## Games Search

## \& <br> Constraint Satisfaction

Thur, July 14, 2016

## Game Search: Exercise 1

In the game tree below it is MAX's turn to move. At each leaf node is the estimated score of that resulting position as returned by the heuristic static evaluator.

Fill in the values of node A-L.


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## Game Search: Exercise 2

Infer the range constraints on nodes A-E.


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Infer the range constraints on nodes A-E.

$A \geq 2$
$B \leq 8$
One of C, D, E $\geq 2$ (C, D, E cannot be all $<2$ )

## Heuristic Evaluation Functions: Exercise

YOU and FRIEND are driving a car from $S$ and notice a rare Pokemon at G. You decide to play a turn based game to decide who should catch it. The rules are:

1. YOU move first, then players (YOU and FRIEND) alternate moves. Making a move is required;
2. On their turn, a player chooses to drive the car to any allowed stops. The other player rides the car;
3. Allowed stops are the stops that are adjacent to the current stop, and are in front. You may not travel back;
4. Whoever gets G on their turns wins.

What could be a reasonable heuristic evaluation function?


## Heuristic Evaluation Functions: Exercise

Assume we use the heuristic evaluation function:

$$
\begin{aligned}
& E(n)=Y(n)-F(n) \\
& Y(n) \text { is the total of YOUR winning path } \\
& F(n) \text { is the total of FRIEND winning path }
\end{aligned}
$$



## Heuristic Evaluation Functions: Exercise

Assume we use the heuristic evaluation function:

```
\(E(n)=Y(n)-F(n)\)
```

$Y(n)$ is the total of YOUR winning path
$F(n)$ is the total of FRIEND winning path

Observation:
On YOU turn: If path length from current node to $G$ is even, FRIEND wins.
If path length from current node to G is odd, YOU win.


## Heuristic Evaluation Functions: Exercise

Assume we use the heuristic evaluation function:
$E(n)=Y(n)-F(n)$
$Y(n)$ is the total of YOUR winning path
$F(n)$ is the total of FRIEND winning path
$\mathrm{E}\left(\mathrm{n}_{\mathrm{r}}, \mathrm{n}\right)=\operatorname{OddPath}\left(\mathrm{n}_{\mathrm{r}}, \mathrm{n}, \mathrm{G}\right)$ - EvenPath( $\left.\mathrm{n}_{\mathrm{r}}, \mathrm{n}, \mathrm{G}\right)$
$\mathrm{n}_{\mathrm{r}}$ is the current node on YOU turn, n is a node on the path.
Observation
On YOU turn: If path length from current node to $G$ is even, FRIEND wins.
If path length from current node to G is odd, YOU win.


## Heuristic Evaluation Functions: Exercise



Travel Direction

## Heuristic Evaluation Functions: Exercise


$E\left(n_{r}, n\right)=\operatorname{OddPath}\left(n_{r}, n, G\right)-\operatorname{EvenPath}\left(n_{r}, n, G\right)$
$\mathrm{n}_{\mathrm{r}}$ is the current node on YOU turn, n is a node on the path.

Travel Direction

## Heuristic Evaluation Functions: Exercise



## YOU picks B



Travel Direction

## Alpha-Beta Pruning: Exercise

If $A=0$, can leaf nodes $B$ and $C$ be pruned by alpha-beta pruning?

If $A=10$, can leaf nodes $B$ and $C$ be pruned by alpha-beta pruning?


## Alpha-Beta Pruning: Exercise

If $A=0$, can leaf nodes $B$ and $C$ be pruned by alpha-beta pruning? Yes

If $A=10$, can leaf nodes $B$ and $C$ be pruned by alpha-beta pruning? No


## Alpha-Beta Pruning: Exercise

Find the interval of $A$ such that $B$ and $C$ will be pruned by alpha-beta pruning.


## Alpha-Beta Pruning: Exercise

Find the interval of $A$ such that $B$ and $C$ will be pruned by alpha-beta pruning.


## Alpha-Beta Pruning: Exercise

Find the interval of $A$ such that $B$ and $C$ will be pruned by alpha-beta pruning.


Child inherits current $\alpha \beta$ Values.

## Alpha-Beta Pruning: Exercise

Find the interval of $A$ such that $B$ and $C$ will be pruned by alpha-beta pruning.


MIN update $\boldsymbol{\beta}$ Value.

## Alpha-Beta Pruning: Exercise

Find the interval of $A$ such that $B$ and $C$ will be pruned by alpha-beta pruning.


