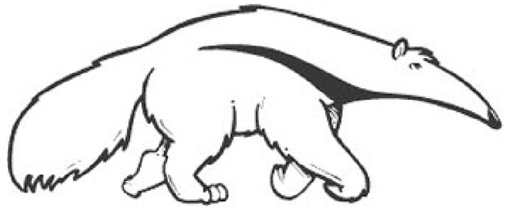
Games & Adversarial Search B: Alpha-Beta Pruning and MCTS

CS171, Summer Session I, 2018

Introduction to Artificial Intelligence

Prof. Richard Lathrop



Read Beforehand: R&N 5.3; Optional: 5.5+

Alpha-Beta pruning

- Exploit the "fact" of an adversary

- Bad = not better than we already know we can get elsewhere
- If a position is provably bad
  - It's NO USE expending search effort to find out just how bad it is
- If the adversary can force a bad position
  - It's NO USE searching to find the good positions the adversary won't let you achieve anyway

- Contrast normal search:
  - ANY node might be a winner, so ALL nodes must be considered.
  - A* avoids this through heuristics that transmit your knowledge.
  - Alpha-Beta pruning avoids this through exploiting the adversary.

Pruning with Alpha/Beta

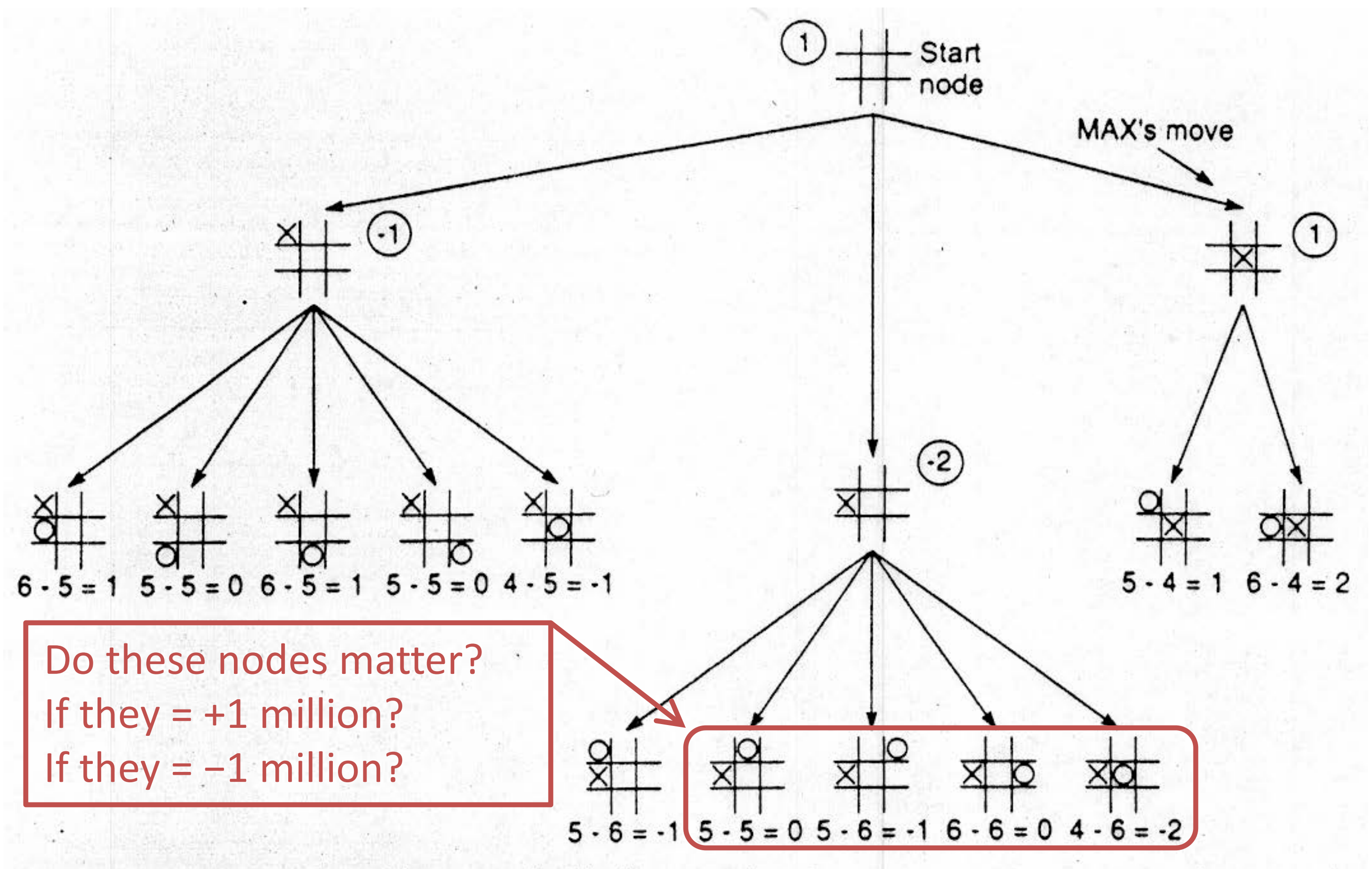
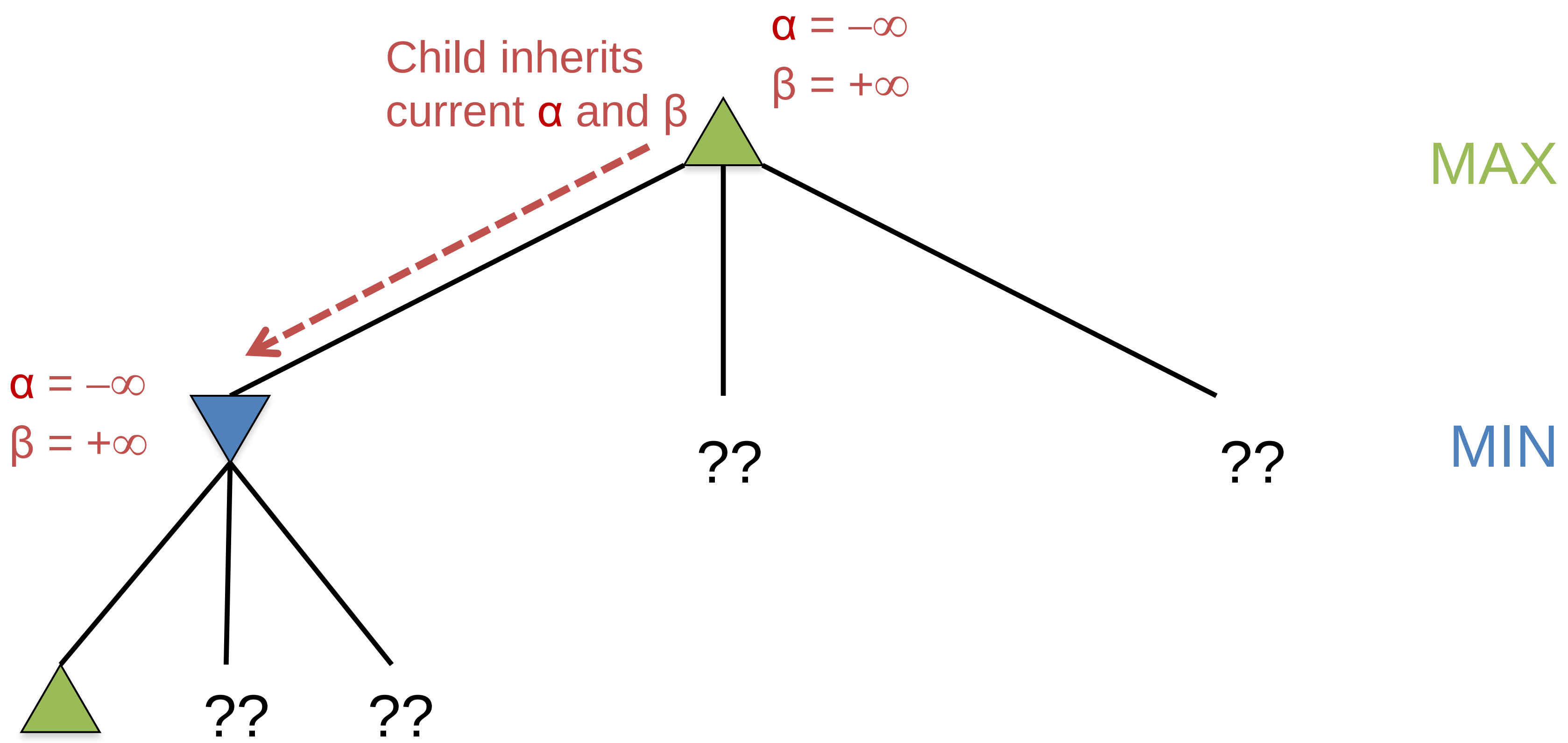


Figure 4.17 Two-ply minimax applied to the opening move of tic-tac-toe.

Alpha-Beta Example

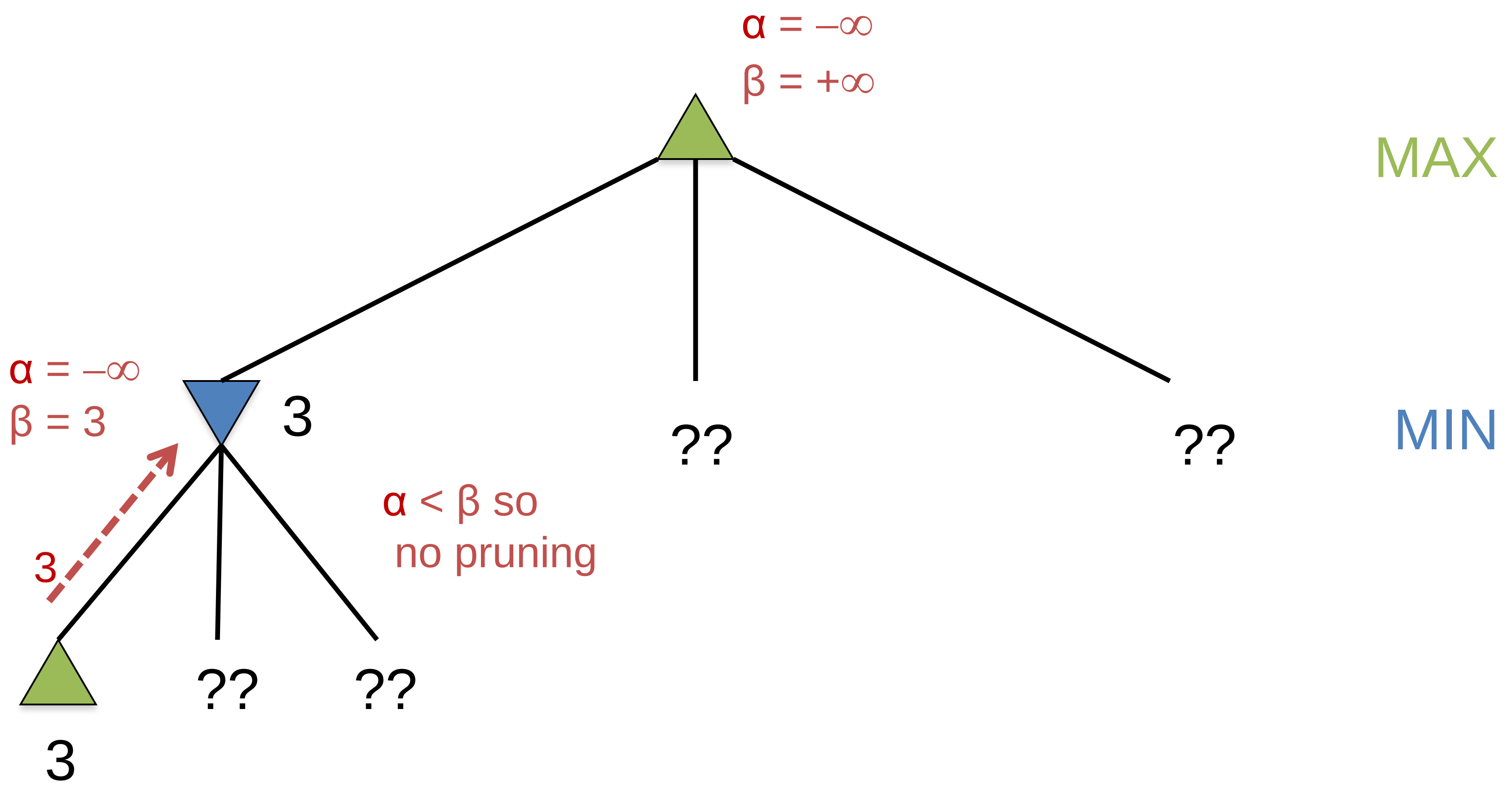
Initially, possibilities are unknown: range (α =-∞, β=+∞)

Do a depth-first search to the first leaf.



Alpha-Beta Example

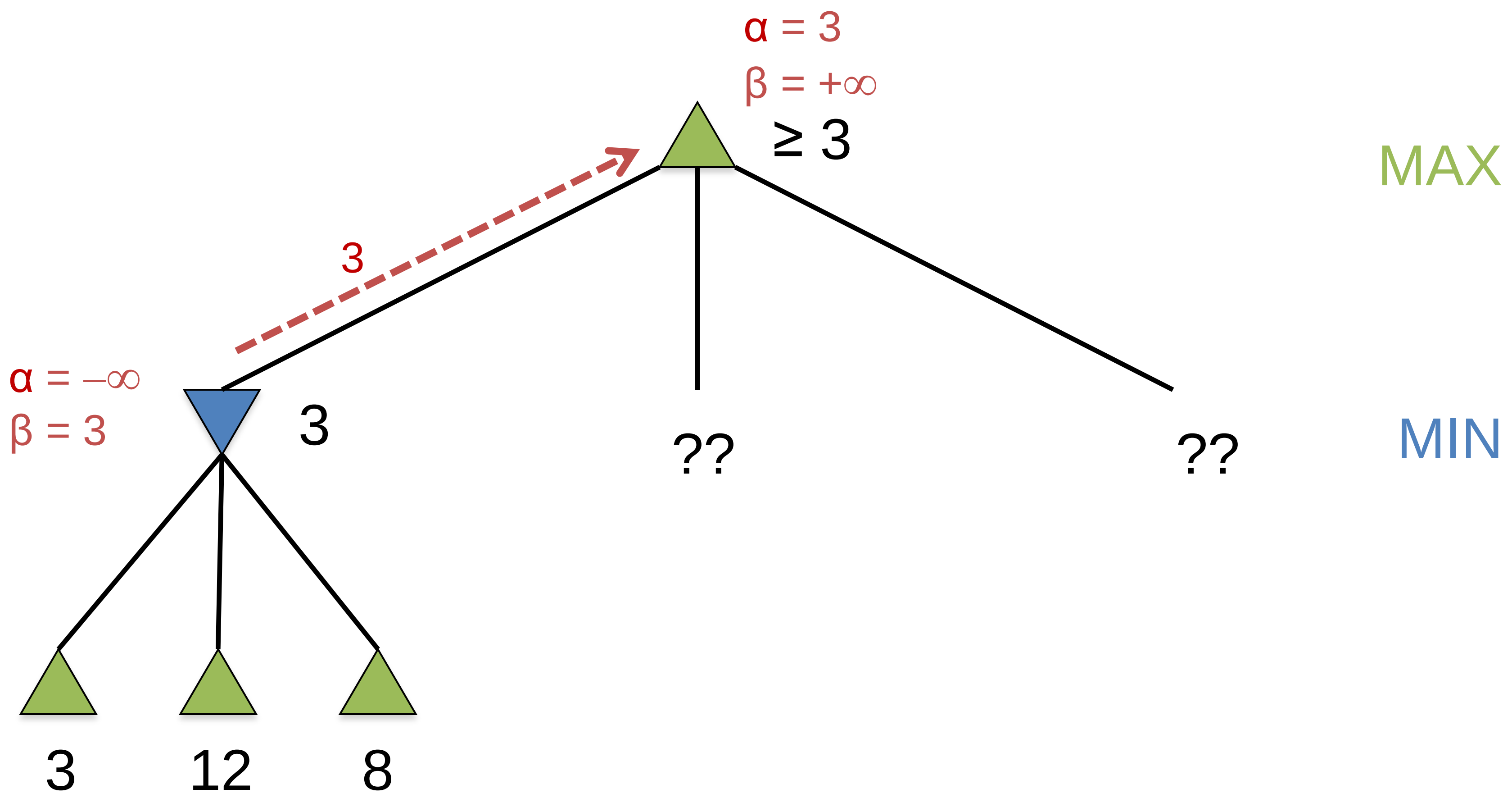
See the first leaf, after MIN's move: MIN updates β



Alpha-Beta Example

See remaining leaves; value is known

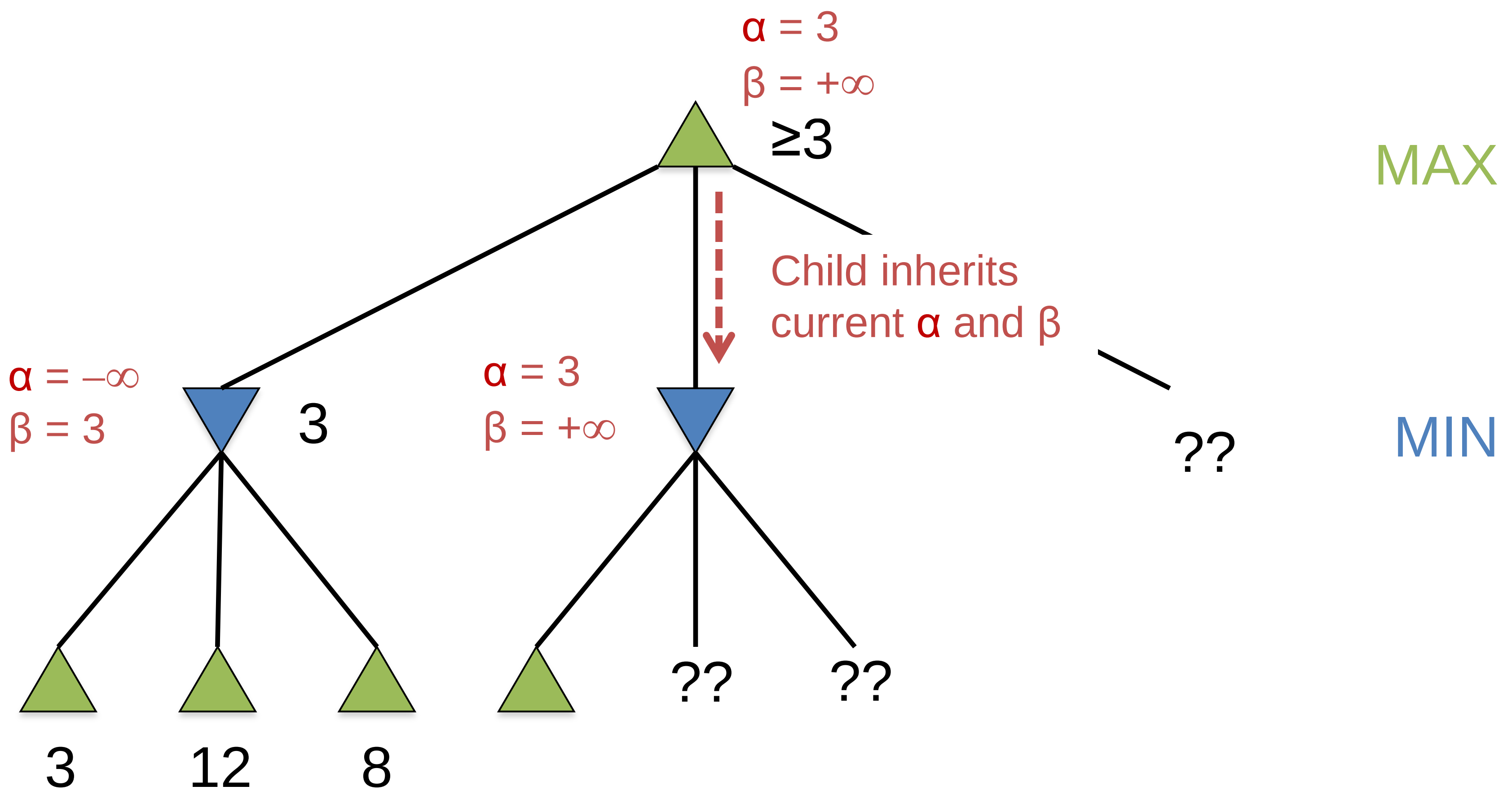
Pass outcome to caller; MAX updates α



Alpha-Beta Example

Continue depth-first search to next leaf.

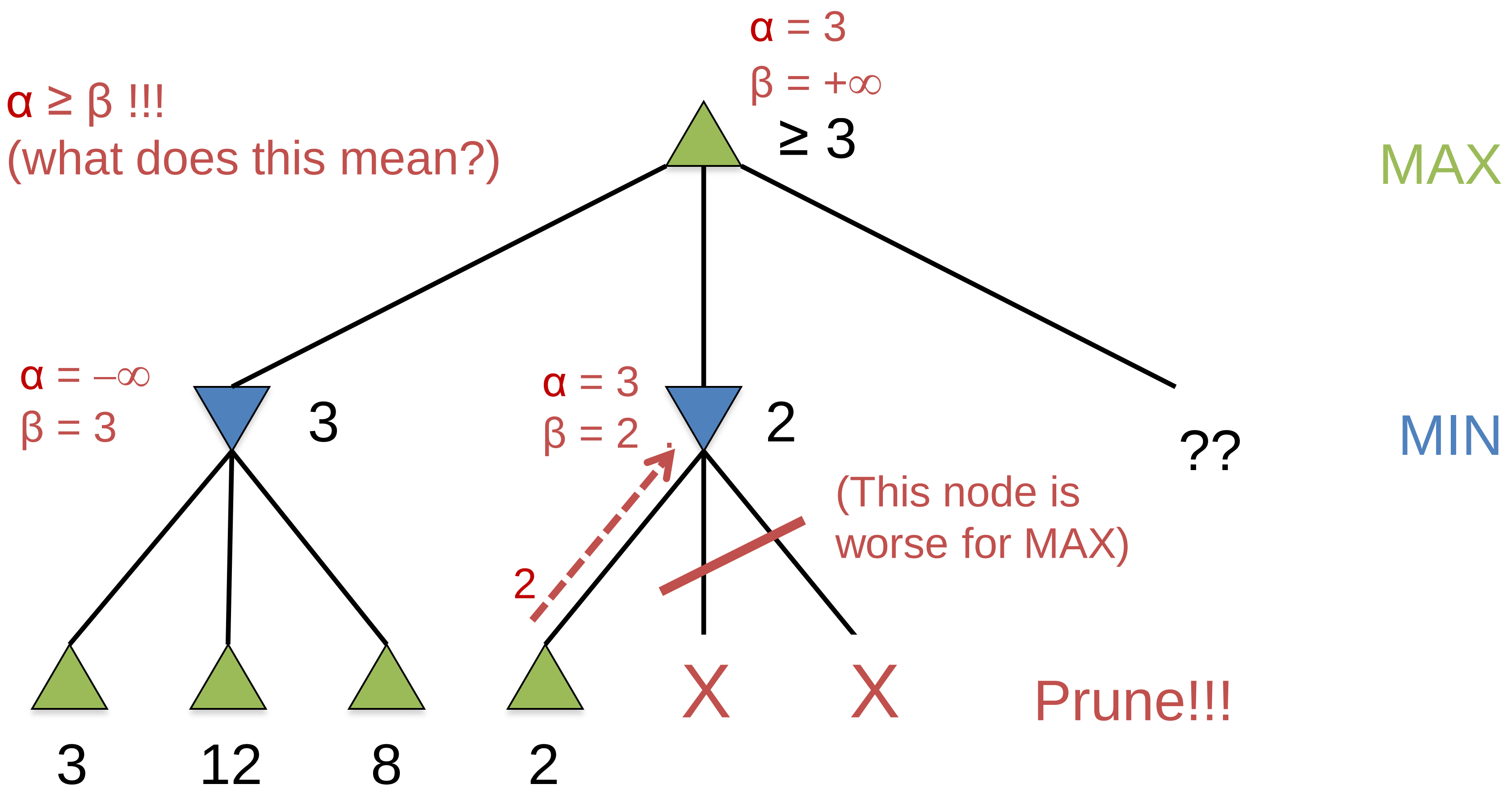
Pass α, β to descendants



Alpha-Beta Example

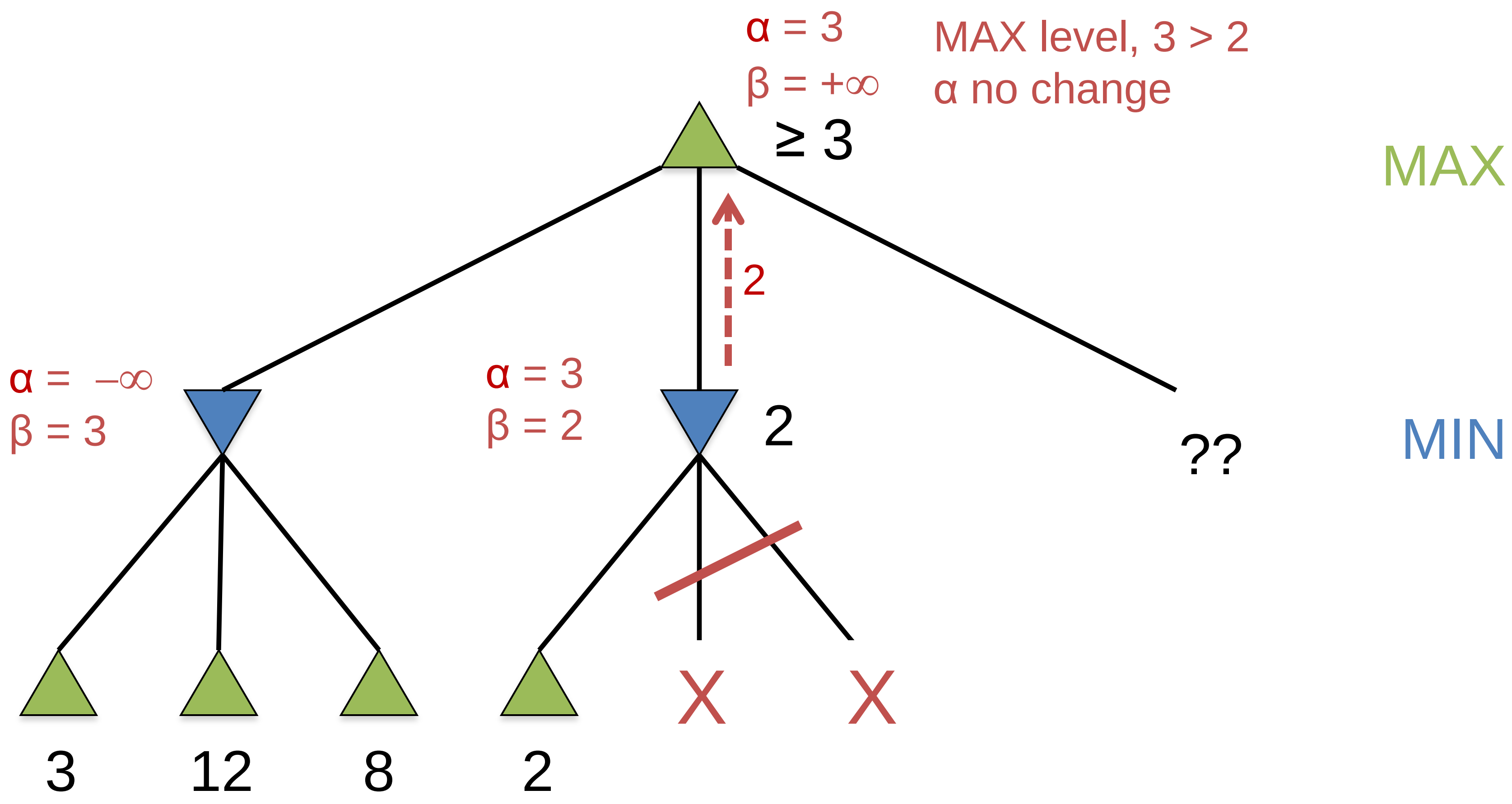
Observe leaf value; MIN's level; MIN updates β

Prune – play will never reach the other nodes!



