

Logic

- Mathematicians prove theorems.
- Can we make machine do it?

Problem # III

- If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is either immortal or a mammal, then it is horned. The unicorn is magical if it is horned.
- $Y = \text{mythical}$ $R = \text{mortal}$
 $M = \text{mammal}$ $H = \text{Horned}$ $G = \text{magical}$

Problem # III

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- Y = mYthical R = moRtal
M = maMmal H = Horned G = maGical

Inference
= weapons in arsenal
+ how to use them

Important rules

- $\alpha \Leftrightarrow \beta$ iff $\alpha \Rightarrow \beta$, $\beta \Rightarrow \alpha$
- $\alpha \Rightarrow \beta$ iff $\neg \alpha \vee \beta$
- $\neg(\alpha \wedge \beta) = \neg \alpha \vee \neg \beta$
- $\neg(\alpha \vee \beta) = \neg \alpha \wedge \neg \beta$
- $(\alpha \wedge \beta) \vee \gamma = (\alpha \vee \gamma) \wedge (\beta \vee \gamma)$
- $(\alpha \vee \beta) \wedge \gamma = (\alpha \wedge \gamma) \vee (\beta \wedge \gamma)$

Can be proved by
truth table.

Important rules

- And many more.
- The more you know, the better you can do inference (Refer to textbook.)

- If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is either immortal or a mammal, then it is horned. The unicorn is magical if it is horned.
- $Y = \text{mythical}$ $R = \text{mortal}$
 $M = \text{mammal}$ $H = \text{Horned}$ $G = \text{magical}$

$$Y \Rightarrow \neg R, \quad \neg Y \Rightarrow (R \wedge M), \\ (\neg R \vee M) \Rightarrow H, \quad H \Rightarrow G$$

- If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is either immortal or a mammal, then it is horned.
The unicorn is magical if it is horned.
- Given these sentences, can we prove
 - “if the unicorn is mortal, it is magical”, i.e. $R \Rightarrow G$

$$\begin{array}{ll}
 Y \Rightarrow \neg R, & \neg Y \Rightarrow (R \wedge M), \\
 (\neg R \vee M) \Rightarrow H, & H \Rightarrow G
 \end{array}$$

Is $R \Rightarrow G$ true?

$$\begin{array}{ll} Y \Rightarrow \neg R, & \neg Y \Rightarrow (R \wedge M), \\ (\neg R \vee M) \Rightarrow H, & H \Rightarrow G \end{array}$$

Knowledge Base

- $KB \vdash (R \Rightarrow G)$?

Is $R \Rightarrow G$ true?

$$\begin{array}{ll} Y \Rightarrow \neg R, & \neg Y \Rightarrow (R \wedge M), \\ (\neg R \vee M) \Rightarrow H, & H \Rightarrow G \end{array}$$

Knowledge Base

- $KB \vdash (R \Rightarrow G)$?
- Yes.

Is $R \Rightarrow G$ true?

$$\begin{array}{ll} Y \Rightarrow \neg R, & \neg Y \Rightarrow (R \wedge M), \\ (\neg R \vee M) \Rightarrow H, & H \Rightarrow G \end{array}$$

Knowledge Base

- $KB \vdash (R \Rightarrow G)$?
- Yes.
- $R \Rightarrow \neg Y \Rightarrow M \Rightarrow H \Rightarrow G$

$$\neg Y \Rightarrow R$$

$$M \Rightarrow H \Rightarrow G$$

Soundness

- $R \Rightarrow \neg Y \Rightarrow M \Rightarrow H \Rightarrow G$
- Our inference is **sound**, since it's based on sound rules. (Our weapons are invincible!)

Completeness

- Inference is complete if it can be used to derive **all possible (true) sentences**.
- This is determined by **truth table**.
- Sound & Complete inference is hard
 - resolution

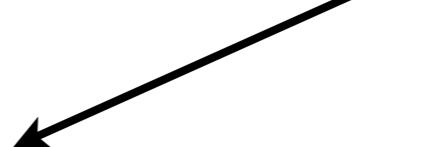
Proof by contradiction

- $\text{KB} \vDash \alpha$ iff

$\text{KB} \Rightarrow \alpha$ iff

$\neg\text{KB} \vee \alpha$

hard to prove.