## Map Coloring: Exercise

Which map would require 4 colors to color?

(A)

(B)

(C)

(D)

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Which map would require 4 colors to color?

(A)


(B)


(C)


(D)


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Which map would require 4 colors to color?

(A)


(B)


(C)


## Constraint Satisfaction: Exercise (Final, Fall 2014, Problem 8)



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\begin{aligned}
& \text { AL }=\text { Alberta } \\
& B C=\text { British Columbia } \\
& M A=\text { Manitoba } \\
& N T=\text { Northwest Territories } \\
& N U=\text { Nunavut } \\
& S A=\text { Saskatchewan } \\
& Y U=\text { Yukon }
\end{aligned}
$$

You are a map-coloring robot assigned to color this western Canada map. Adjacent regions must be colored a different color ( $R=$ Red, $B=B l u e, G=G r e e n$ ). The constraint graph is shown.
8.a. (2 pts) FORWARD CHECKING. Cross out all values that would be eliminated by Forward Checking, after variable NT has just been assigned value G, as shown:

| $A L$ | $B C$ | MA | NT | NU | SA | YU |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $R G B$ | $R G B$ | $R G B$ | $G$ | $R G B$ | $R G B$ | $R G B$ |

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| $A L$ | $B C$ | $M A$ | $N T$ | $N U$ | $S A$ | $Y U$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $R \& B$ | $R \& B$ | $R G B$ | $G$ | $R \& B$ | $R \& B$ | $R \& B$ |

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## 8.b. (2 pts) ARC CONSISTENCY.

AL and MA have been assigned values, but no constraint propagation has been done.
Cross out all values that would be eliminated by Arc Consistency (AC-3 in your book).

| $A L$ | $B C$ | $M A$ | $N T$ | $N U$ | $S A$ | $Y U$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $B$ | $R G B$ | $R$ | $R G B$ | $R G B$ | $R G B$ | $R G B$ |

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| $A L$ | $B C$ | $M A$ | $N T$ | $N U$ | $S A$ | $Y U$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $B$ | $R G B$ | $R$ | $R G B$ | $R Q B$ | $R G B$ | $R G B$ |

## Constraint Satisfaction: Exercise (Final, Fall 2014, Problem 8)



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8.c. (2 pts) MINIMUM-REMAINING-VALUES HEURISTIC. Consider the assignment below. YU is assigned and constraint propagation has been done. List all unassigned variables that might be selected by the Minimum-Remaining-Values (MRV) Heuristic:

| $A L$ | $B C$ | $M A$ | $N T$ | $N U$ | $S A$ | $Y U$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $R G B$ | $G B$ | $R G B$ | $G B$ | $R G B$ | $R G B$ | $R$ |

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8.c. (2 pts) MINIMUM-REMAINING-VALUES HEURISTIC. Consider the assignment below. YU is assigned and constraint propagation has been done. List all unassigned variables that might be selected by the Minimum-Remaining-Values (MRV) Heuristic: BC, NT

| $A L$ | $B C$ | $M A$ | $N T$ | $N U$ | $S A$ | $Y U$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $R G B$ | $G B$ | $R G B$ | $G B$ | $R G B$ | $R G B$ | $R$ |

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8.d. (2 pts) DEGREE HEURISTIC. Consider the assignment below. (It is the same assignment as in problem 8.c. above.) YU is assigned and constraint propagation has been done. List all unassigned variables that might be selected by the Degree Heuristic: $\qquad$ .

| $A L$ | $B C$ | $M A$ | $N T$ | $N U$ | $S A$ | $Y U$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $R G B$ | $G B$ | $R G B$ | $G B$ | $R G B$ | $R G B$ | $R$ |

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| $A L$ | $B C$ | $M A$ | $N T$ | $N U$ | $S A$ | $Y U$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $R G B$ | $G B$ | $R G B$ | $G B$ | $R G B$ | $R G B$ | $R$ |

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8.e. (2 pts) MIN-CONFLICTS HEURISTIC. Consider the complete but inconsistent assignment below. AL has just been selected to be assigned a new value during local search for a complete and consistent assignment. What new value would be chosen below for AL by the Min-Conflicts Heuristic? $\qquad$

| AL | BC | MA | NT | NU | SA | YU |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $?$ | B | G | R | G | B | B |

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| AL | BC | MA | NT | NU | SA | YU |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $?$ | B | G | R | G | B | B |