Alpha-Beta Example

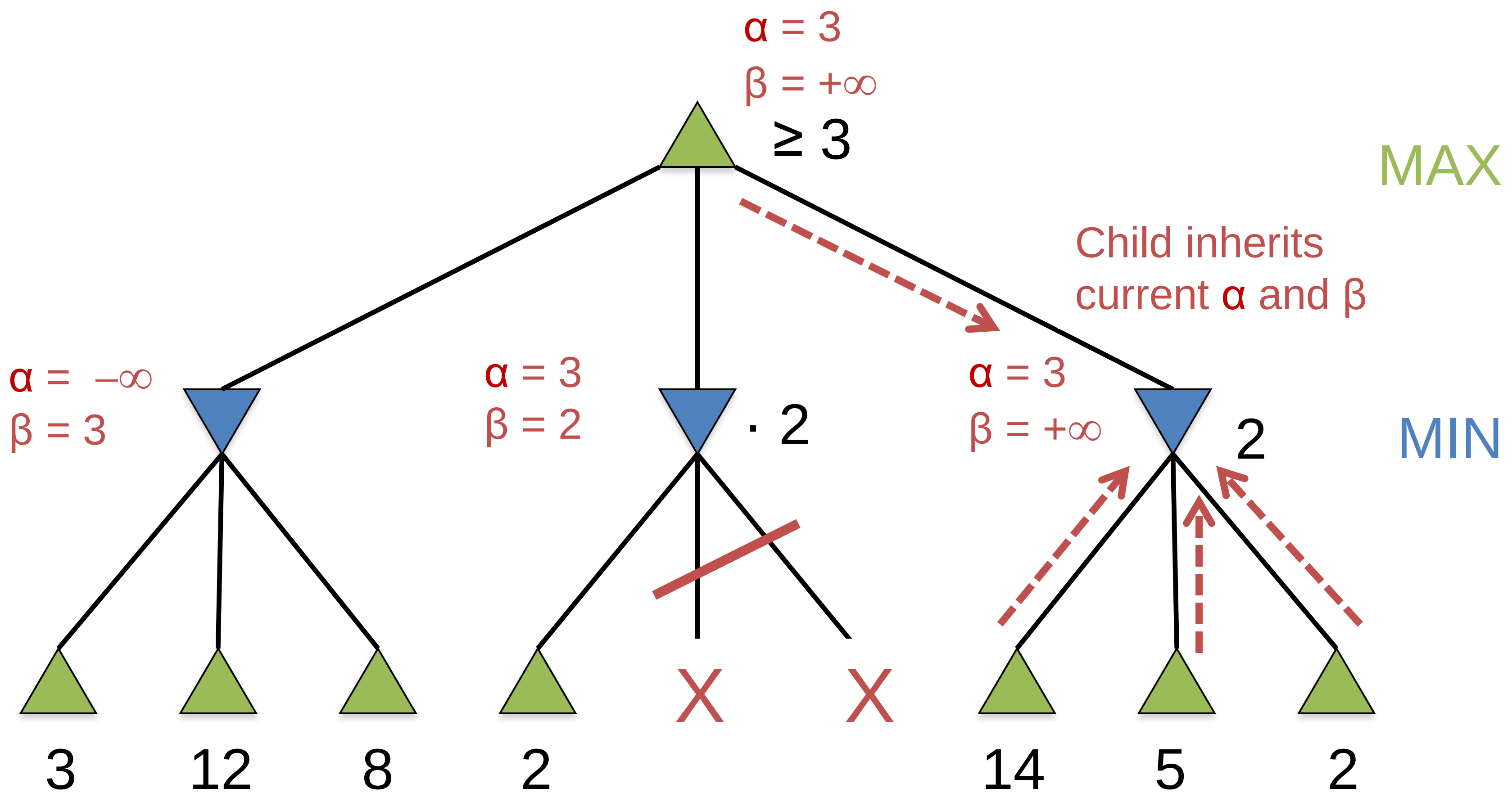
Pass outcome to caller & update caller:



Alpha-Beta Example

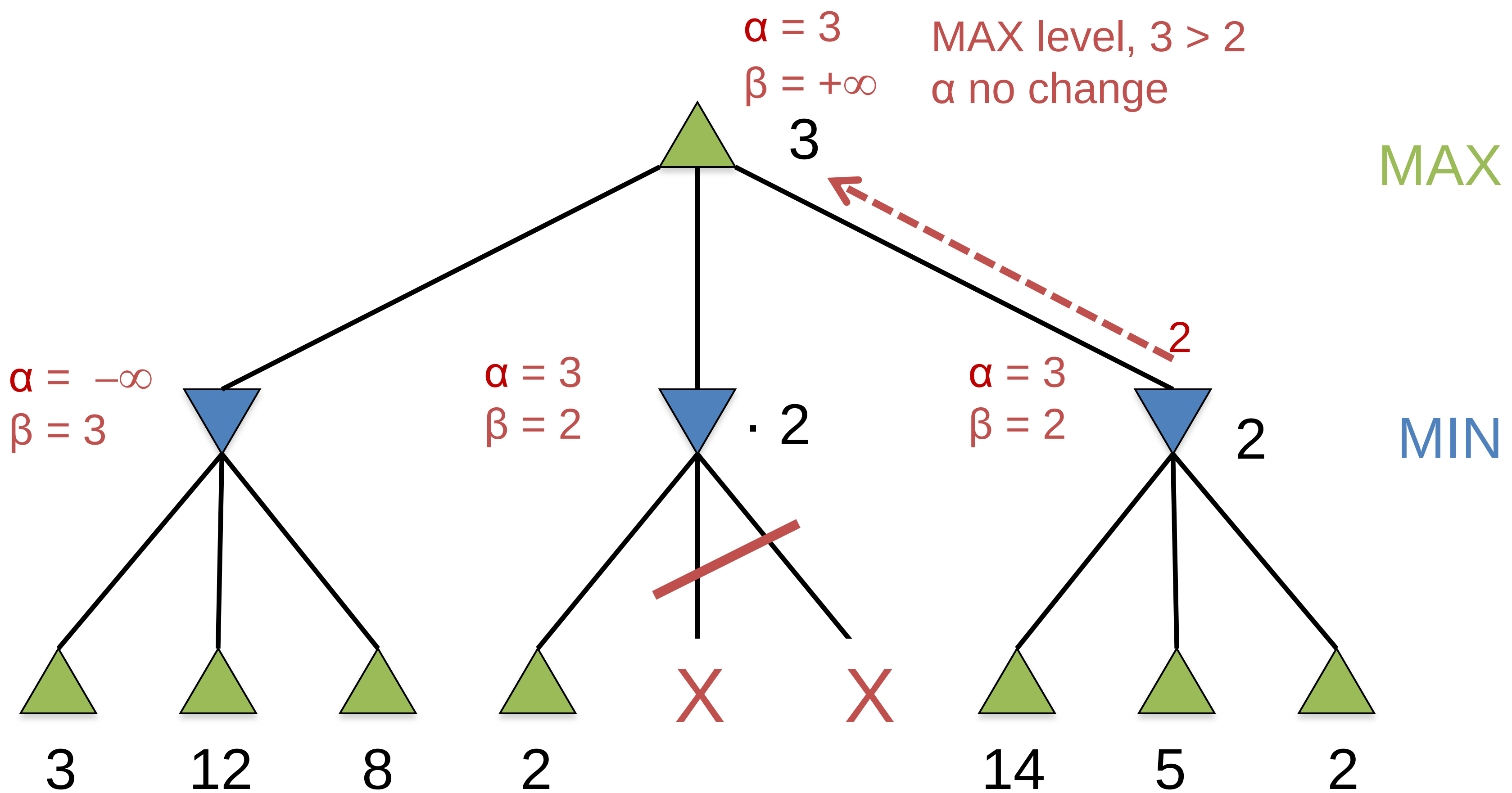
Continue depth-first exploration…

No pruning here; value is not resolved until final leaf.



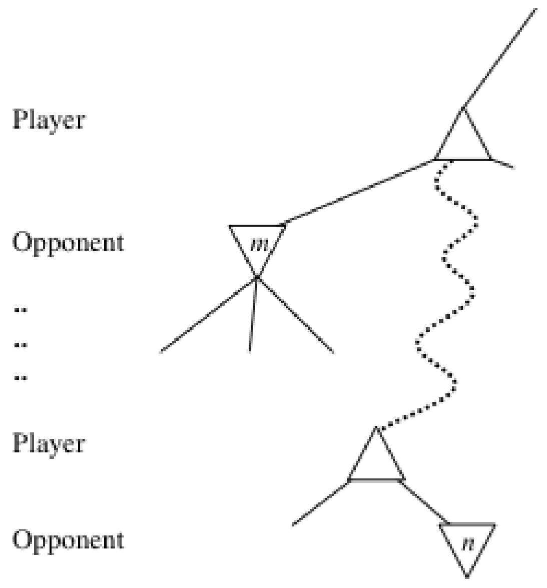
Alpha-Beta Example

Pass outcome to caller & update caller.
Value at the root is resolved.



General alpha-beta pruning

- Consider a node n in the tree:
- If player has a better choice at
  - Parent node of n
  - Or, any choice further up!
- Then n is never reached in play
- So:
  - When that much is known about n, it can be pruned



Recursive α-β pruning (expands on Fig. 5.7)

```
Alpha-Beta-Search(state)                                 Simple stub to call recursion functions
   alpha = –infty, beta = +infty, act = None             Initialize alpha, beta; no move found
   for each a in Actions(state) do                       Score each action; update alpha & best action
       val = Min-Value( Result(state, a), alpha, beta )
       if ( val > alpha ) then alpha = val, act = a
   return act

MaxValue(state, al, be)
   if (Cutoff(state)) then return Eval(state)            If Cutoff reached, return Eval heuristic
   val = –infty                                          Otherwise, find our best child:
   for each a in Actions(state) do                       If our options are too good, our min
      val = max( val,  MinValue( Result(state, a), al, be )   ancestor will never let us come this way
      if ( val ≥ be ) then return val                    Otherwise return the best we can find
      al = max( al, val )
  return val

MinValue(state, al, be)
   if (Cutoff(state)) then return Eval(state)            If Cutoff reached, return Eval heuristic
   val = +infty                                          Otherwise, find the worst child:
   for each a in Actions(state) do                       If our options are too bad, our max
      val = min( val,  MaxValue( Result(state, a), al, be )   ancestor will never let us come this way
      if ( val ≤ al) then return val                     Otherwise return the worst we can find
      be = min( be, val )
   return be
```

Effectiveness of α-β Search

- Worst-Case
  - Branches are ordered so that no pruning takes place. In this case alpha-beta gives no improvement over exhaustive search
- Best-Case
  - Each player’s best move is the left-most alternative (i.e., evaluated first)
  - In practice, performance is closer to best rather than worst-case
- In practice often get $O(b^{(d/2)})$ rather than $O(b^d)$
  - This is the same as having a branching factor of sqrt(b),
    - since $(\text{sqrt}(b))^d = b^{(d/2)}$ (i.e., we have effectively gone from b to square root of b)
  - In chess go from b ~ 35 to b ~ 6
    - permiting much deeper search in the same amount of time

Iterative deepening

- In real games, there is usually a time limit T to make a move

- How do we take this into account?
- Minimax cannot use “partial” results with any confidence, unless the full tree has been searched
  - Conservative: set small depth limit to guarantee finding a move in time < T
  - But, we may finish early – could do more search!

- Added benefit with Alpha-Beta Pruning:
  - Remember node values found at the previous depth limit
  - Sort current nodes so that each player’s best move is left-most child
  - Likely to yield good Alpha-Beta Pruning => better, faster search
  - Only a heuristic: node values will change with the deeper search
  - Usually works well in practice

Comments on alpha-beta pruning

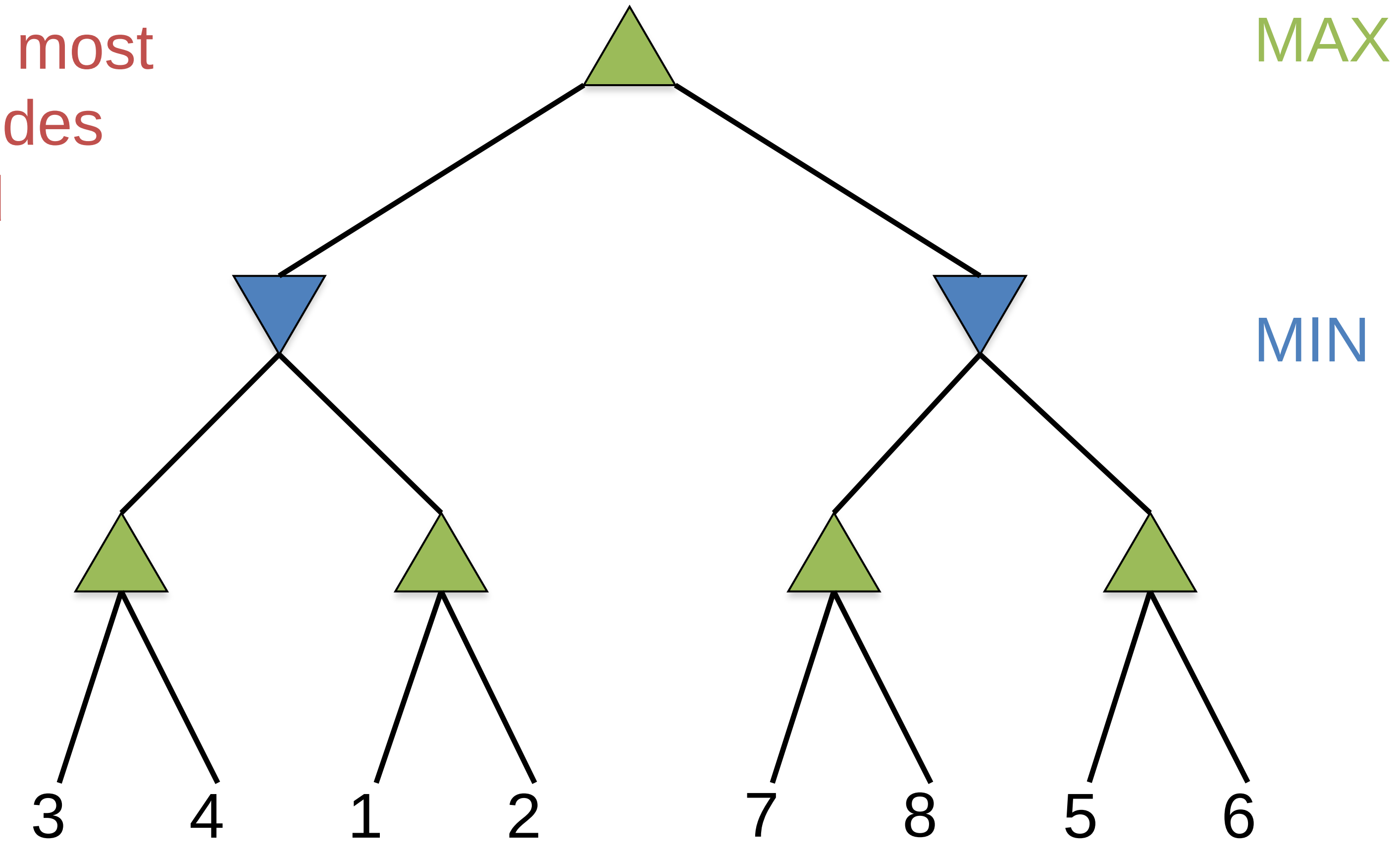
- Pruning does not affect final results
- Entire subtrees can be pruned
- Good move ordering improves pruning
  - Order nodes so player's best moves are checked first
- Repeated states are still possible
  - Store them in memory = transposition table

Iterative deepening reordering

Which leaves can be pruned?

None!

because the most favorable nodes are explored last…

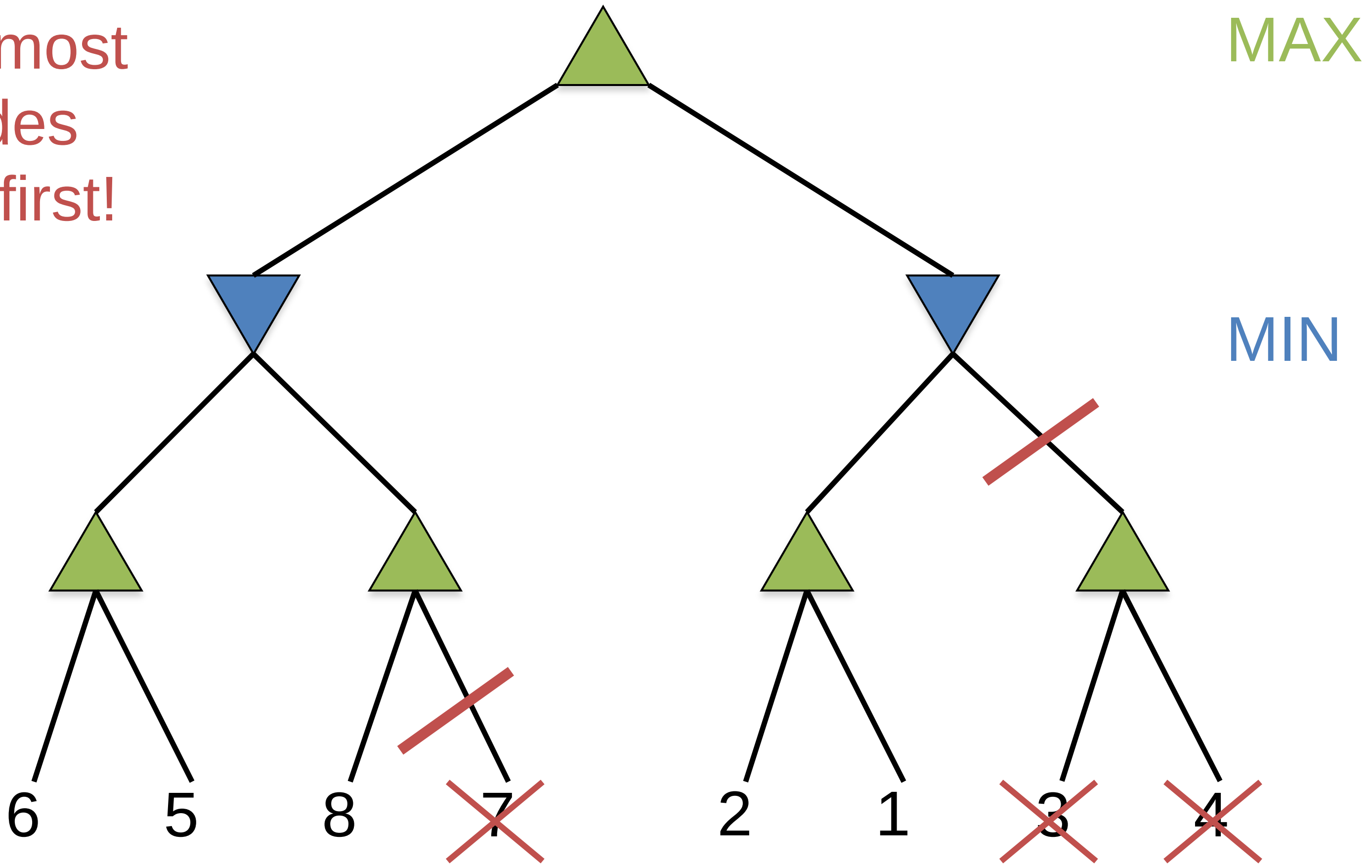


Iterative deepening reordering

Different exploration order: now which leaves can be pruned?

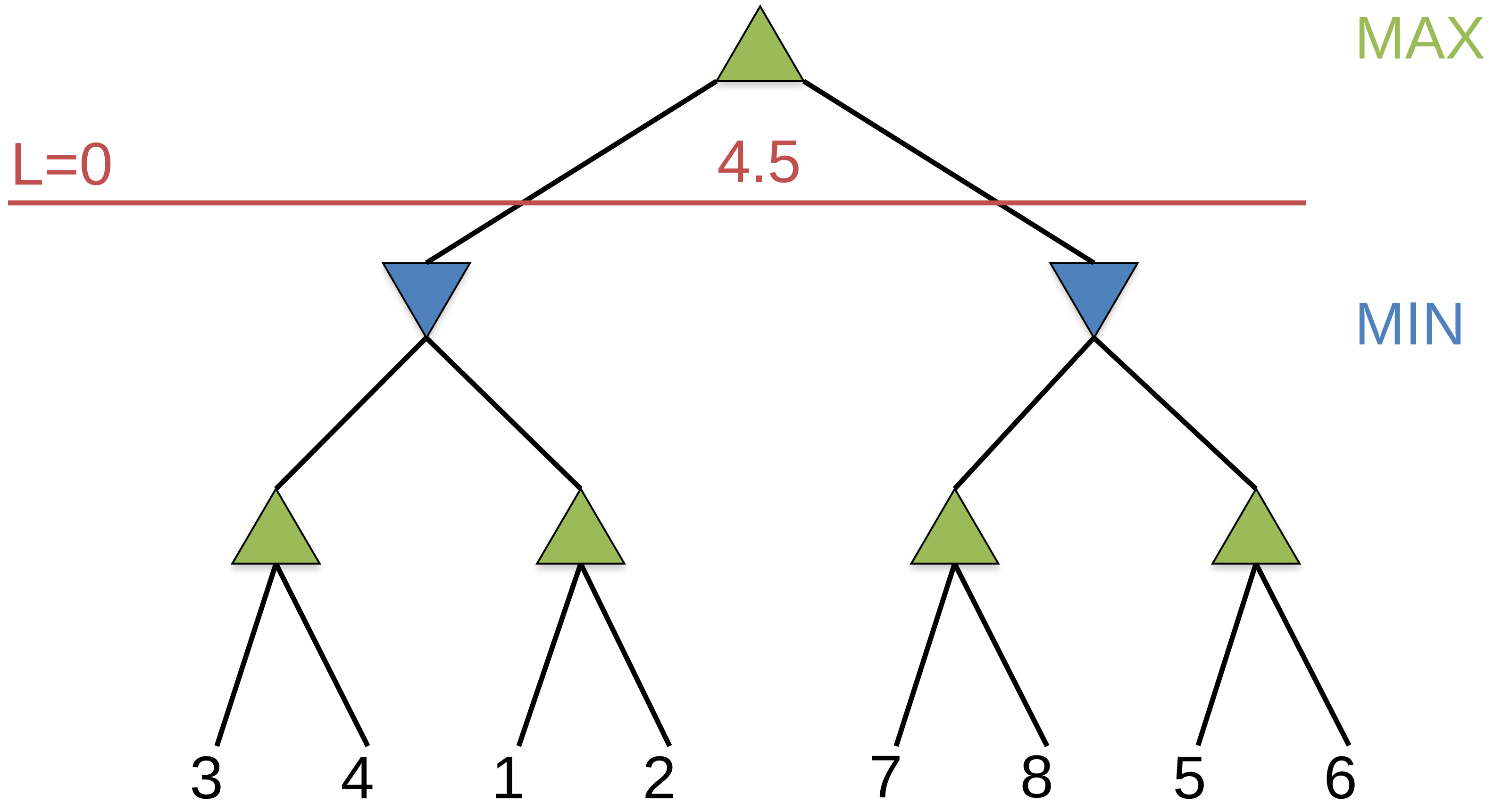
Lots!

because the most favorable nodes are explored first!



Iterative deepening reordering

Order with no pruning; use iterative deepening approach.
Assume node score is the average of leaf values below.



