - ColorOf(Ball-5) = Red
 - Ball-5 and Red are objects, and ColorOf() is a function.
 - HasColor(Ball-5(), Red())
 - Ball-5() and Red() are functions of zero arguments that both return an object, which objects are related by HasColor.
 - ...
- This can GREATLY confuse a pattern-matching reasoner.
 - Especially if multiple people collaborate to build the KB, and they all have different representational conventions.

Summary

- First-order logic:
 - Much more expressive than propositional logic
 - Allows objects and relations as semantic primitives
 - Universal and existential quantifiers
 - syntax: constants, functions, predicates, equality, quantifiers
 -
- Knowledge engineering using FOL
 - Capturing domain knowledge in logical form
- Inference and reasoning in FOL
 - Next lecture
- Required Reading:
 - All of Chapter 8
 - Next lecture: Chapter 9