- Let's prove

$\neg(\neg\text{KB} \vee \alpha)$ i.e. $\text{KB} \wedge \neg\alpha$ is **false**.

easy to prove using resolution

Digress: Problem #10 (a)

From Mid-term Winter 2012

- $(A \vee B \vee \neg C \vee D) \wedge (A \vee \neg D \vee E \vee F)$

Problem #10 (a)

From Mid-term Winter 2012

- $(A \vee B \vee \neg C \vee D) \wedge (A \vee \neg D \vee E \vee F)$

Problem #10 (a)

From Mid-term Winter 2012

$$\bullet (A \vee B \vee \neg C \vee D) \wedge (A \vee \neg D \vee E \vee F)$$

$$\Leftrightarrow (A \vee B \vee \neg C \vee A \vee E \vee F)$$

Problem #10 (a)

From Mid-term Winter 2012

$$\bullet (A \vee B \vee \neg C \vee D) \wedge (A \vee \neg D \vee E \vee F)$$

$$\Leftrightarrow (\textcolor{red}{A} \vee B \vee \neg C \vee \textcolor{red}{A} \vee E \vee F)$$

Problem #10 (a)

From Mid-term Winter 2012

$$\bullet (A \vee B \vee \neg C \vee D) \wedge (A \vee \neg D \vee E \vee F)$$

$$\Leftrightarrow (A \vee B \vee \neg C \vee A \vee E \vee F)$$

$$\Leftrightarrow (A \vee B \vee \neg C \vee E \vee F)$$

Problem #10 (c)

From Mid-term Winter 2012

- $(\neg C) \wedge (C) \Leftrightarrow F$

Problem #10 (d)

From Mid-term Winter 2012

- $(A \vee B \vee \neg C \vee D) \wedge (A \vee C \vee \neg D \vee E \vee F)$

Problem #10 (d)

From Mid-term Winter 2012

- $(A \vee B \vee \neg C \vee D) \wedge (A \vee \textcolor{red}{C} \vee \neg D \vee E \vee F)$

Problem #10 (d)

From Mid-term Winter 2012

- $(A \vee B \vee \neg C \vee D) \wedge (A \vee C \vee \neg D \vee E \vee F)$
 $\Leftrightarrow (A \vee B \vee D \vee \neg D \vee E \vee F)$

Problem #10 (d)

From Mid-term Winter 2012

- $(A \vee B \vee \neg C \vee D) \wedge (A \vee C \vee \neg D \vee E \vee F)$
 $\Leftrightarrow (A \vee B \vee \textcolor{red}{D} \vee \neg D \vee E \vee F)$

Problem #10 (d)

From Mid-term Winter 2012

- $(A \vee B \vee \neg C \vee D) \wedge (A \vee C \vee \neg D \vee E \vee F)$
 $\Leftrightarrow (A \vee B \vee D \vee \neg D \vee E \vee F)$
 $\Leftrightarrow (A \vee B \vee T \vee E \vee F)$

Problem #10 (d)

From Mid-term Winter 2012

- $(A \vee B \vee \neg C \vee D) \wedge (A \vee C \vee \neg D \vee E \vee F)$
 $\Leftrightarrow (A \vee B \vee D \vee \neg D \vee E \vee F)$
 $\Leftrightarrow (A \vee B \vee T \vee E \vee F)$
 $\Leftrightarrow T$

Resolution

- Resolution requires CNF
- $\text{KB} \wedge \neg \alpha \quad \text{CNF}$, if KB is CNF
- Turn KB into CNF!

- Given these sentences, can we prove “unicorn is both magical and horned”, i.e. $G \wedge H$?

$$\begin{array}{ll} Y \Rightarrow \neg R, & \neg Y \Rightarrow (R \wedge M), \\ (\neg R \vee M) \Rightarrow H, & H \Rightarrow G \end{array}$$

$$Y \Rightarrow \neg R, \quad \neg Y \Rightarrow (R \wedge M),$$

$$(\neg R \vee M) \Rightarrow H, \quad H \Rightarrow G$$

- turn into CNF
- $Y \Rightarrow \neg R$ iff $(\neg Y \vee \neg R)$
- $\neg Y \Rightarrow (R \wedge M)$ iff $Y \vee (R \wedge M)$
iff $(Y \vee R) \wedge (Y \vee M)$
- $(\neg R \vee M) \Rightarrow H$ iff $(R \wedge \neg M) \vee H$
iff $(R \vee H) \wedge (\neg M \vee H)$
- $H \Rightarrow G$ iff $(\neg H \vee G)$

$\neg Y \vee \neg R$ $Y \vee R$ $Y \vee M$ $R \vee H$ $\neg M \vee H$ $\neg H \vee G$ $\neg G \vee \neg H$

KB

$\neg \alpha$

$\neg Y \vee \neg R$ $Y \vee R$ $Y \vee M$ $R \vee H$ $\neg M \vee H$ $\neg H \vee G$ $\neg G \vee \neg H$ $\neg H$

$\neg Y \vee \neg R$ $Y \vee R$ $Y \vee M$ $R \vee H$ $\neg M \vee H$ $\neg H \vee G$ $\neg G \vee \neg H$ $\neg H$ $\neg M$