Iterative deepening reordering

Order with no pruning; use iterative deepening approach.
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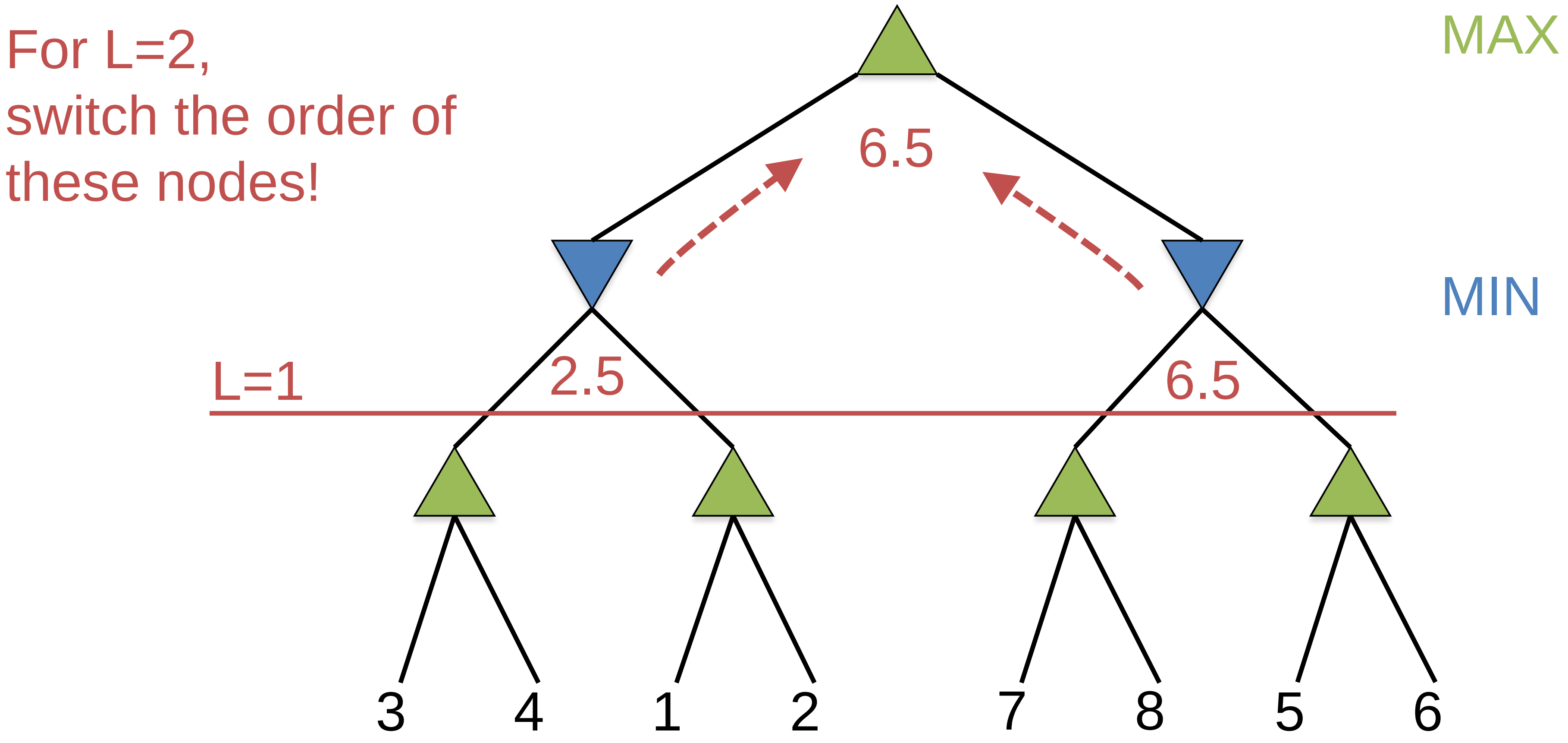

Iterative deepening reordering

Order with no pruning; use iterative deepening approach.
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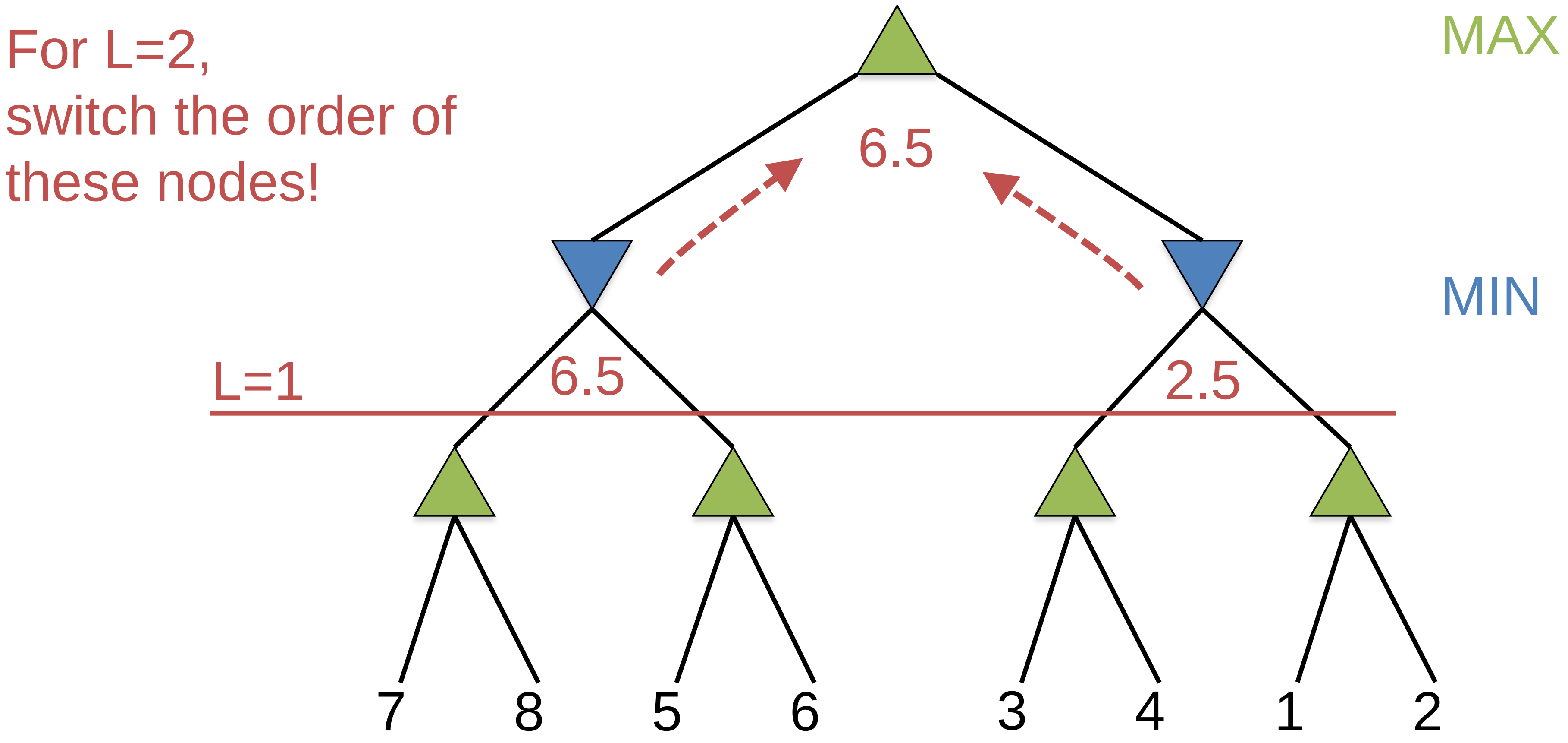
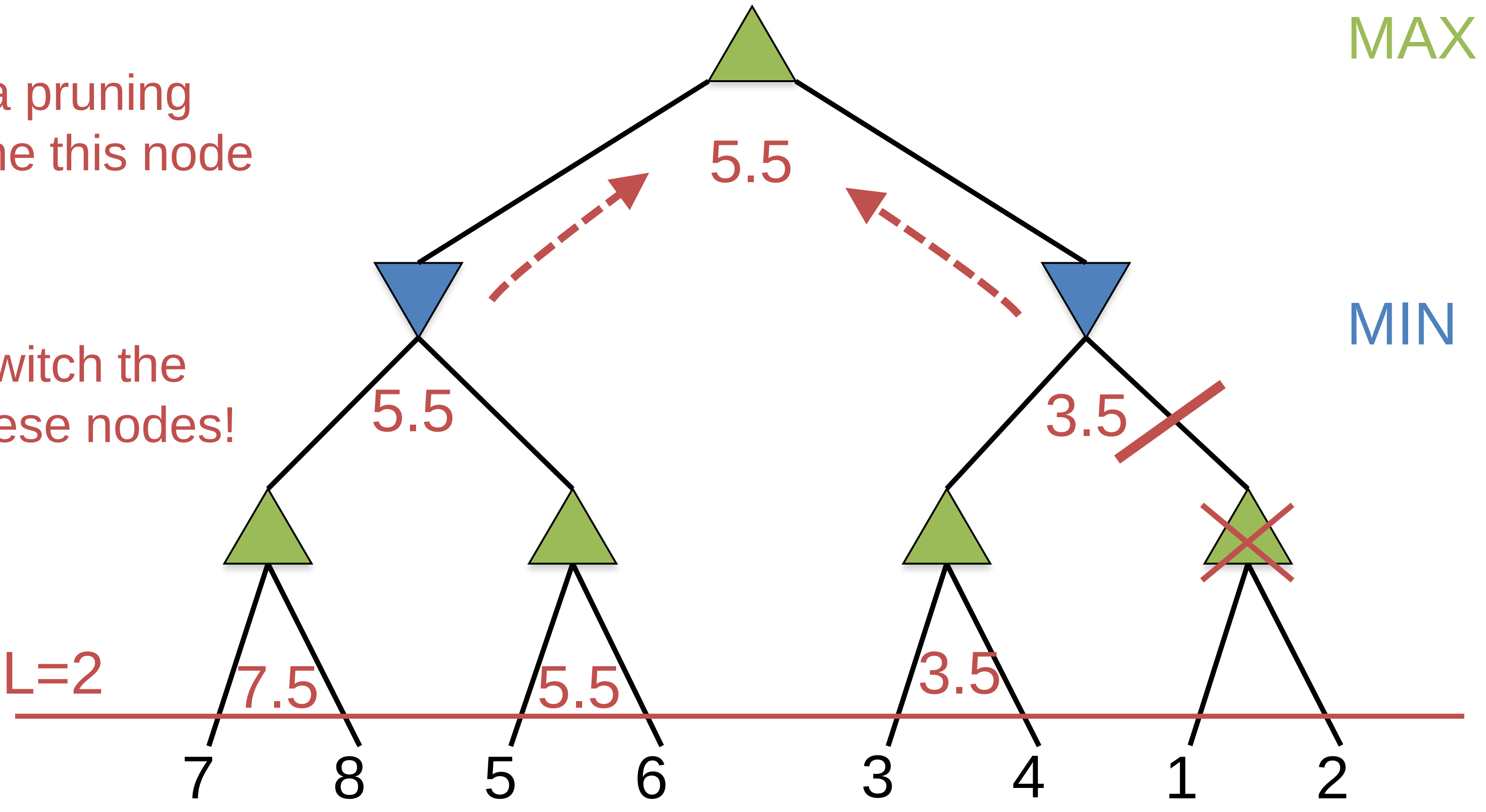


Iterative deepening reordering

Order with no pruning; use iterative deepening approach.
Assume node score is the average of leaf values below.

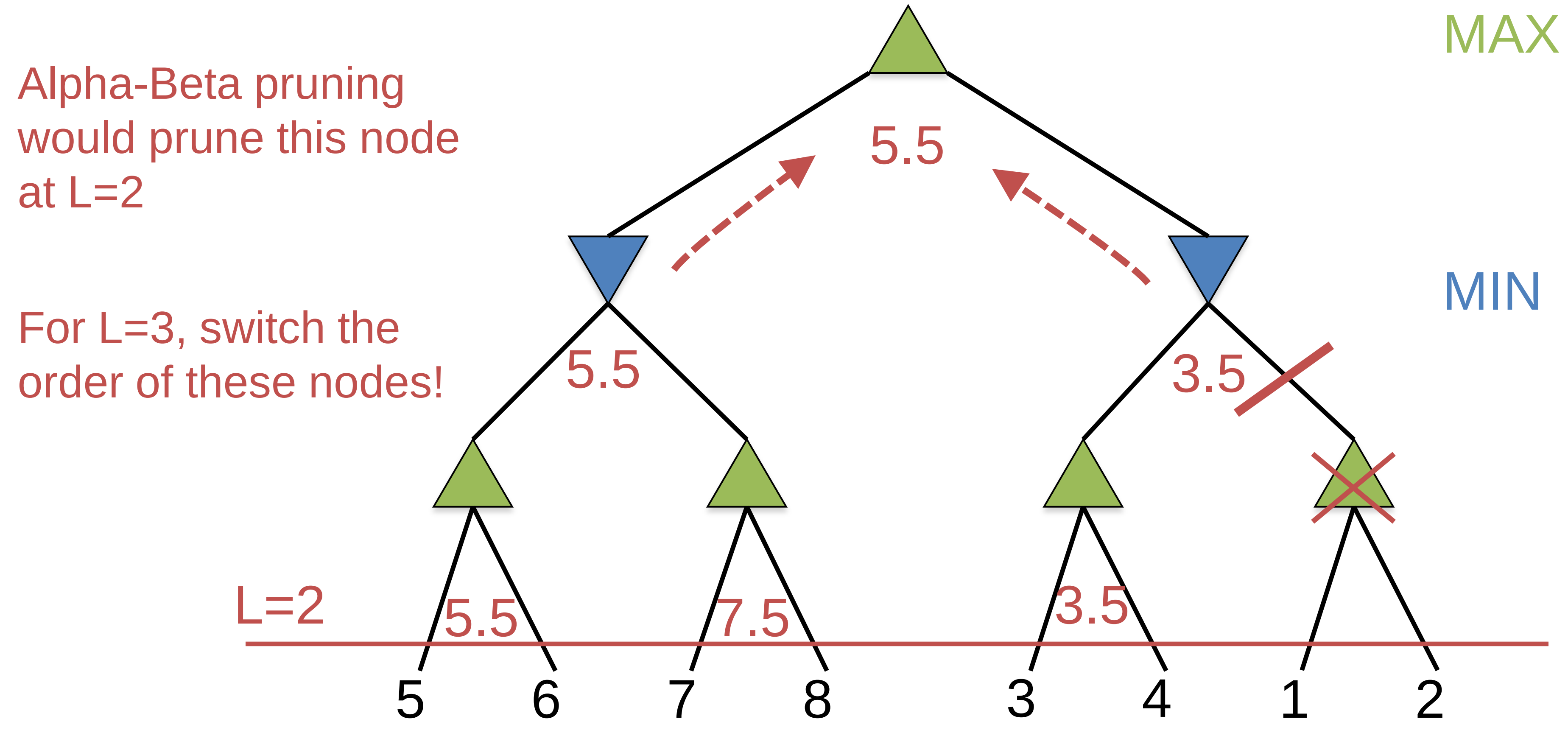


Alpha-Beta pruning would prune this node at L=2

For L=3, switch the order of these nodes!

Iterative deepening reordering

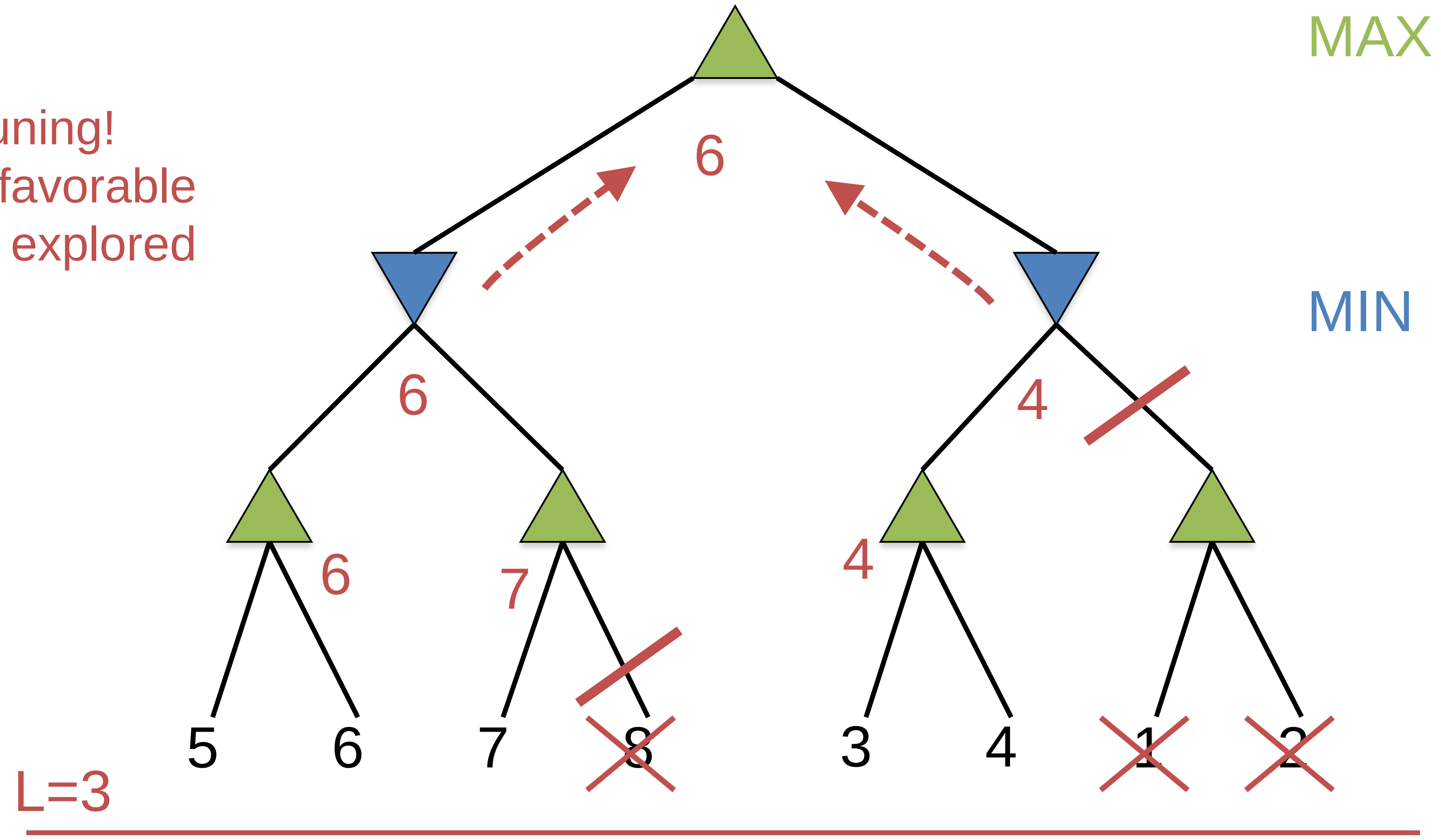
Order with no pruning; use iterative deepening approach.
Assume node score is the average of leaf values below.


MAX
MIN
Alpha-Beta pruning would prune this node at L=2
For L=3, switch the order of these nodes!
5.5
5.5
3.5
L=2
5.5
7.5
3.5
5
6
7
8
3
4
1
2

Iterative deepening reordering

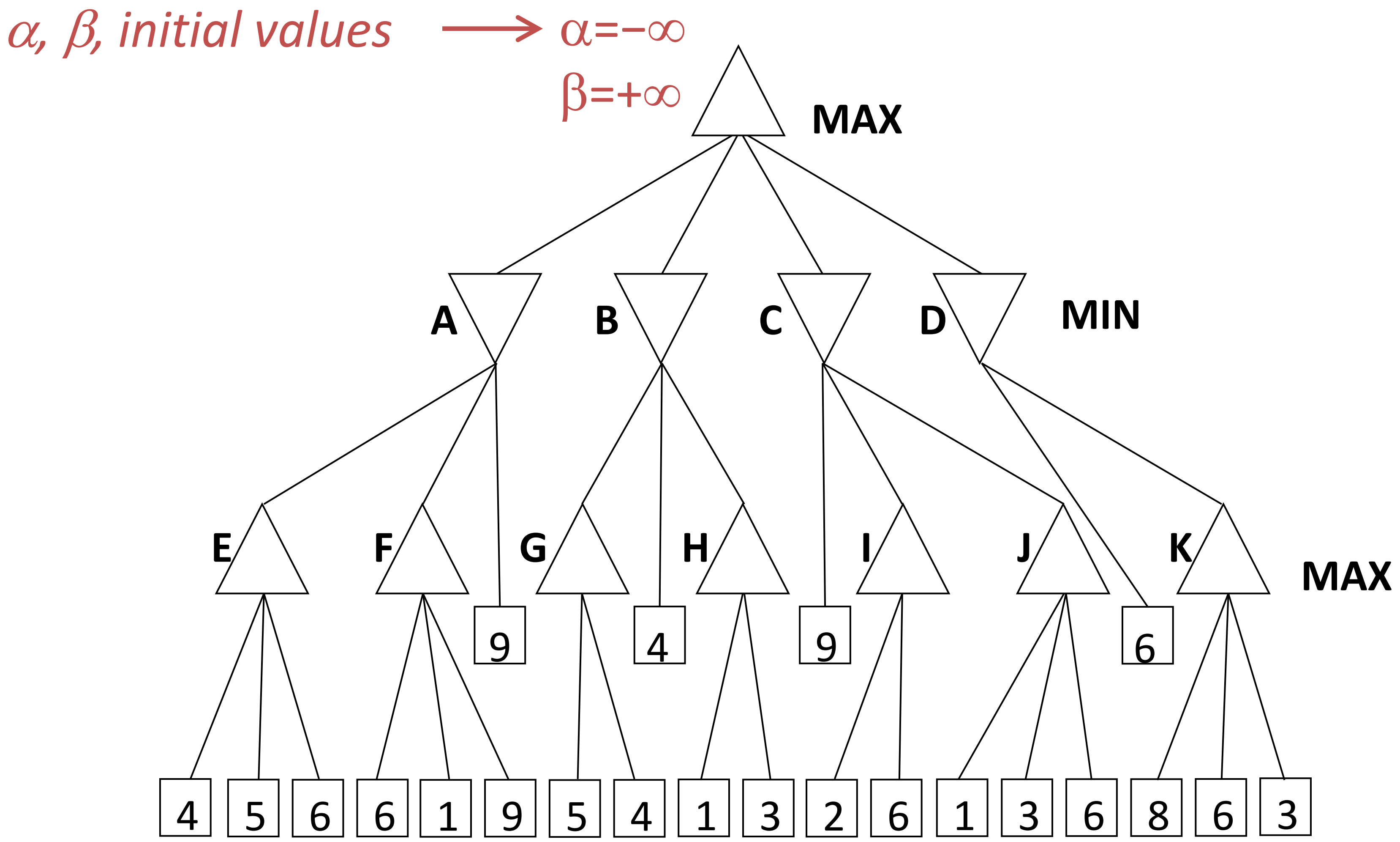
Order with no pruning; use iterative deepening approach.
Assume node score is the average of leaf values below.

Lots of pruning!
The most favorable nodes are explored earlier.



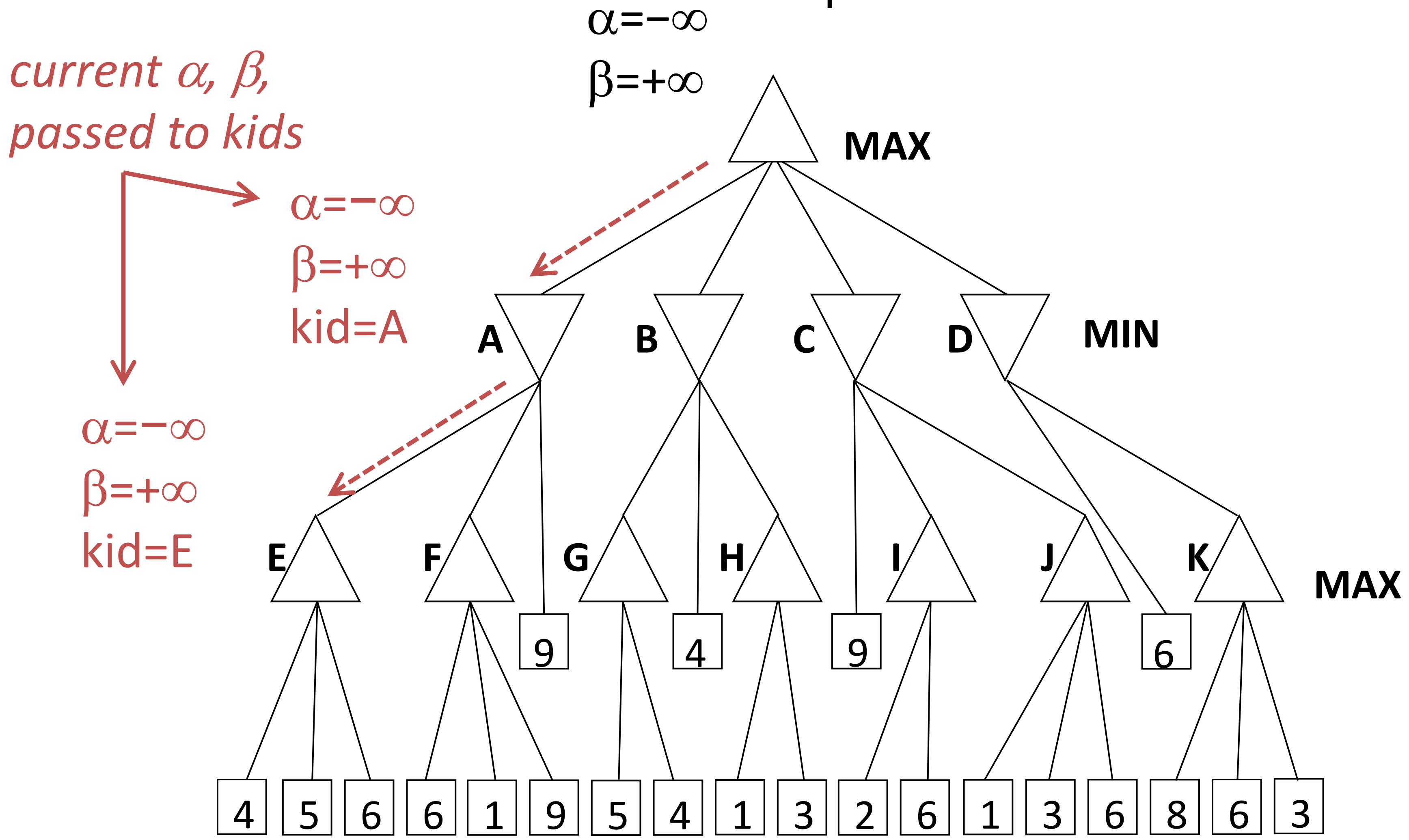
Longer Alpha-Beta Example

Branch nodes are labelel A..K for easy discussion

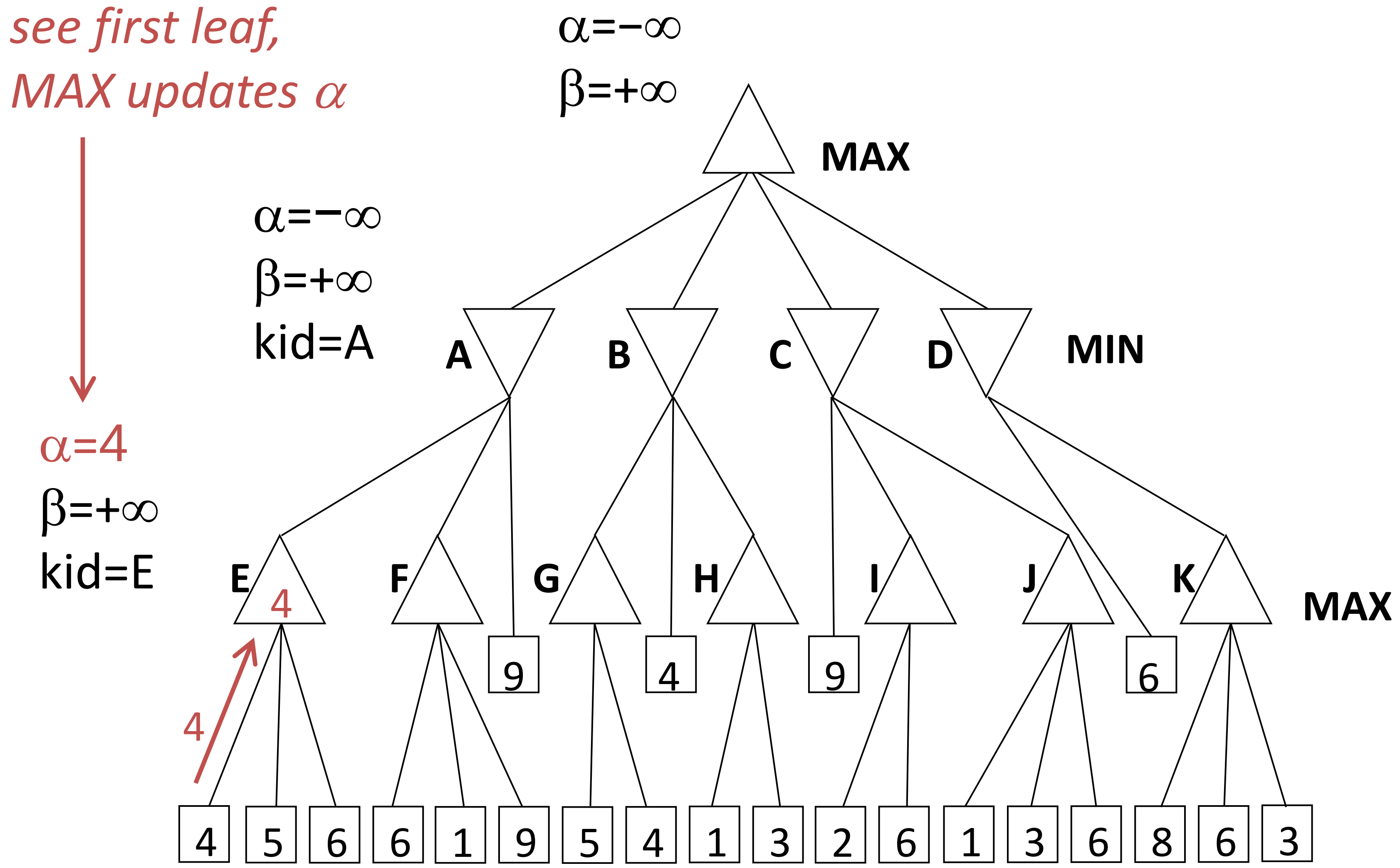


Longer Alpha-Beta Example

Note that cut-off occurs at different depths…

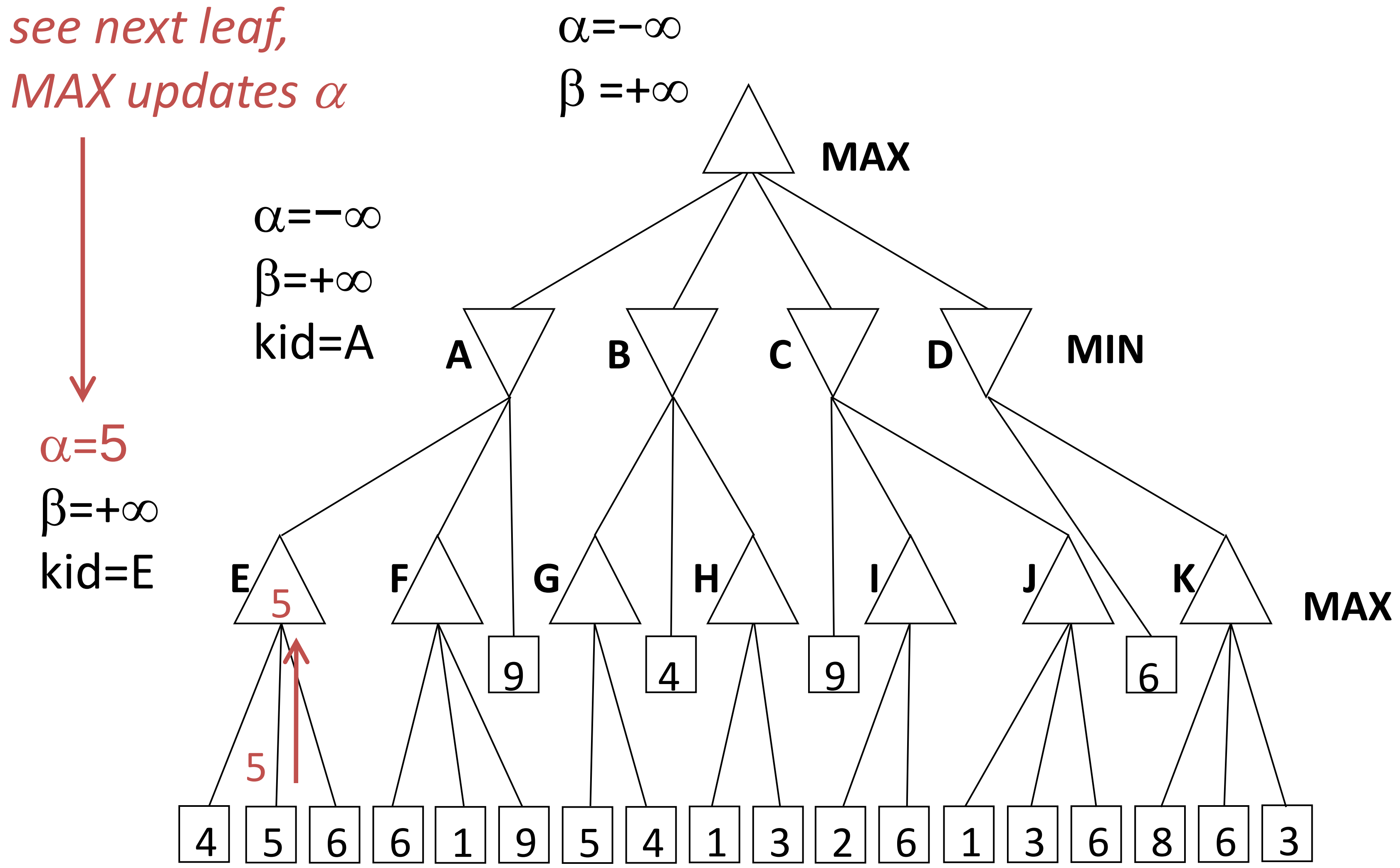


Longer Alpha-Beta Example



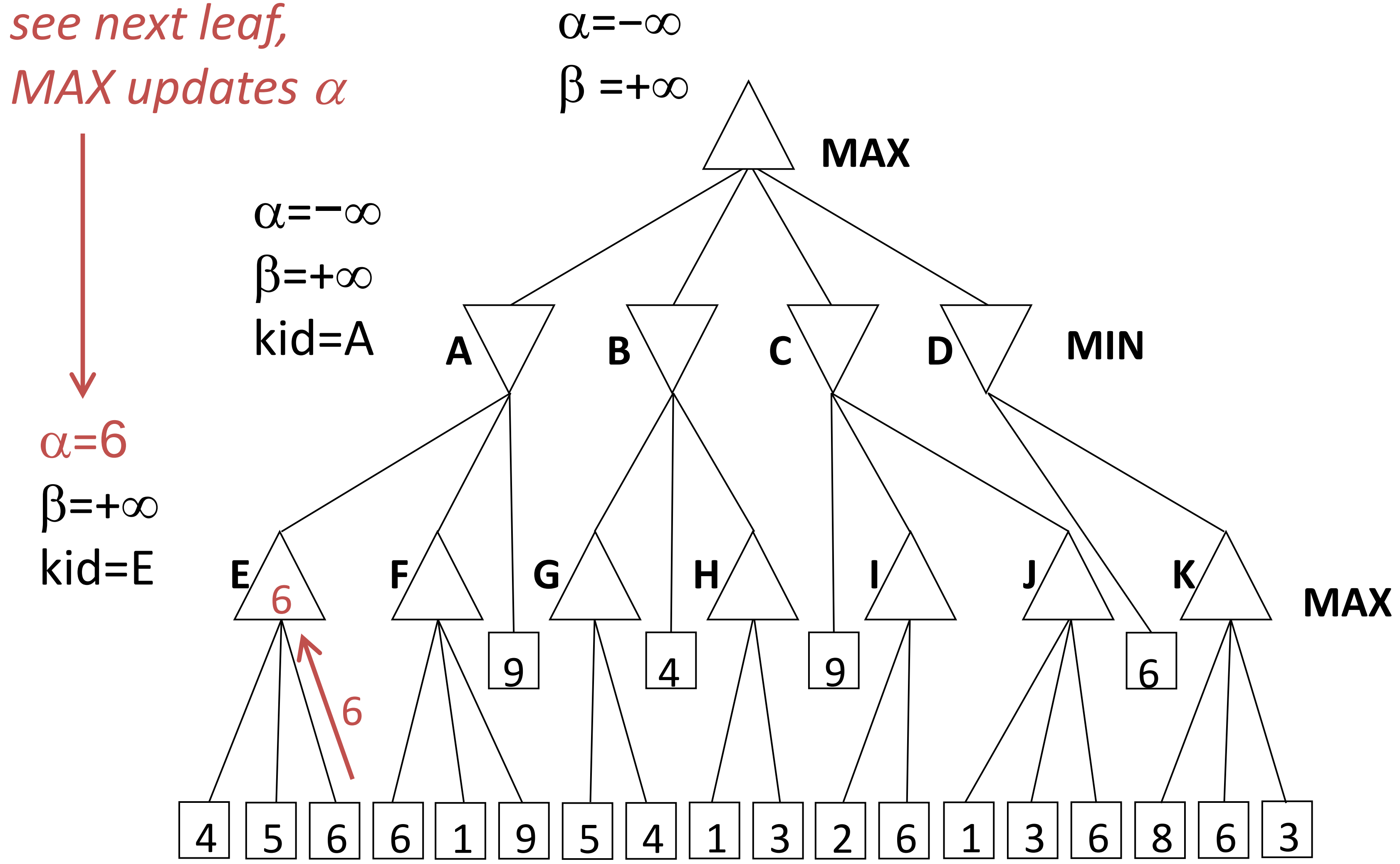
We also are running MiniMax search and recording node values within the triangles, without explicit comment.