$\neg Y \vee \neg R$ $Y \vee R$ $Y \vee M$ $R \vee H$ $\neg M \vee H$ $\neg H \vee G$ $\neg G \vee \neg H$ $\neg H$ $\neg M$ R

$\neg Y \vee \neg R$ $Y \vee R$ $Y \vee M$ $R \vee H$ $\neg M \vee H$ $\neg H \vee G$ $\neg G \vee \neg H$ $\neg H$
 $\neg M$
 R
 Y

$\neg Y \vee \neg R$

$Y \vee R$

$Y \vee M$

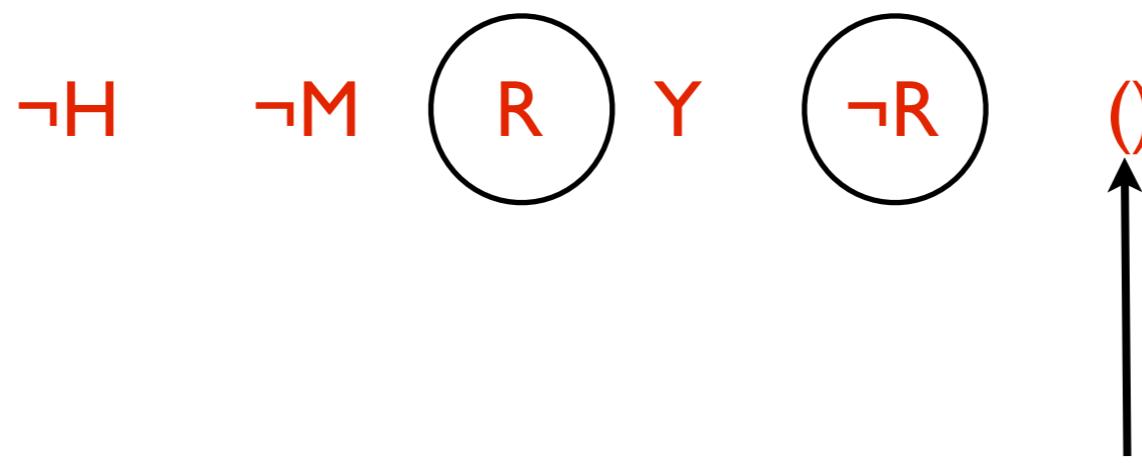
$R \vee H$

$\neg M \vee H$

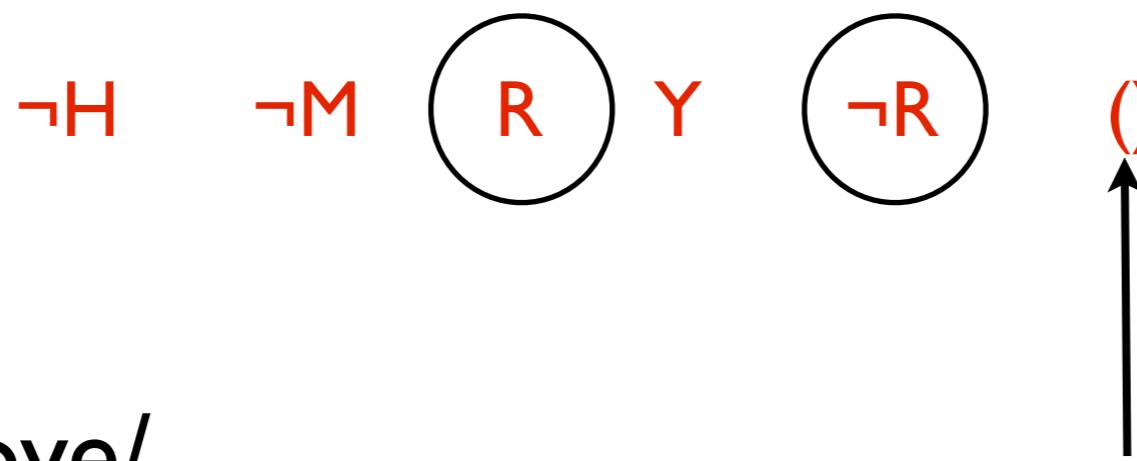
$\neg H \vee G$

$\neg G \vee \neg H$

$\neg H$ $\neg M$ R Y $\neg R$

$$\neg Y \vee \neg R \quad Y \vee R \quad Y \vee M \quad R \vee H \quad \neg M \vee H \quad \neg H \vee G \quad \neg G \vee \neg H$$


“empty” means false, and this makes the entire proposition false, since all clauses are linked by conjunction.

$$\neg Y \vee \neg R \quad Y \vee R \quad Y \vee M \quad R \vee H \quad \neg M \vee H \quad \neg H \vee G \quad \neg G \vee \neg H$$


We can prove/
disprove “any” α
in this way, hence
this inference is
complete.

“empty” means false, and this
makes the entire proposition
false, since all clauses are linked
by conjunction.