Longer Alpha-Beta Example



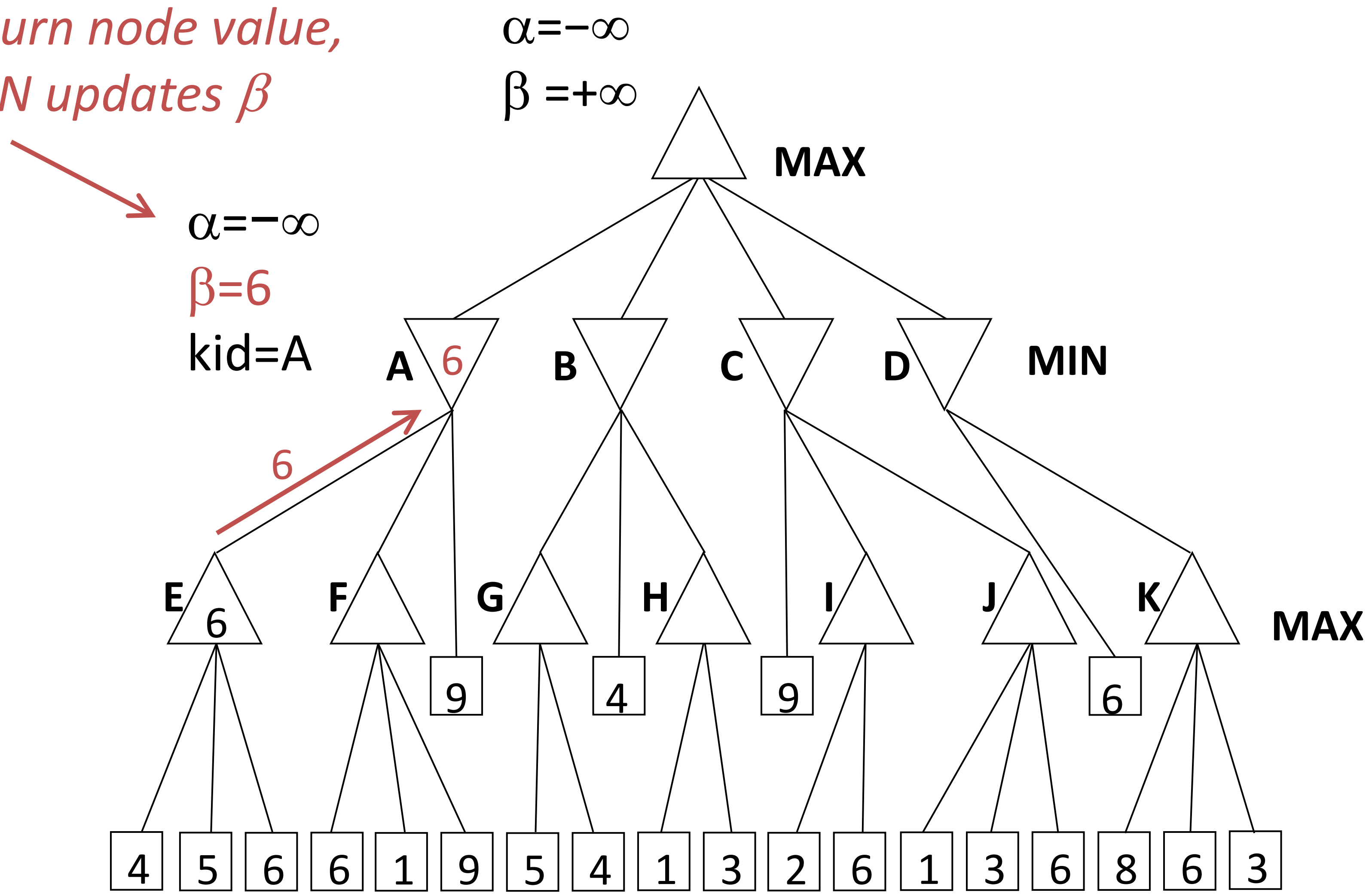
Longer Alpha-Beta Example

see next leaf,
MAX updates α



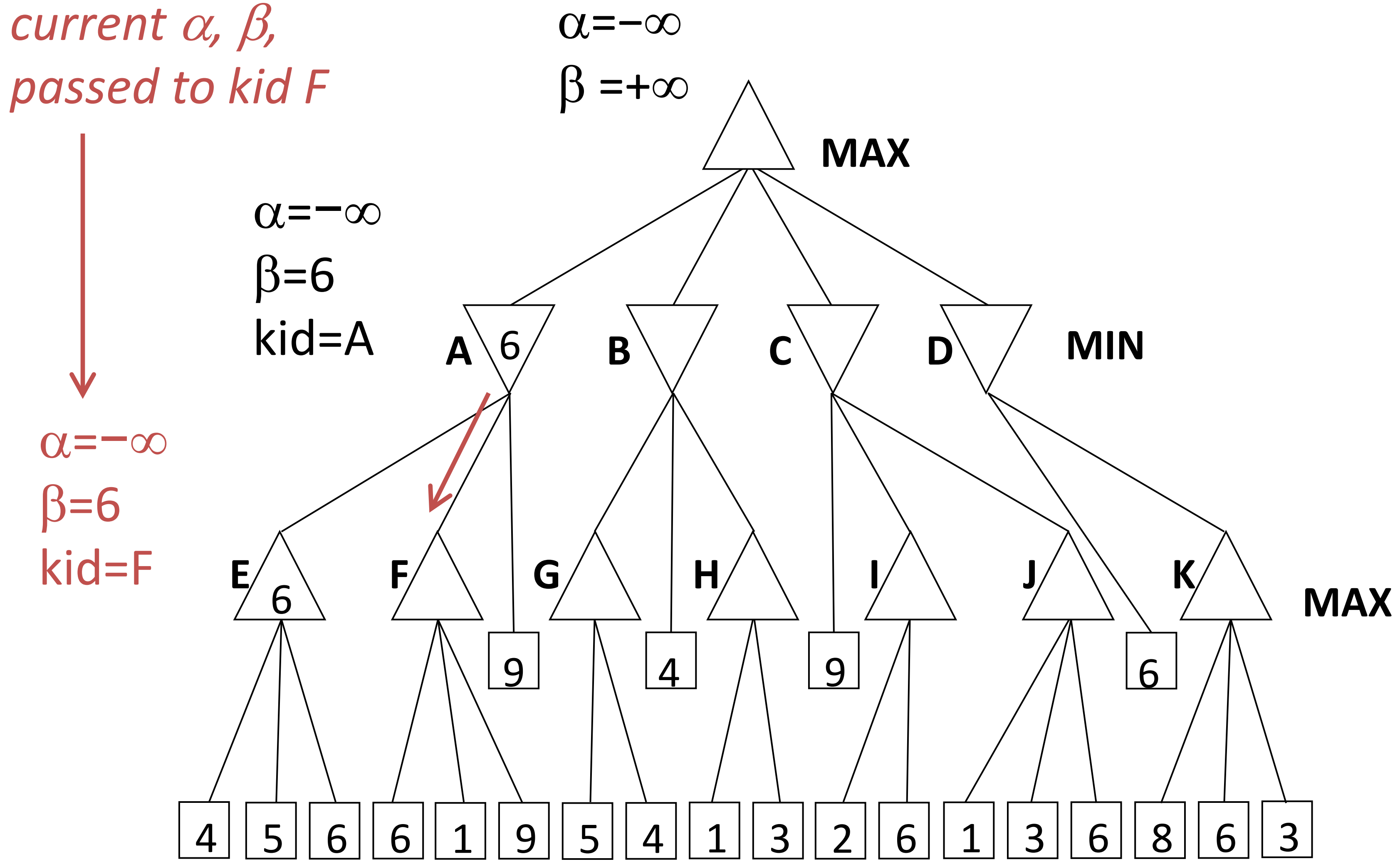
Longer Alpha-Beta Example

return node value,
MIN updates β



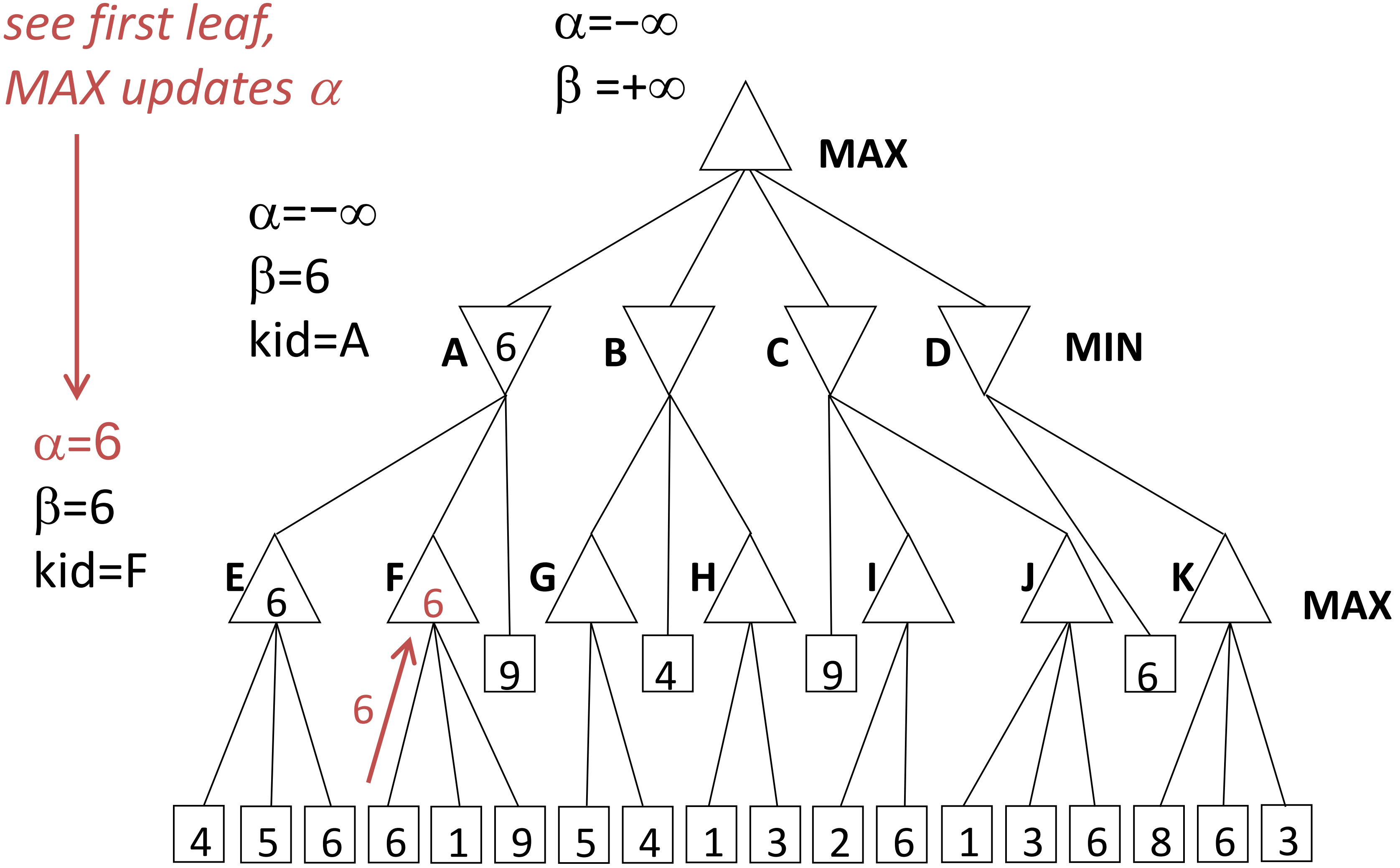
Longer Alpha-Beta Example

current α, β,
passed to kid F

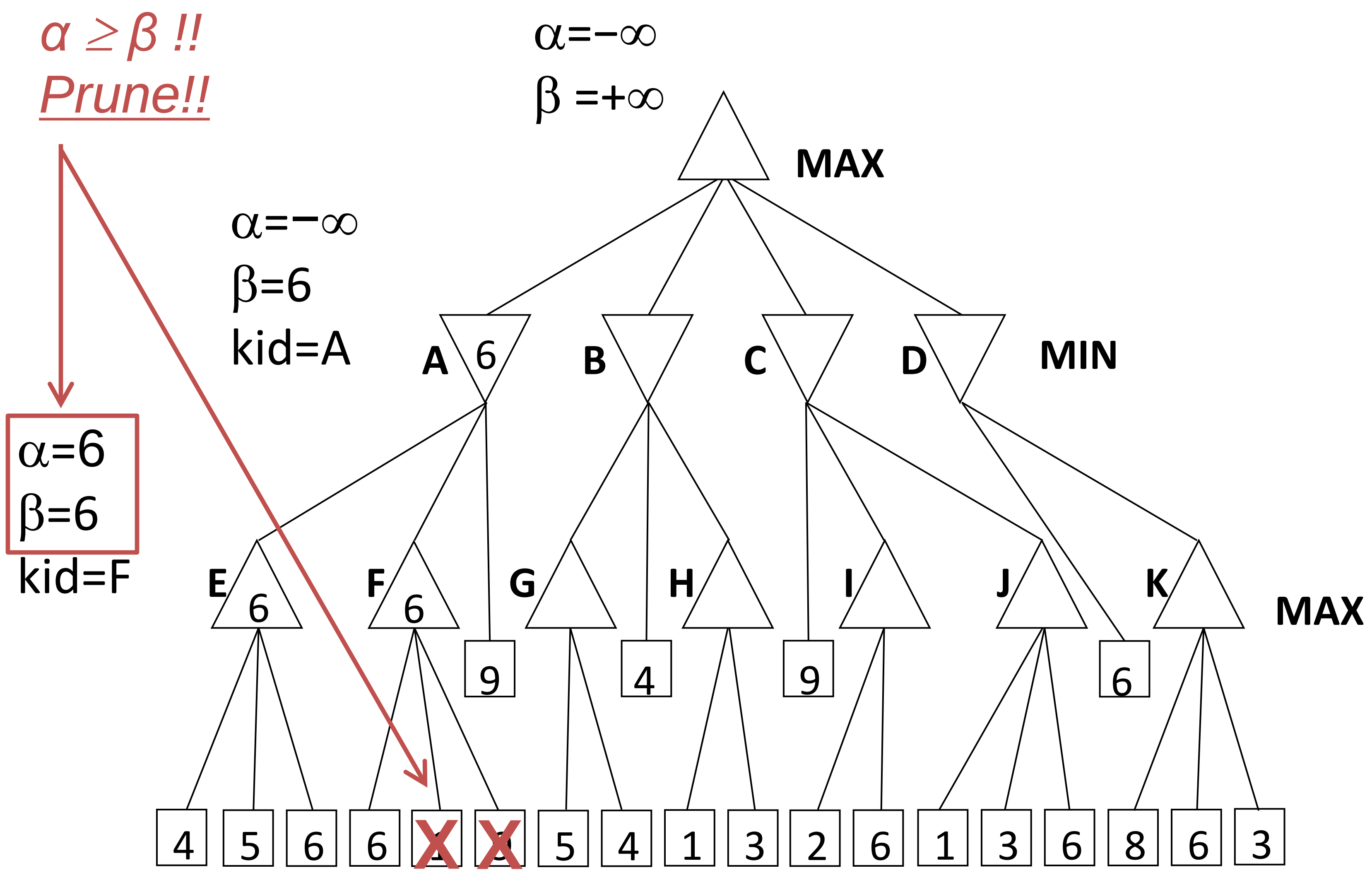


Longer Alpha-Beta Example

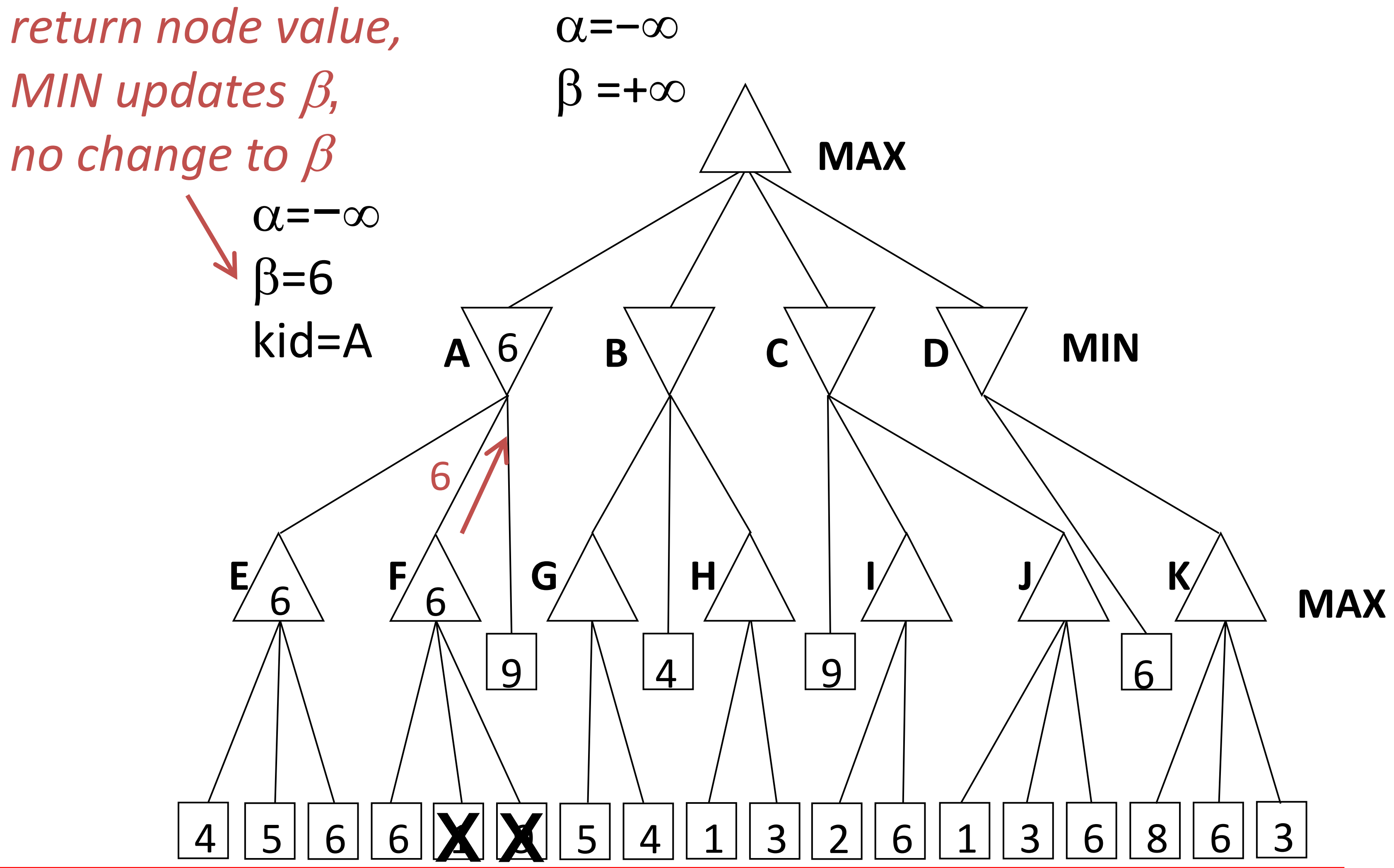
see first leaf,
MAX updates α



Longer Alpha-Beta Example



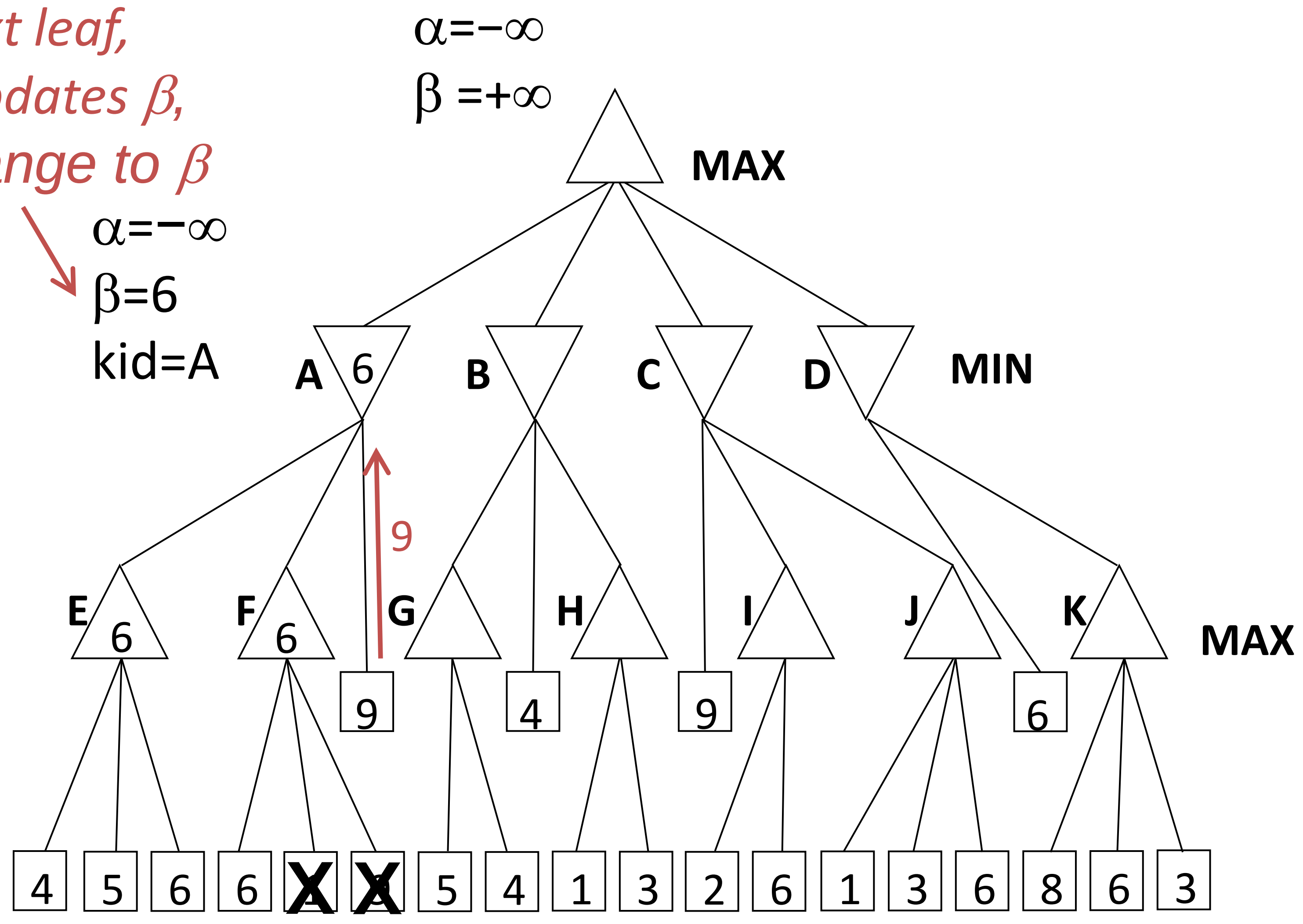
Longer Alpha-Beta Example



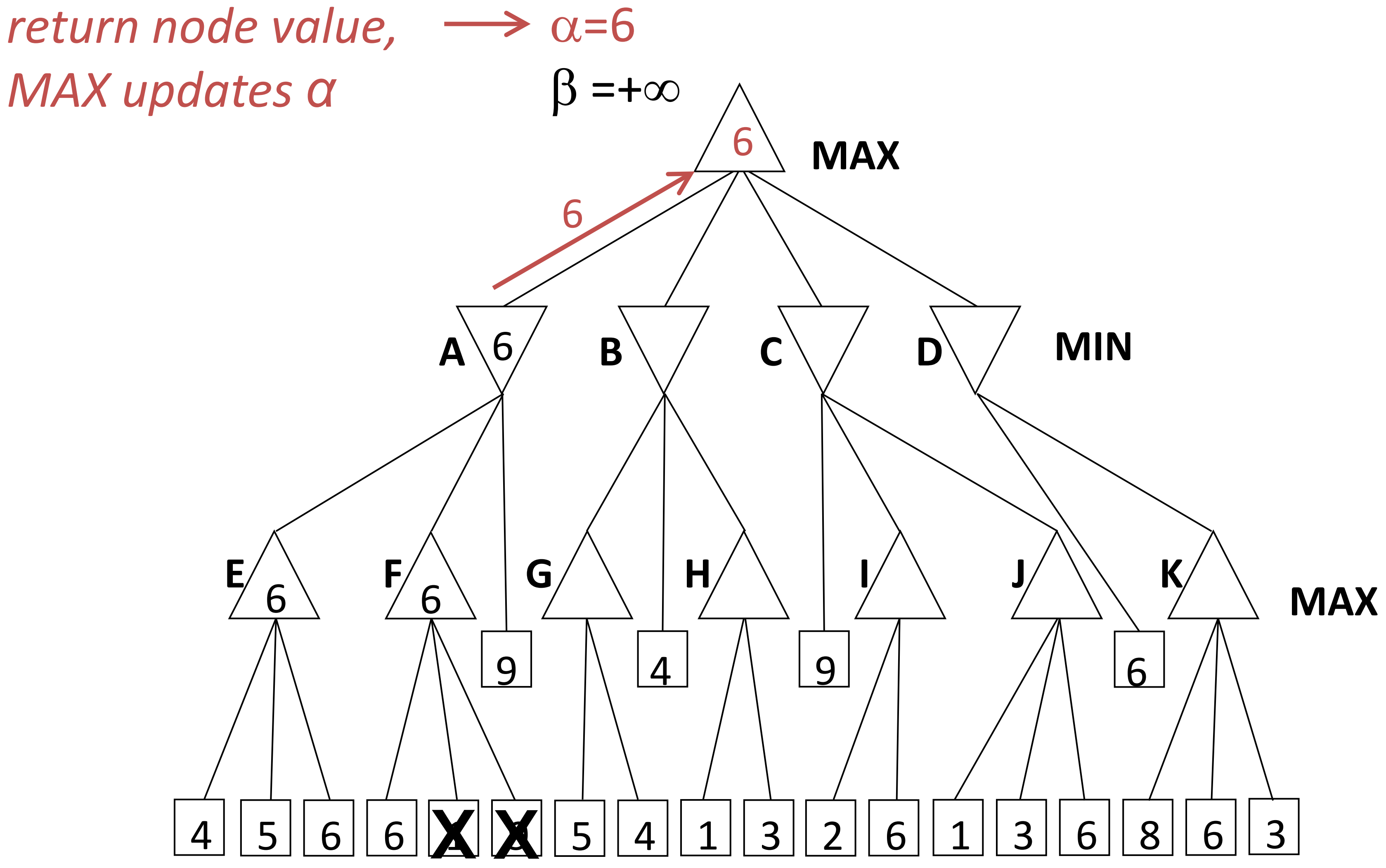
If we had continued searching at node F, we would see the 9 from its third leaf. Our returned value would be 9 instead of 6. But at A, MIN would choose E(=6) instead of F(=9). Internal values may change; root values do not.

Longer Alpha-Beta Example

see next leaf,
MIN updates β,
no change to β

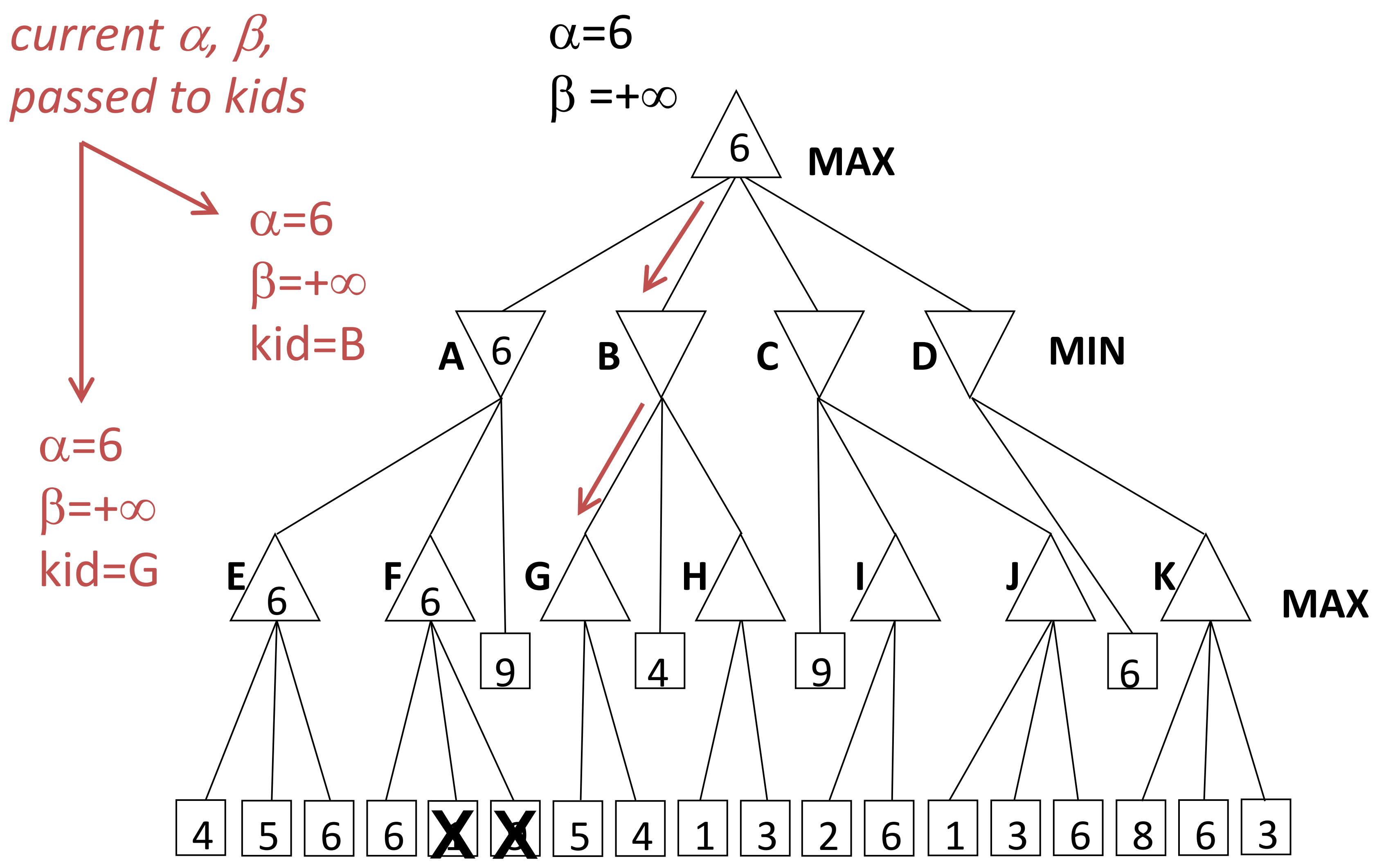


Longer Alpha-Beta Example



Longer Alpha-Beta Example

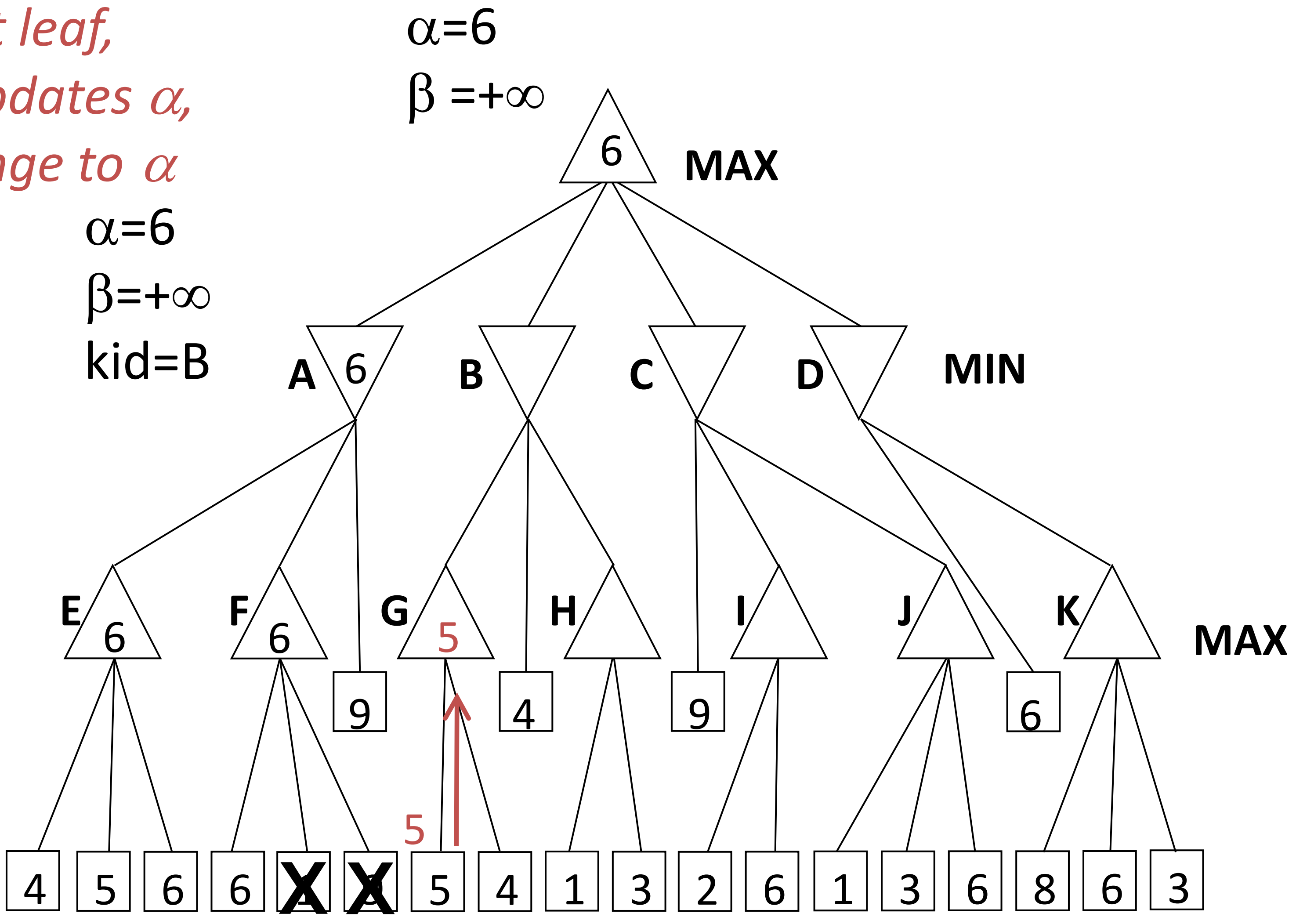
current α, β, passed to kids



Longer Alpha-Beta Example

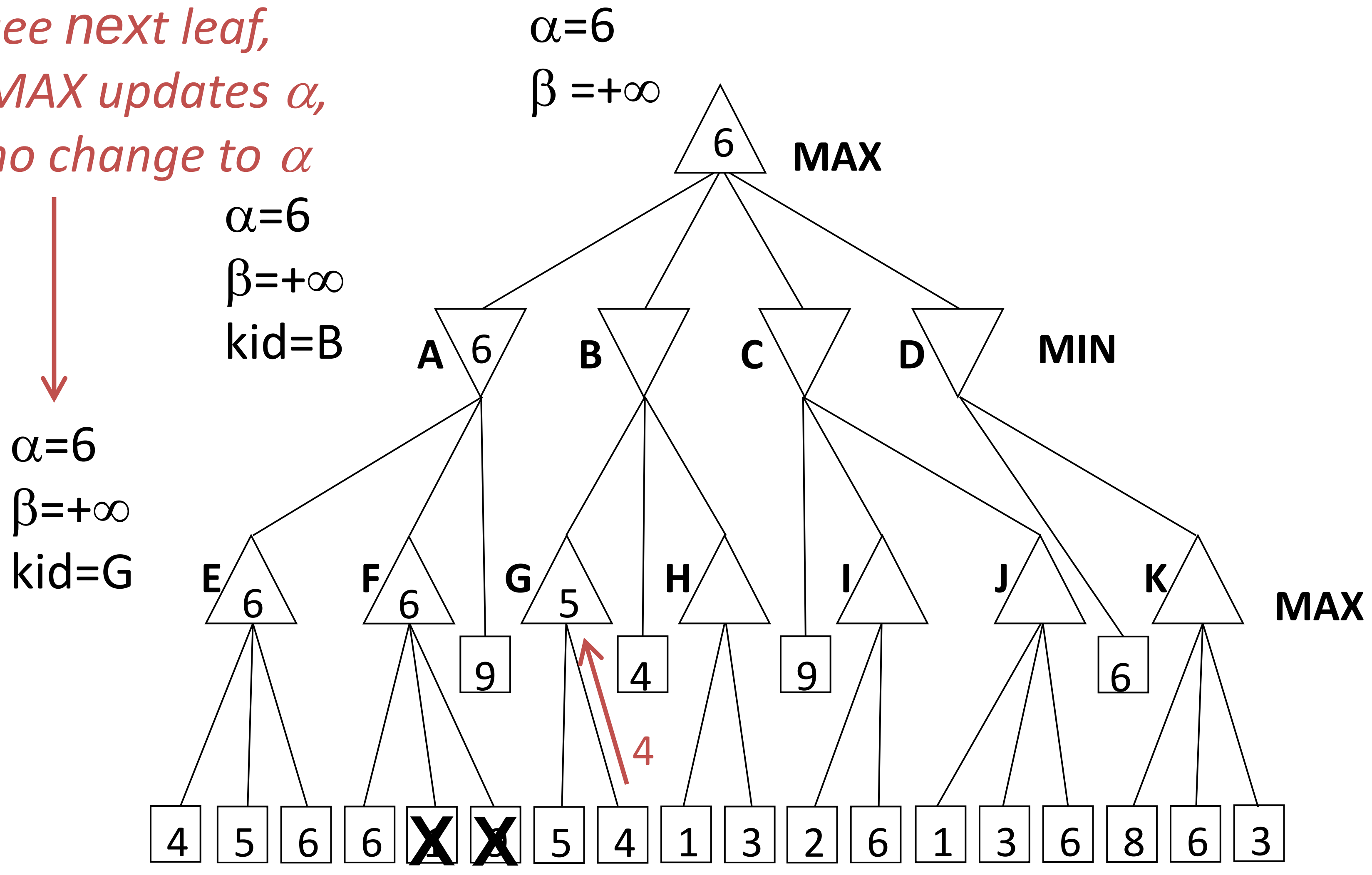
see first leaf,
MAX updates α,
no change to α

α=6
β=+∞
kid=G

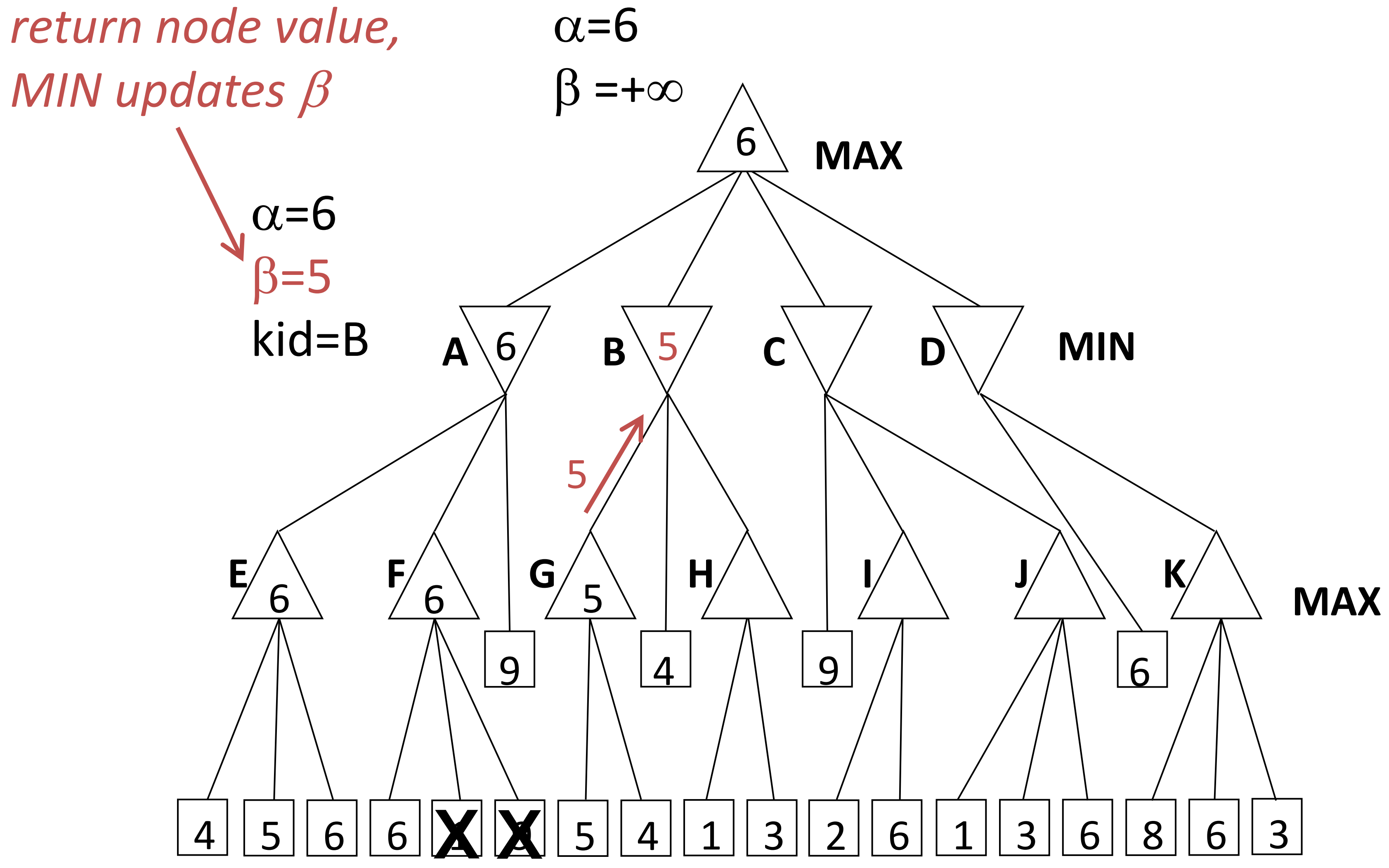


Longer Alpha-Beta Example

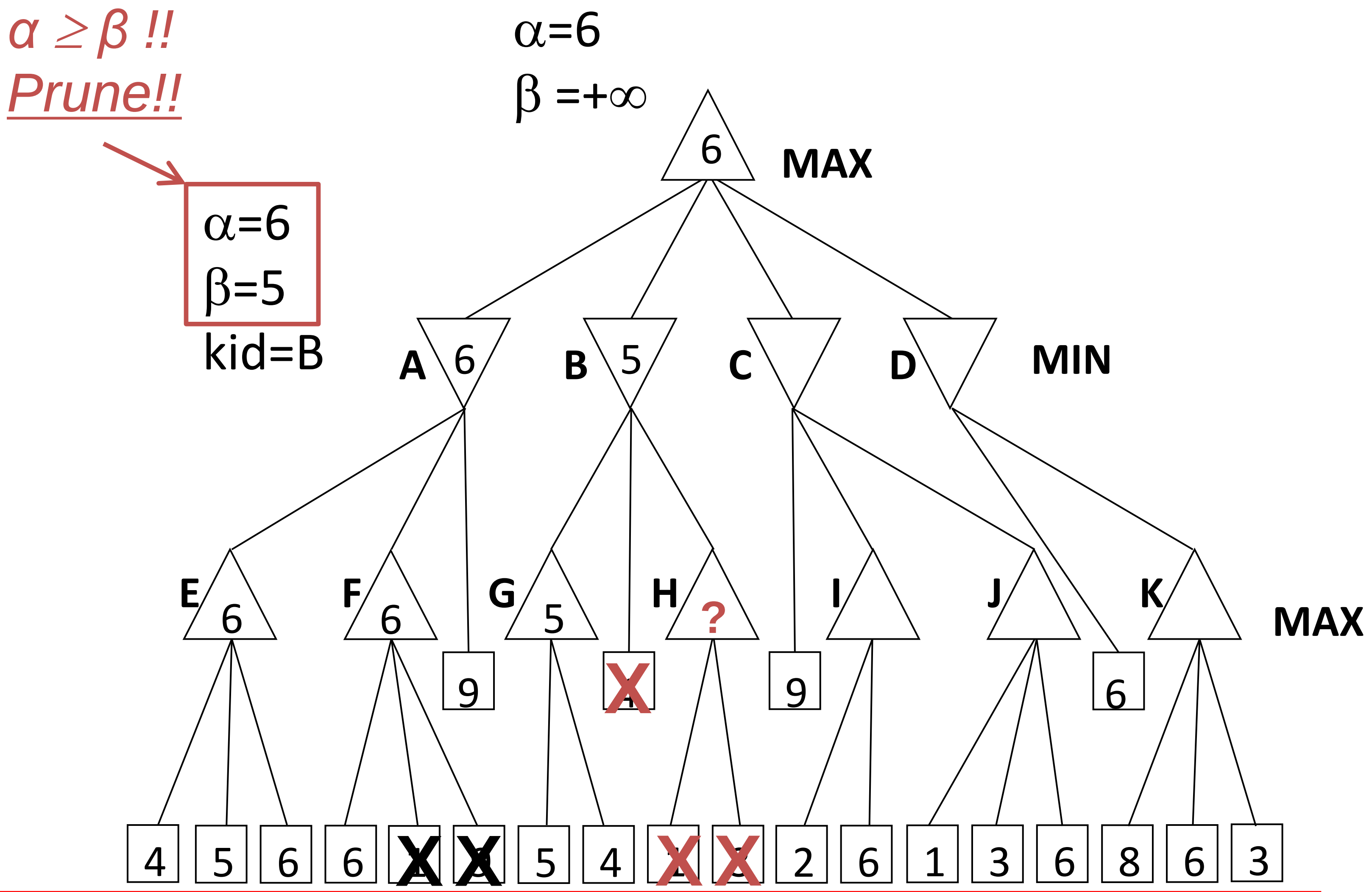
see next leaf,
MAX updates α,
no change to α



Longer Alpha-Beta Example



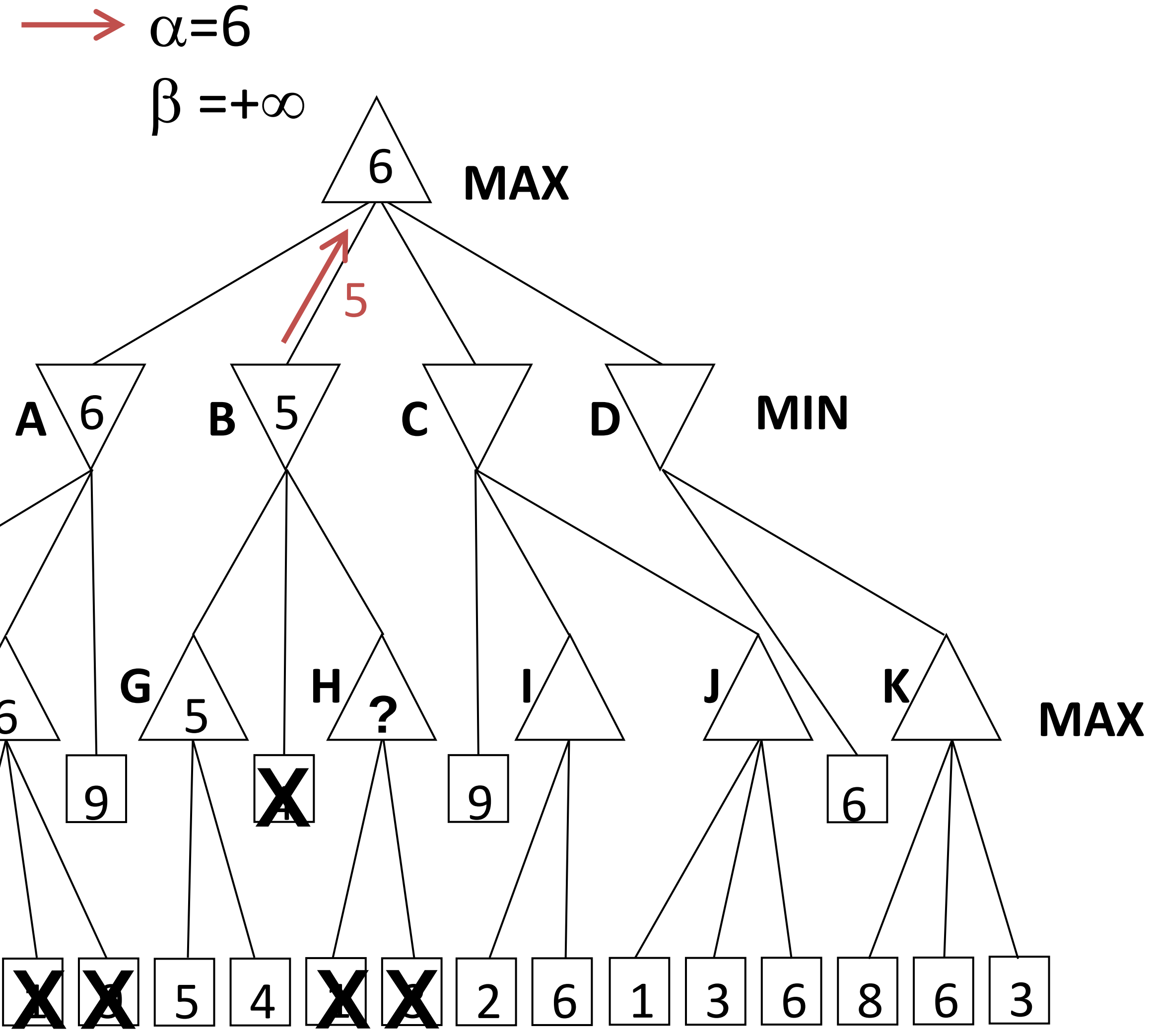
Longer Alpha-Beta Example



Note that we never find out, what is the node value of H? But we have proven it doesn't matter, so we don't care.

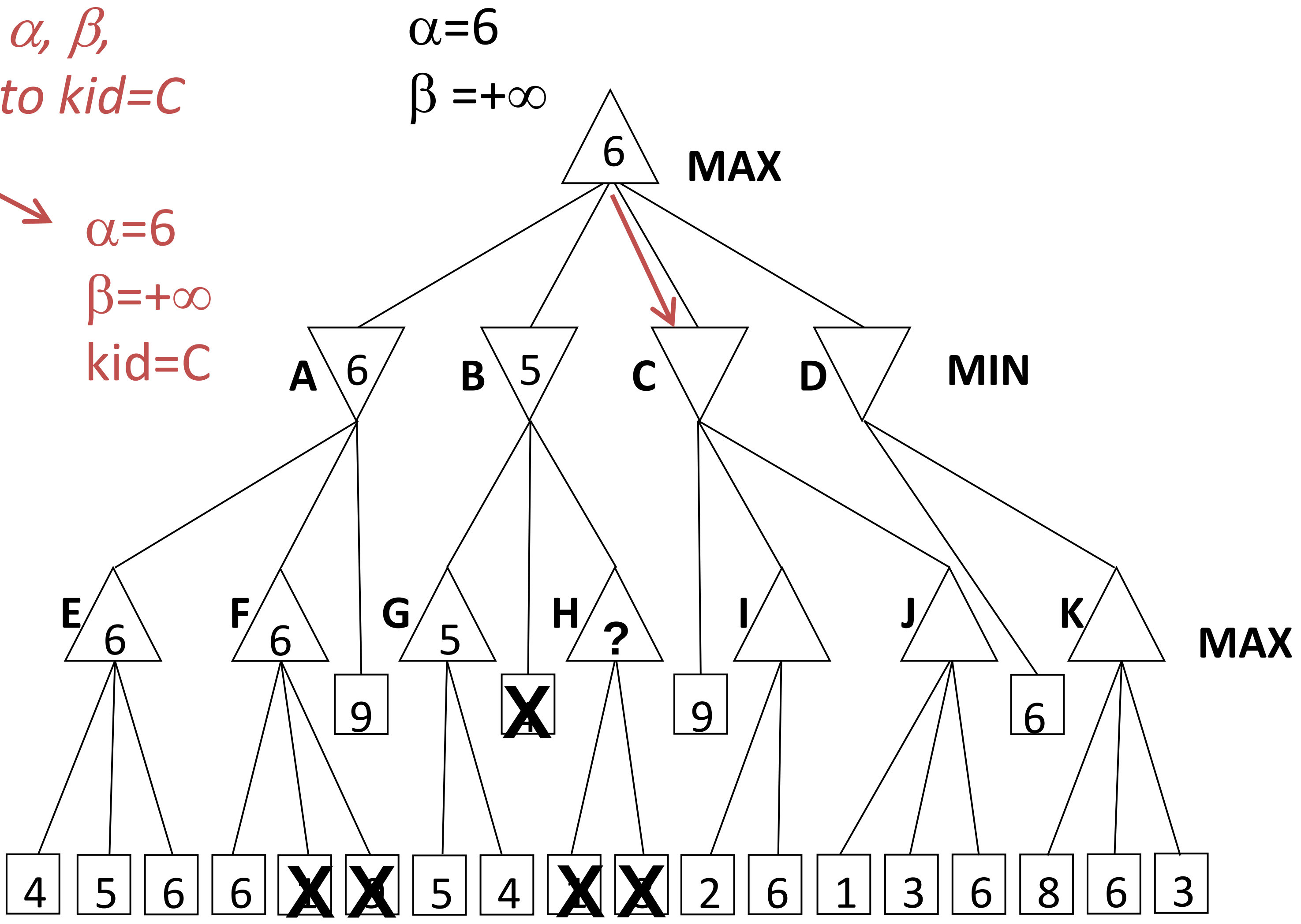
Longer Alpha-Beta Example

return node value,
MAX updates α,
no change to α



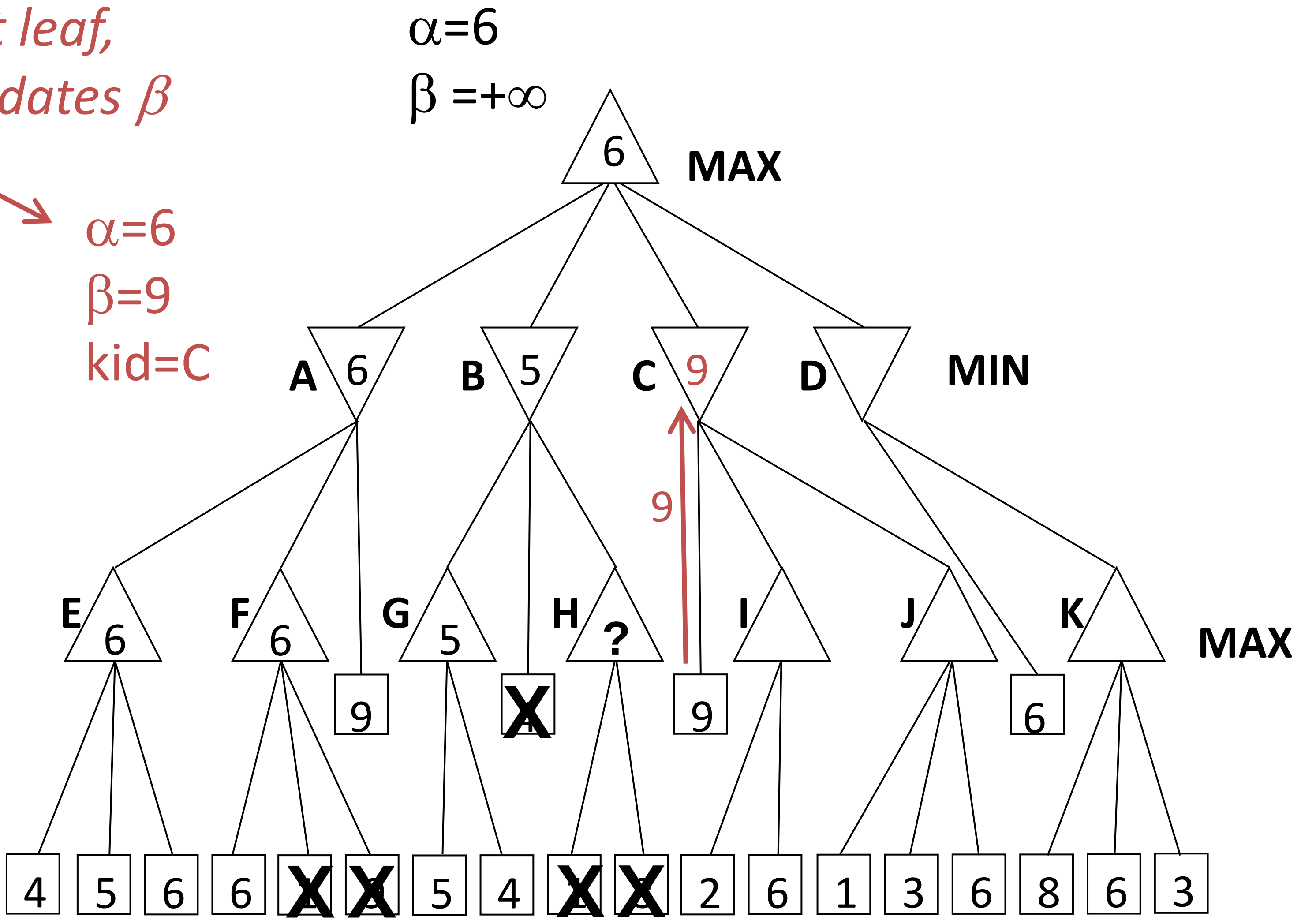
Longer Alpha-Beta Example

current α, β,
passed to kid=C



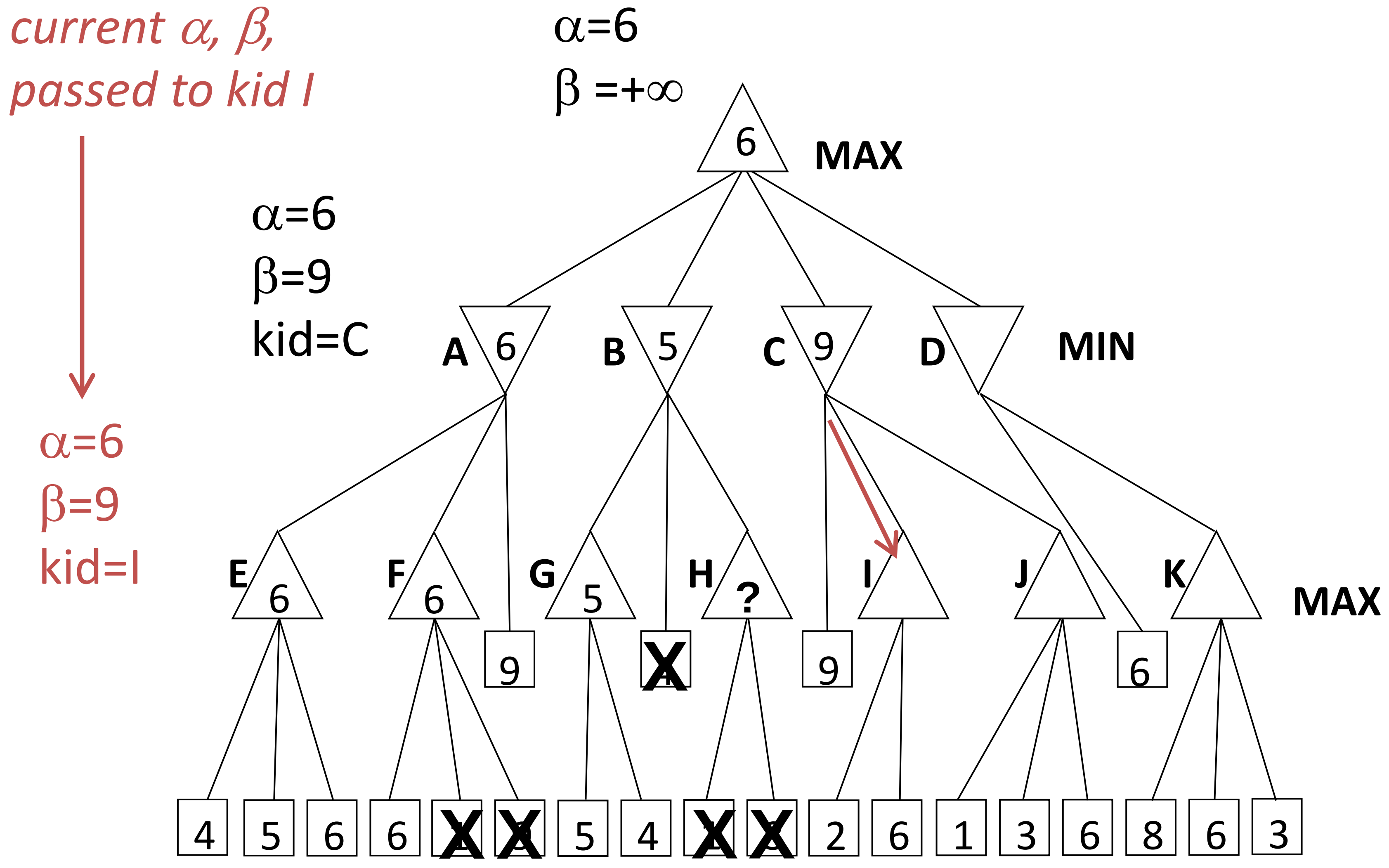
Longer Alpha-Beta Example

see first leaf,
MIN updates β



Longer Alpha-Beta Example

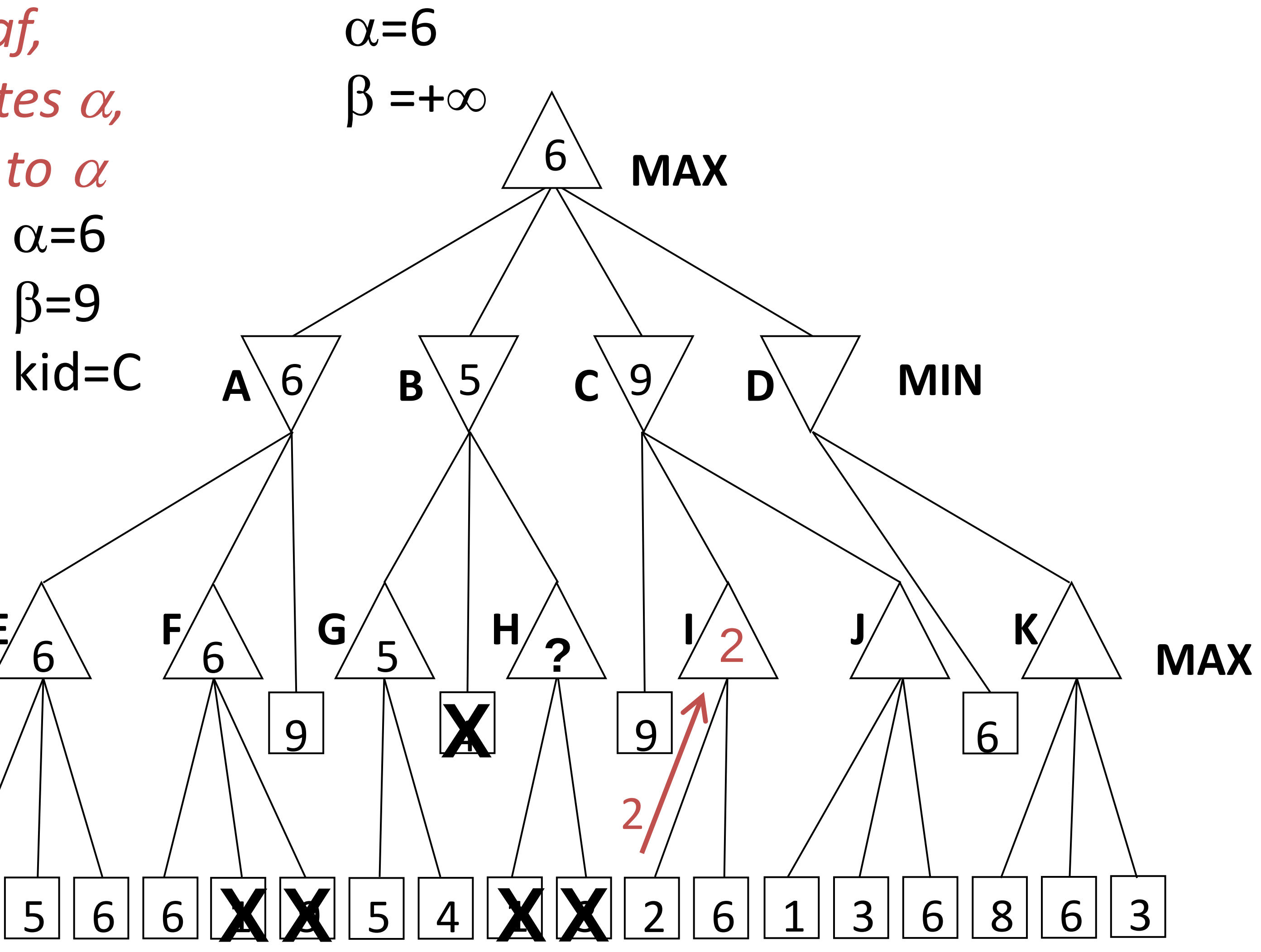
current α, β, passed to kid I



Longer Alpha-Beta Example

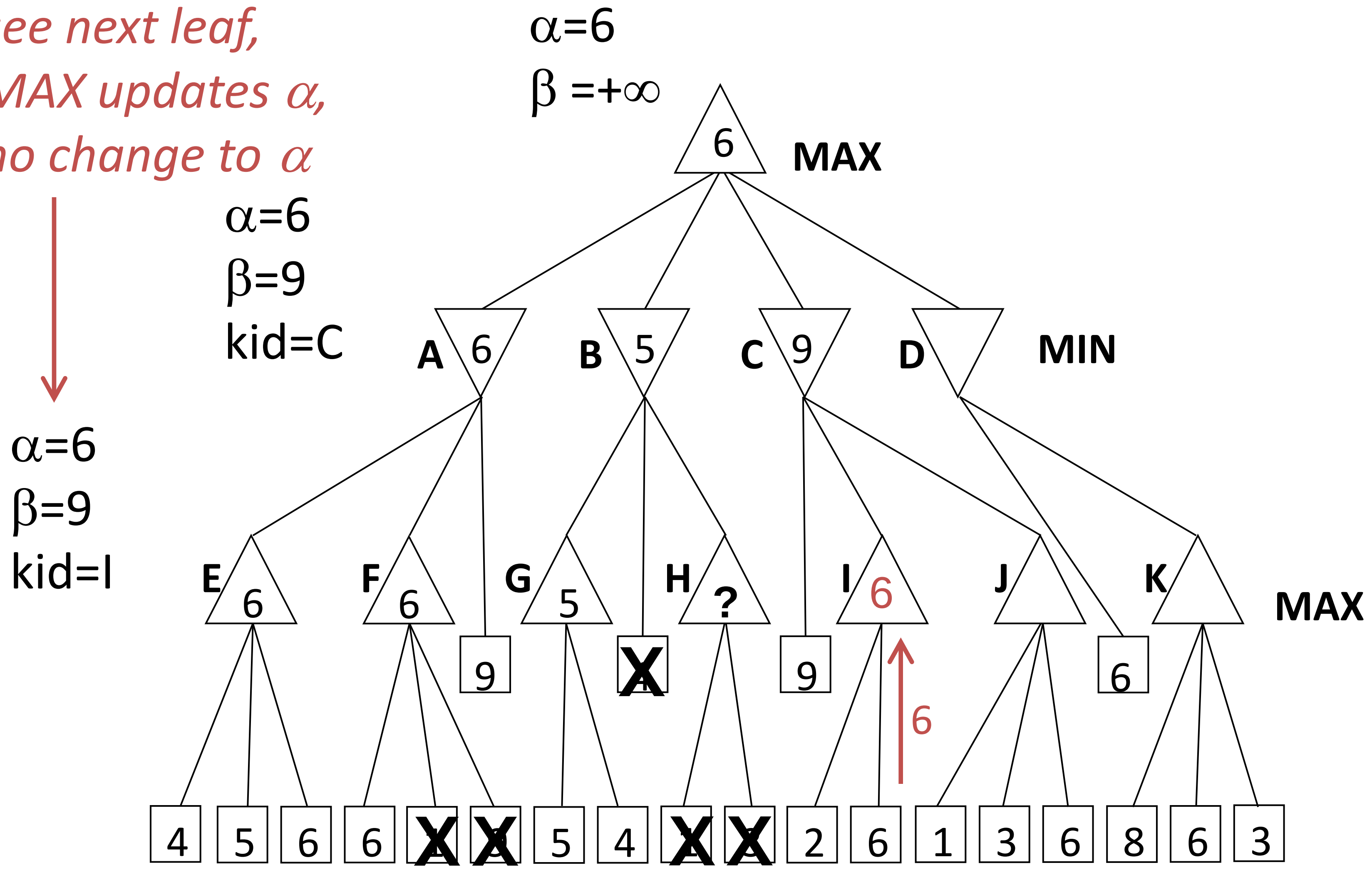
see first leaf,
MAX updates α,
no change to α

α=6
β=9
kid=I



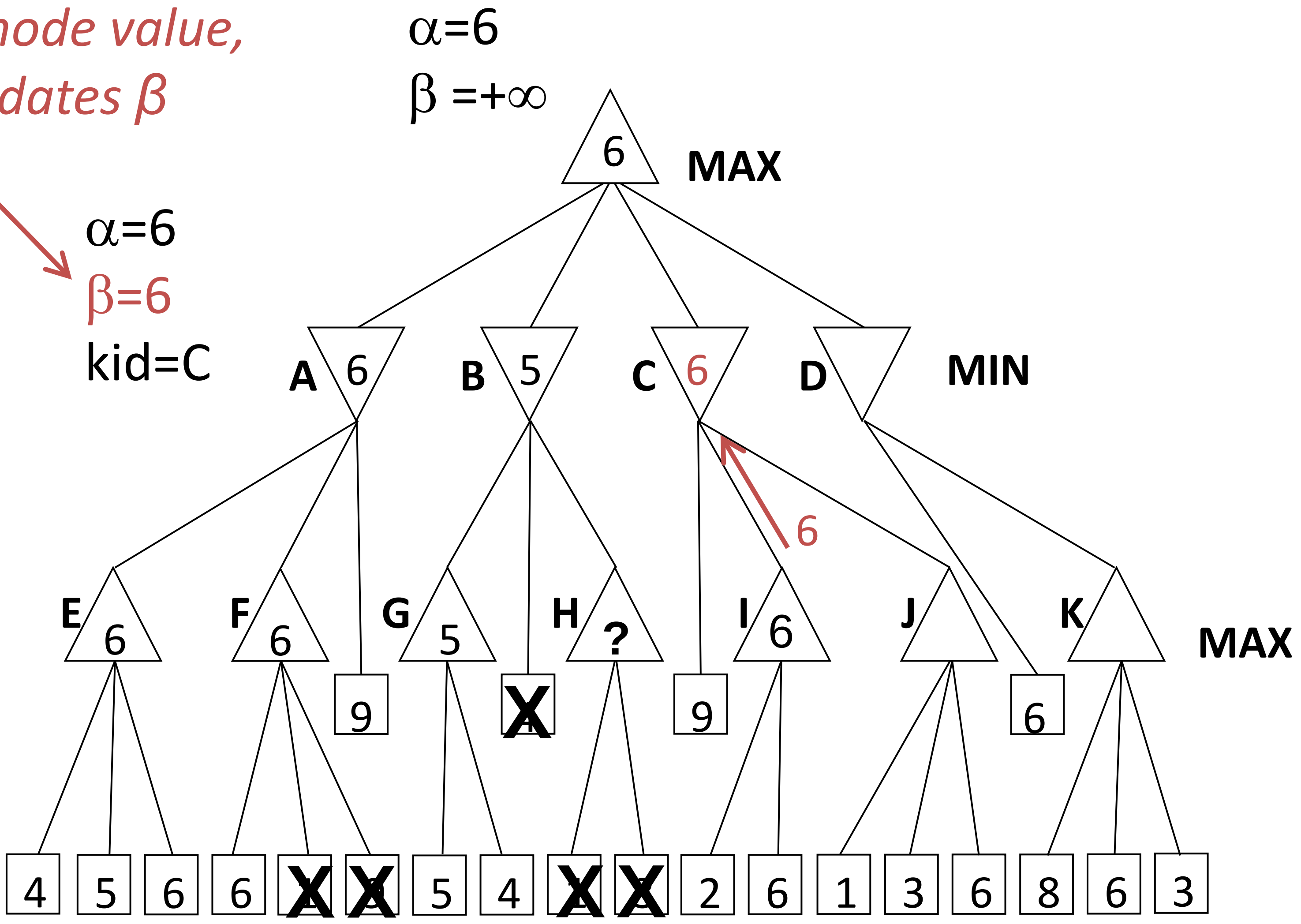
Longer Alpha-Beta Example

see next leaf,
MAX updates α,
no change to α

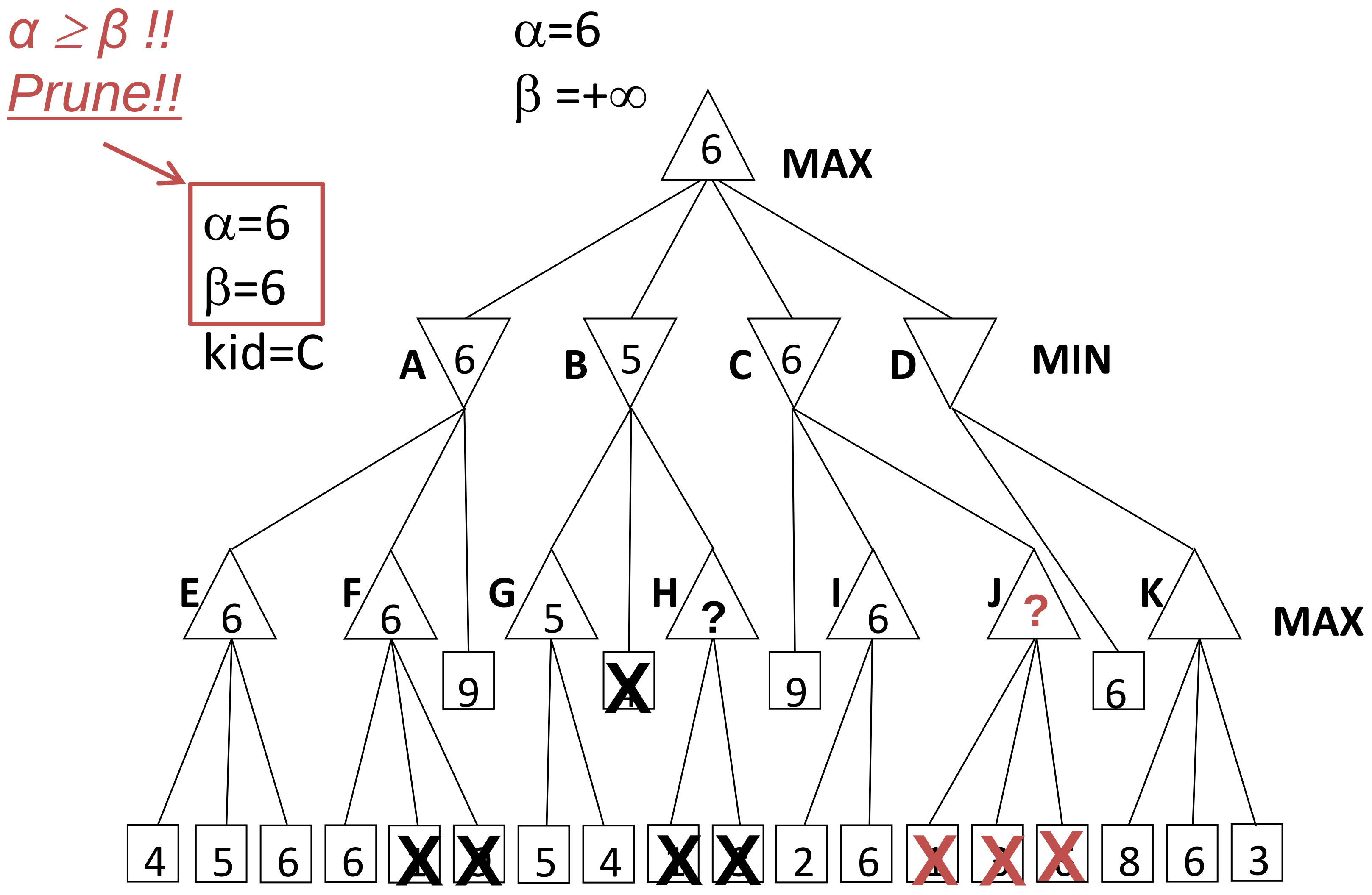


Longer Alpha-Beta Example

return node value, MIN updates β

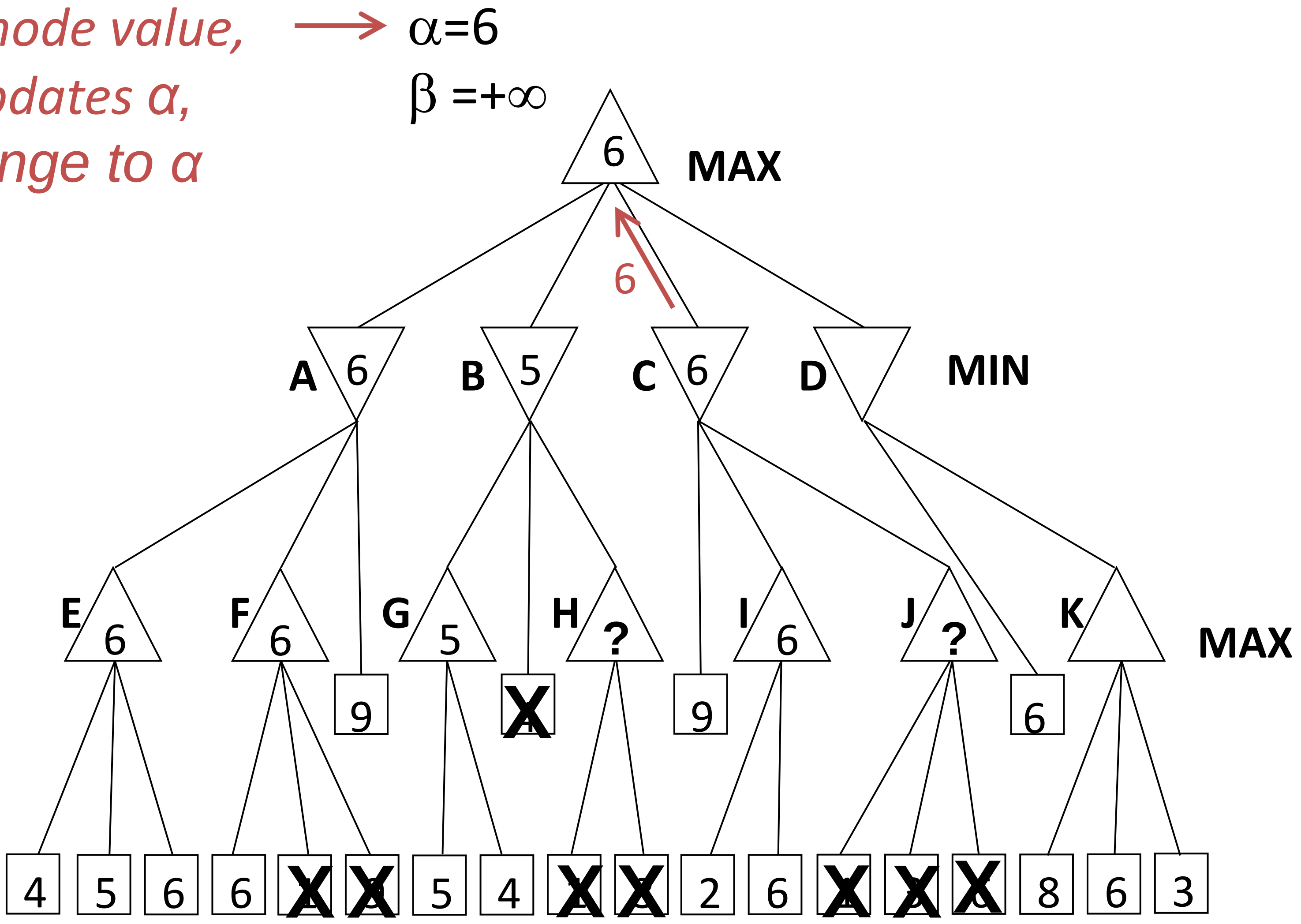


Longer Alpha-Beta Example



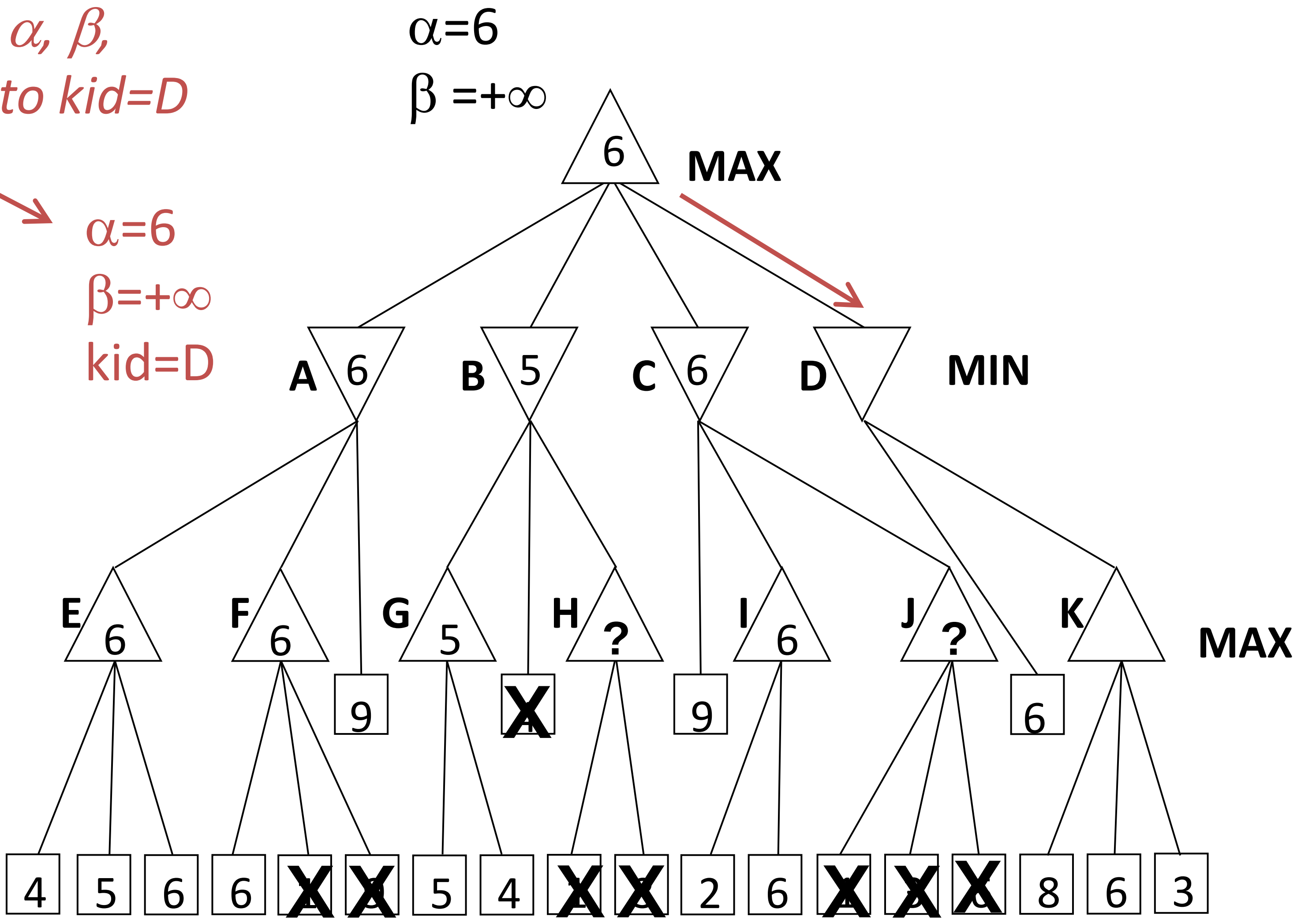
Longer Alpha-Beta Example

return node value,
MAX updates α,
no change to α

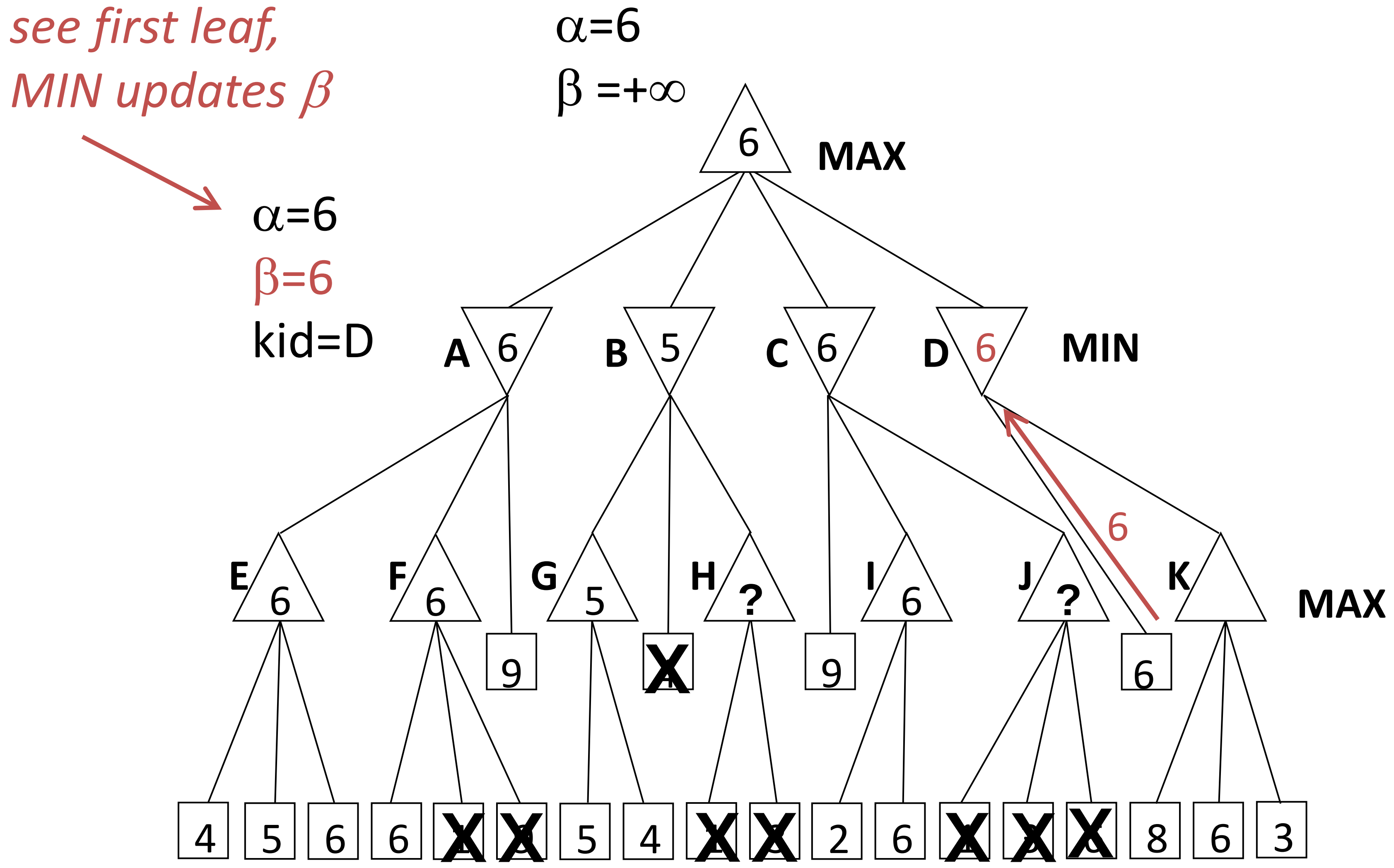


Longer Alpha-Beta Example

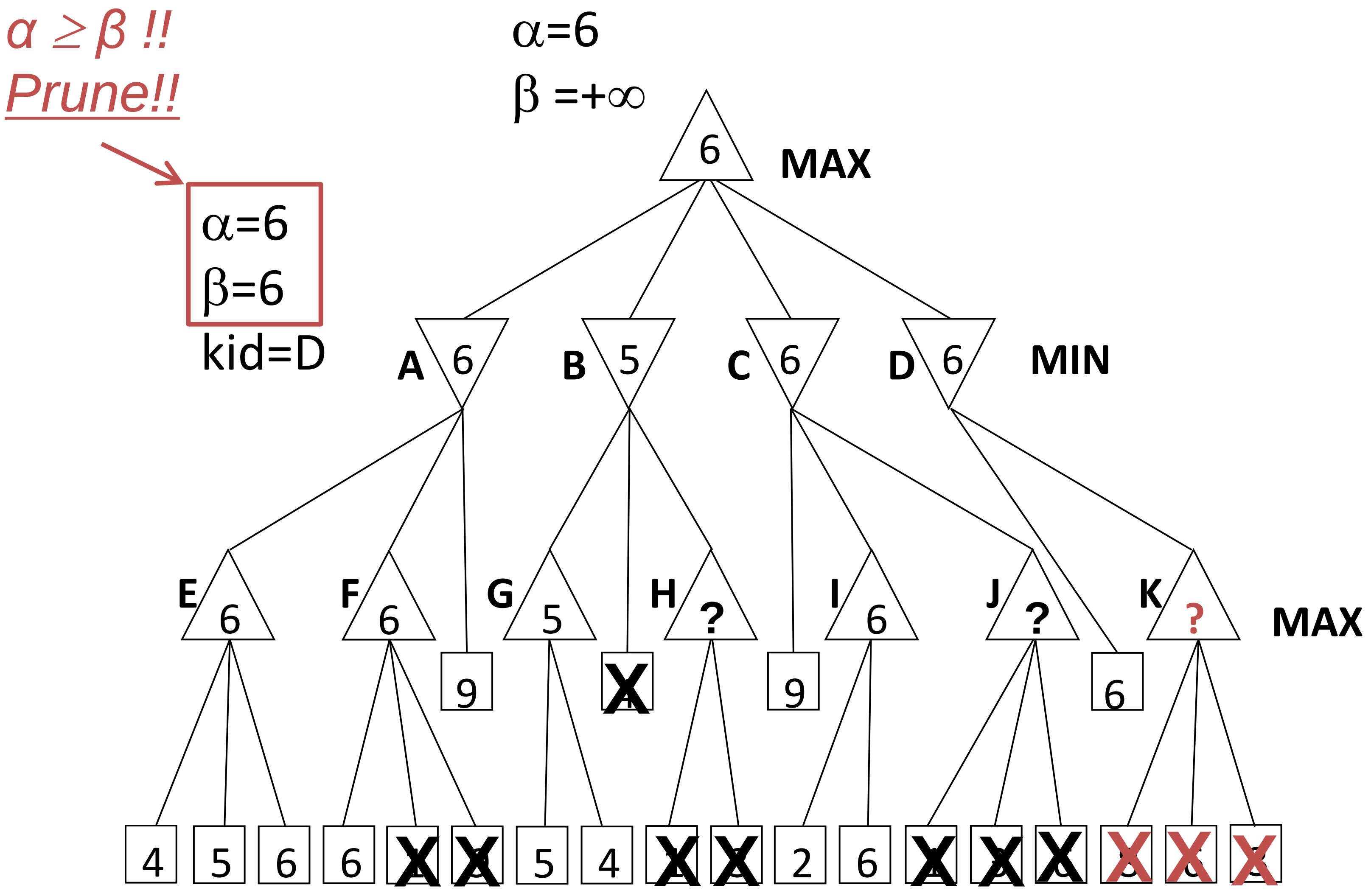
current α, β,
passed to kid=D



Longer Alpha-Beta Example



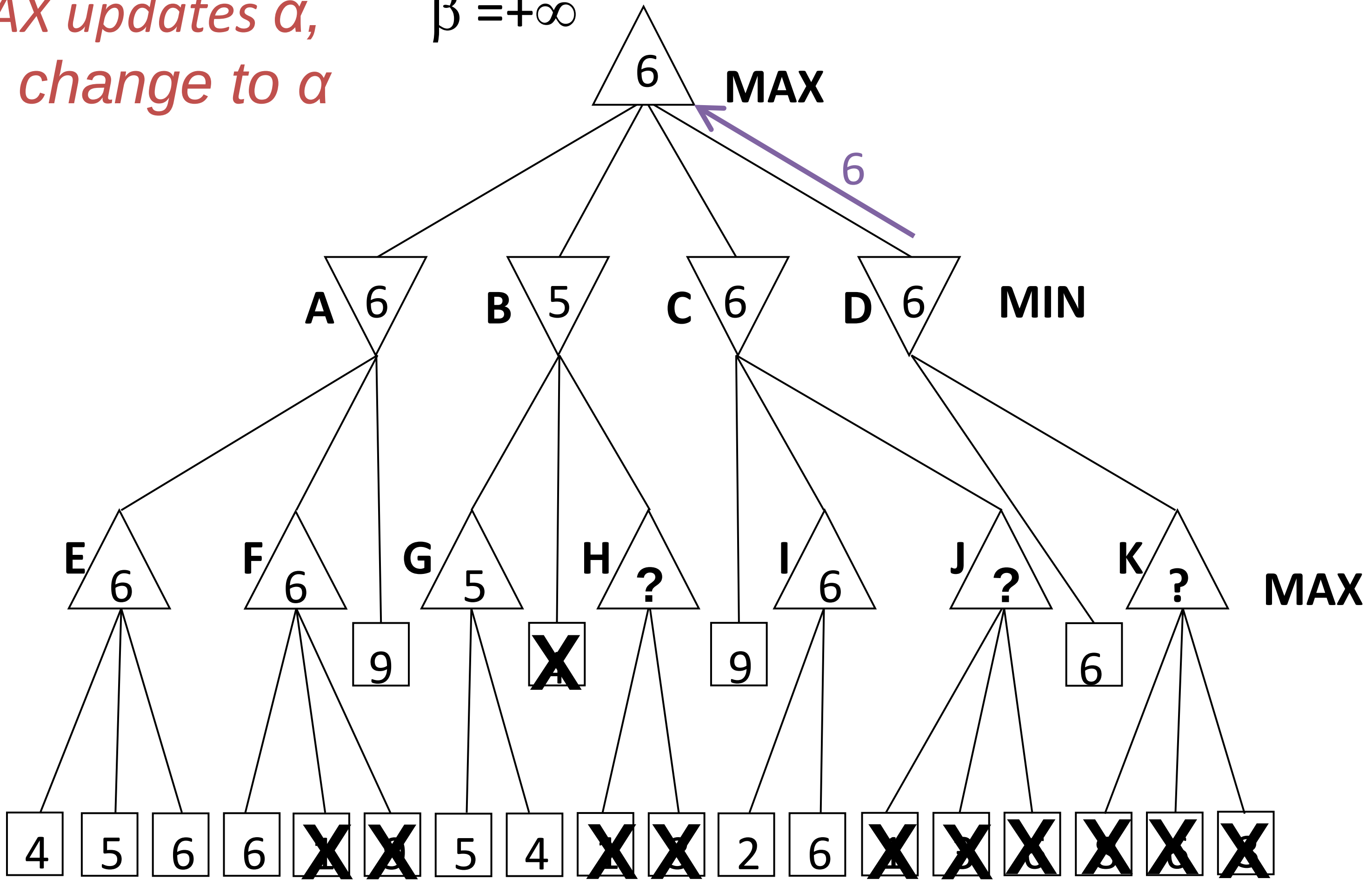
Longer Alpha-Beta Example



Alpha-Beta Example #2

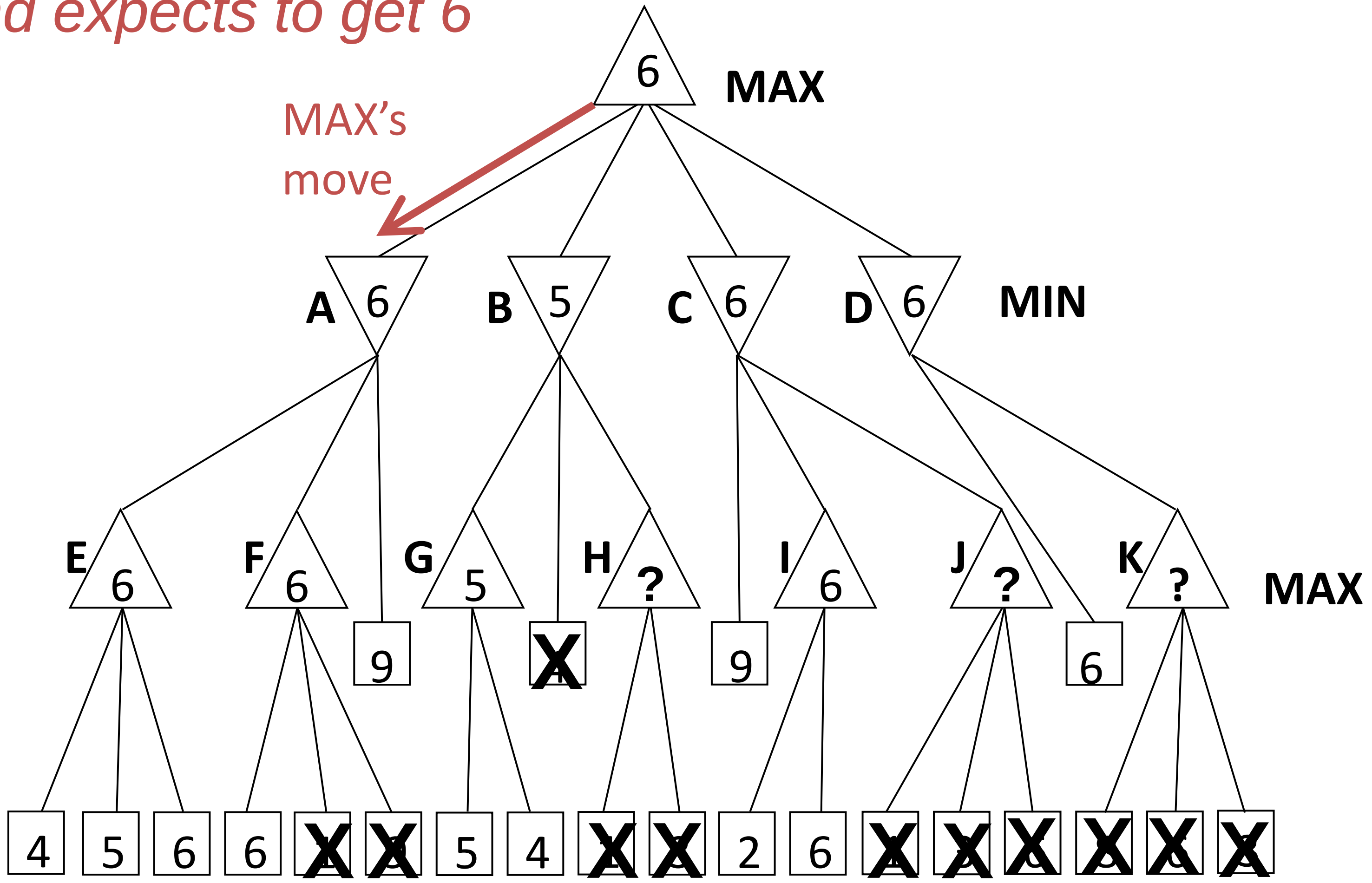
return node value,
MAX updates α,
no change to α

α=6
β =+∞



Alpha-Beta Example #2

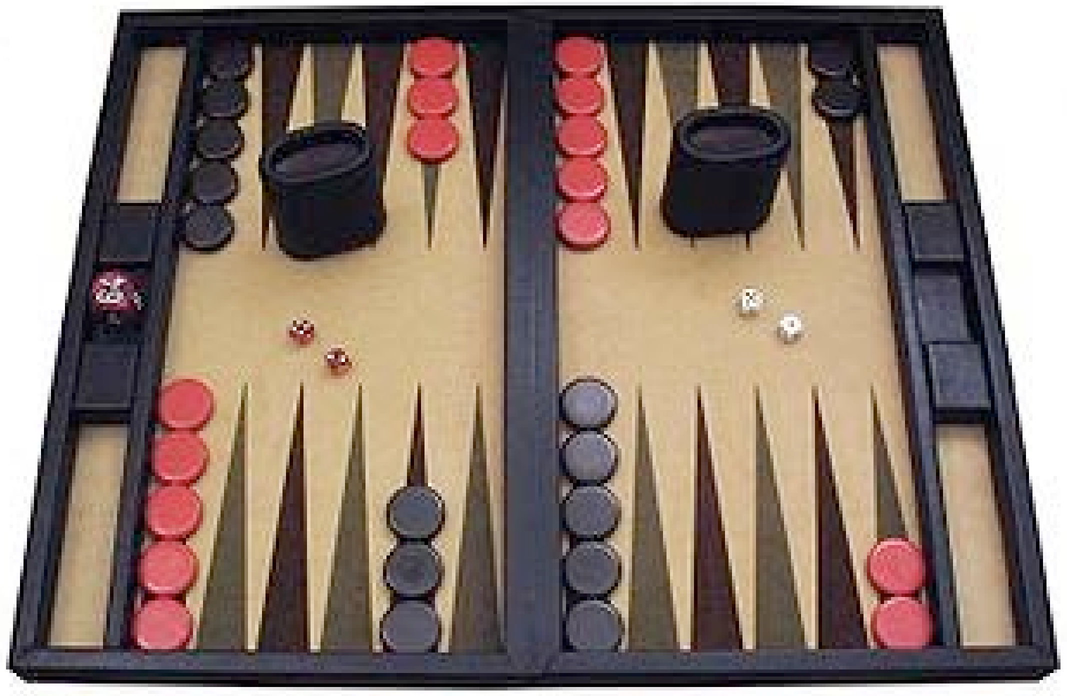
MAX moves to A,
and expects to get 6



Although we may have changed some internal branch node return values, the final root action and expected outcome are identical to if we had not done alpha-beta pruning. Internal values may change; root values do not.

Nondeterministic games

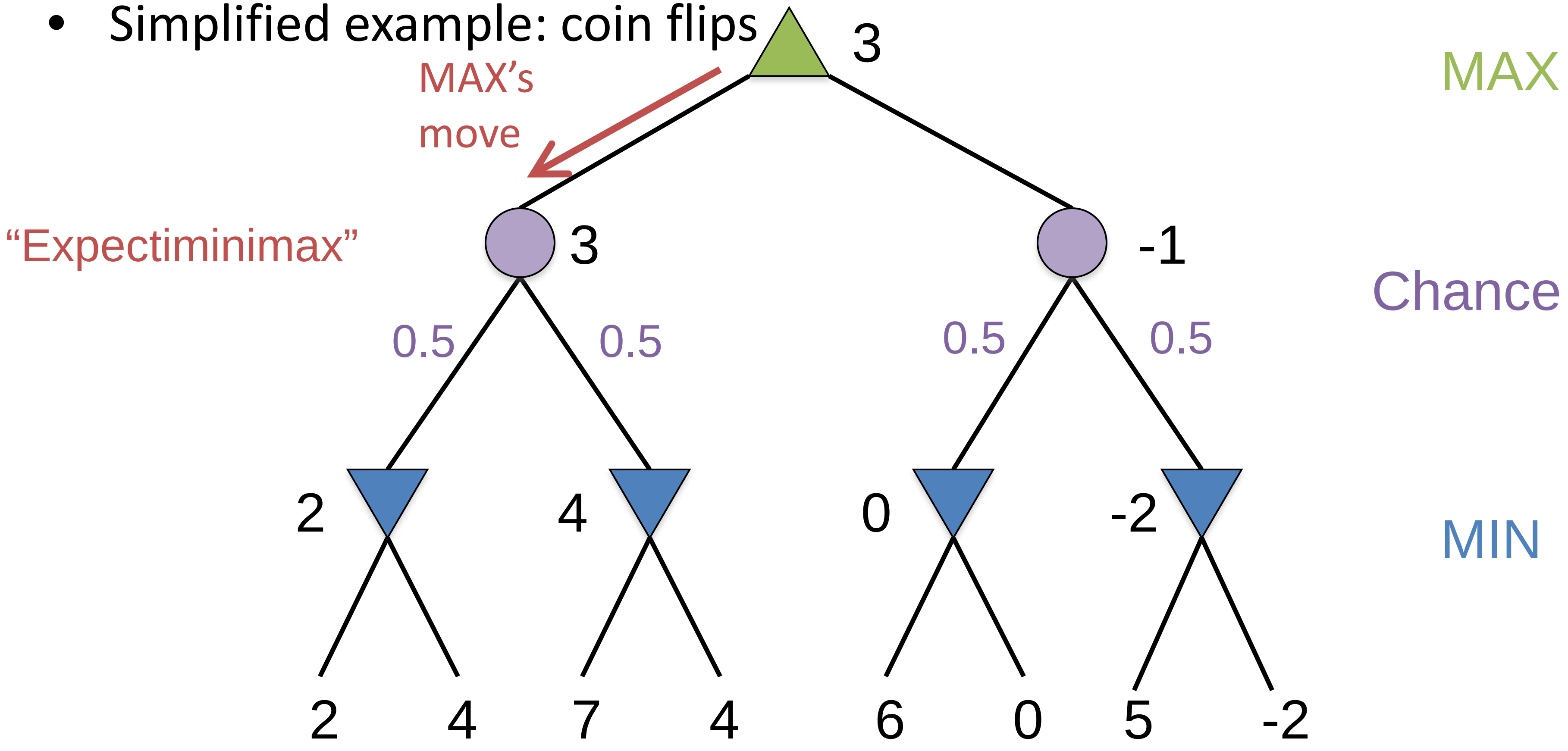
- Ex: Backgammon
  - Roll dice to determine how far to move (random)
  - Player selects which checkers to move (strategy)



https://commons.wikimedia.org/wiki/File:Backgammon_lg.jpg

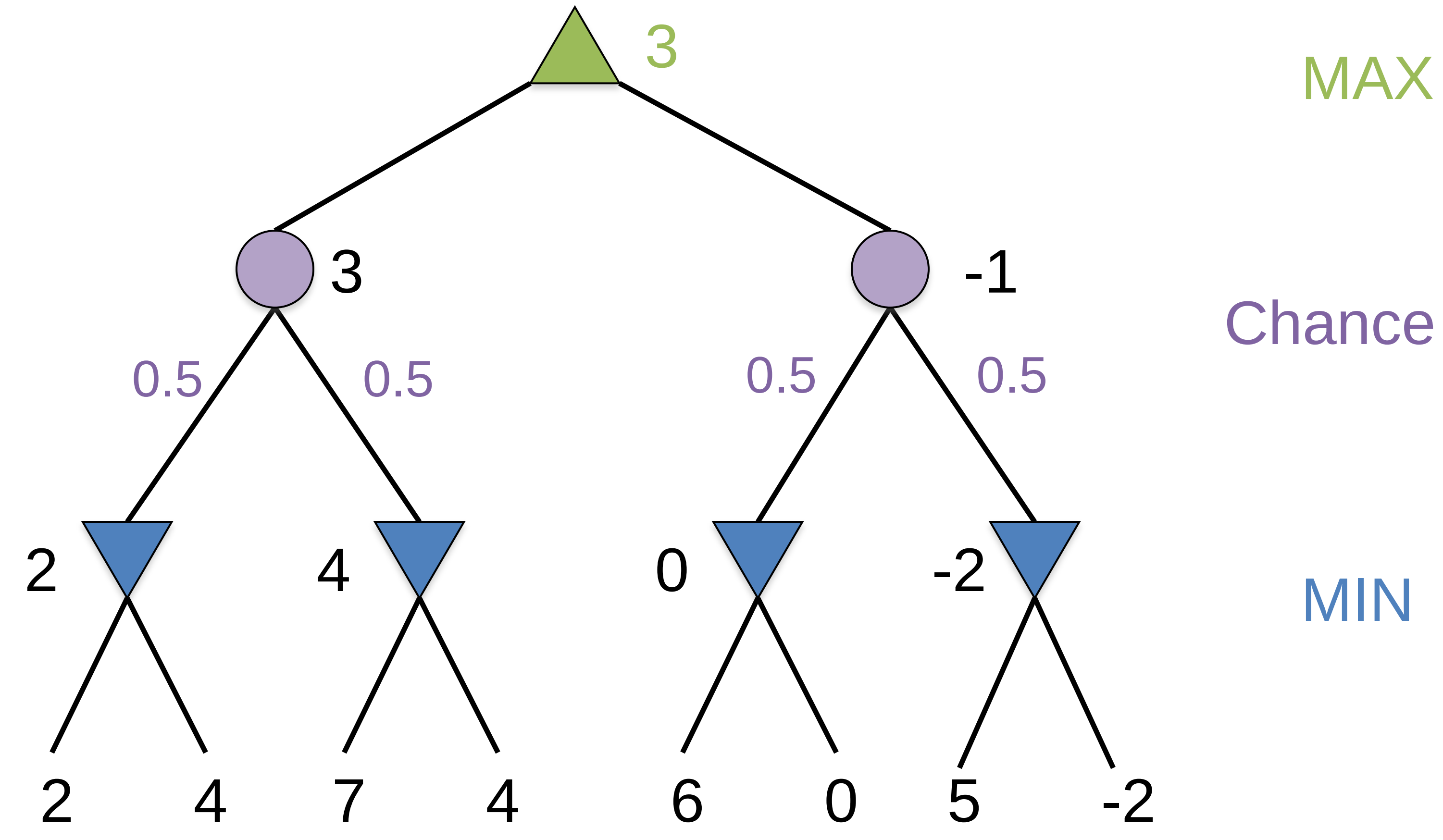
Nondeterministic games

- Chance (random effects) due to dice, card shuffle, …
- Chance nodes: expectation (weighted average) of successors
- Simplified example: coin flips



Pruning in nondeterministic games

- Can still apply a form of alpha-beta pruning



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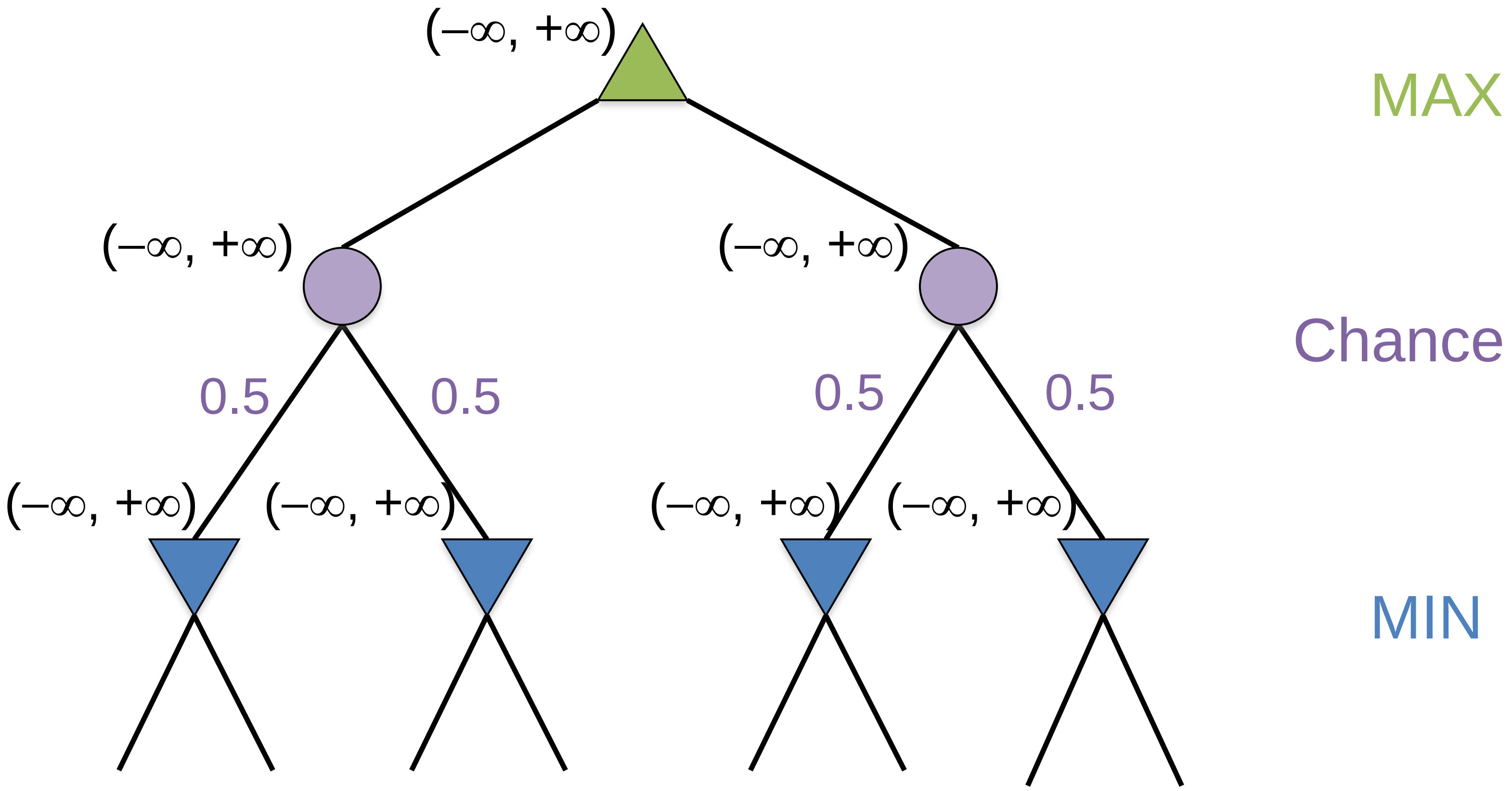

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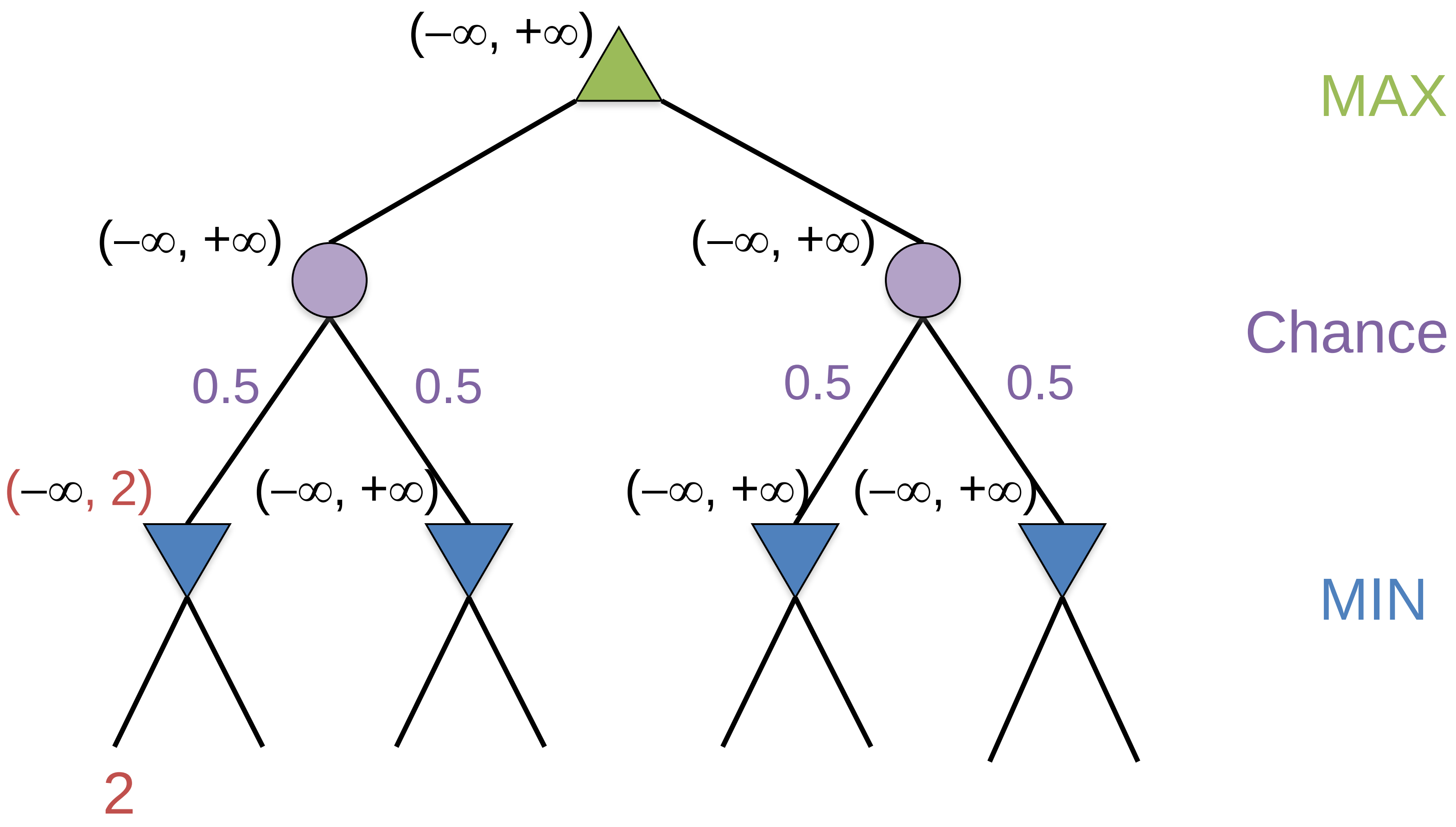

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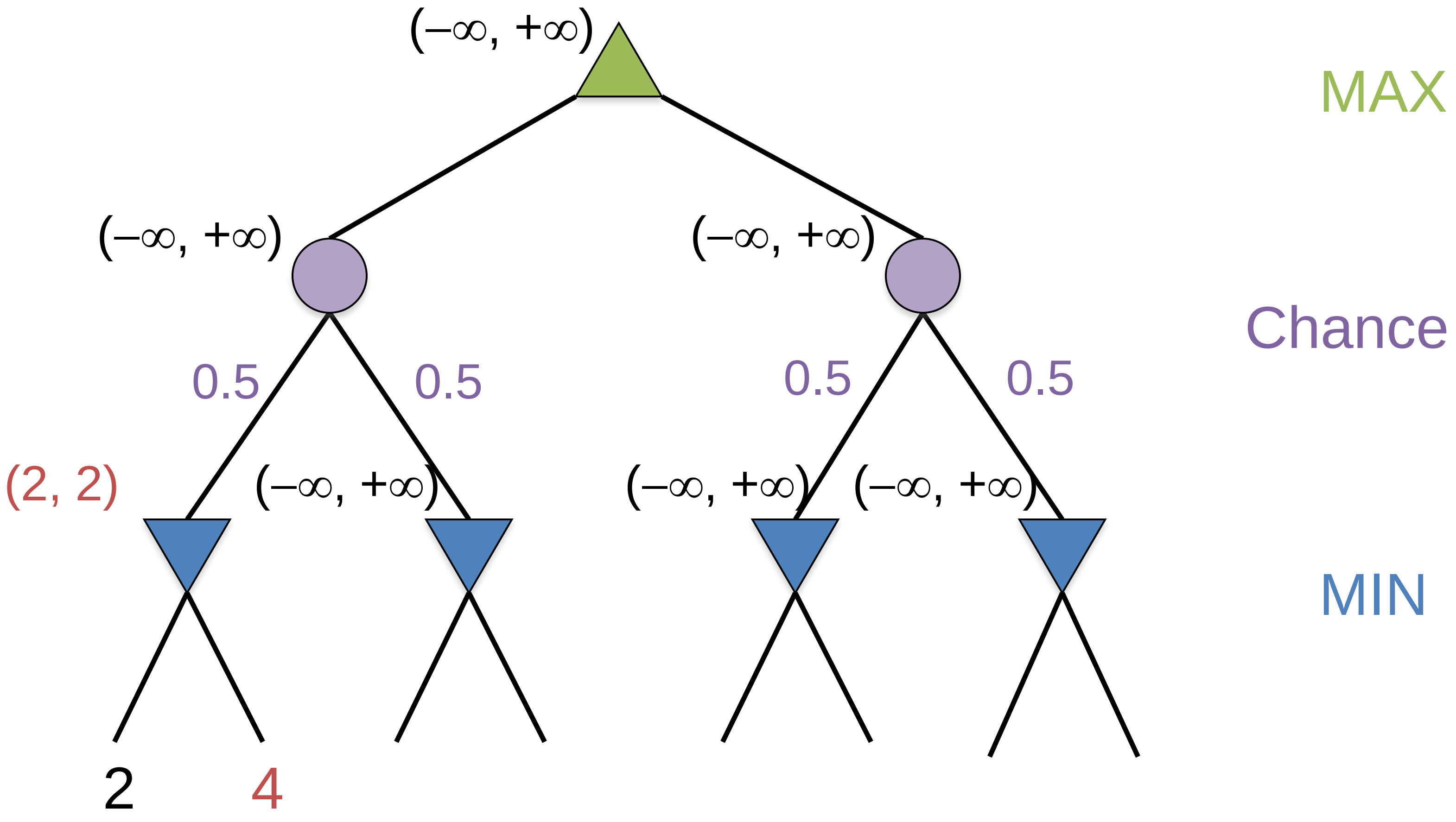

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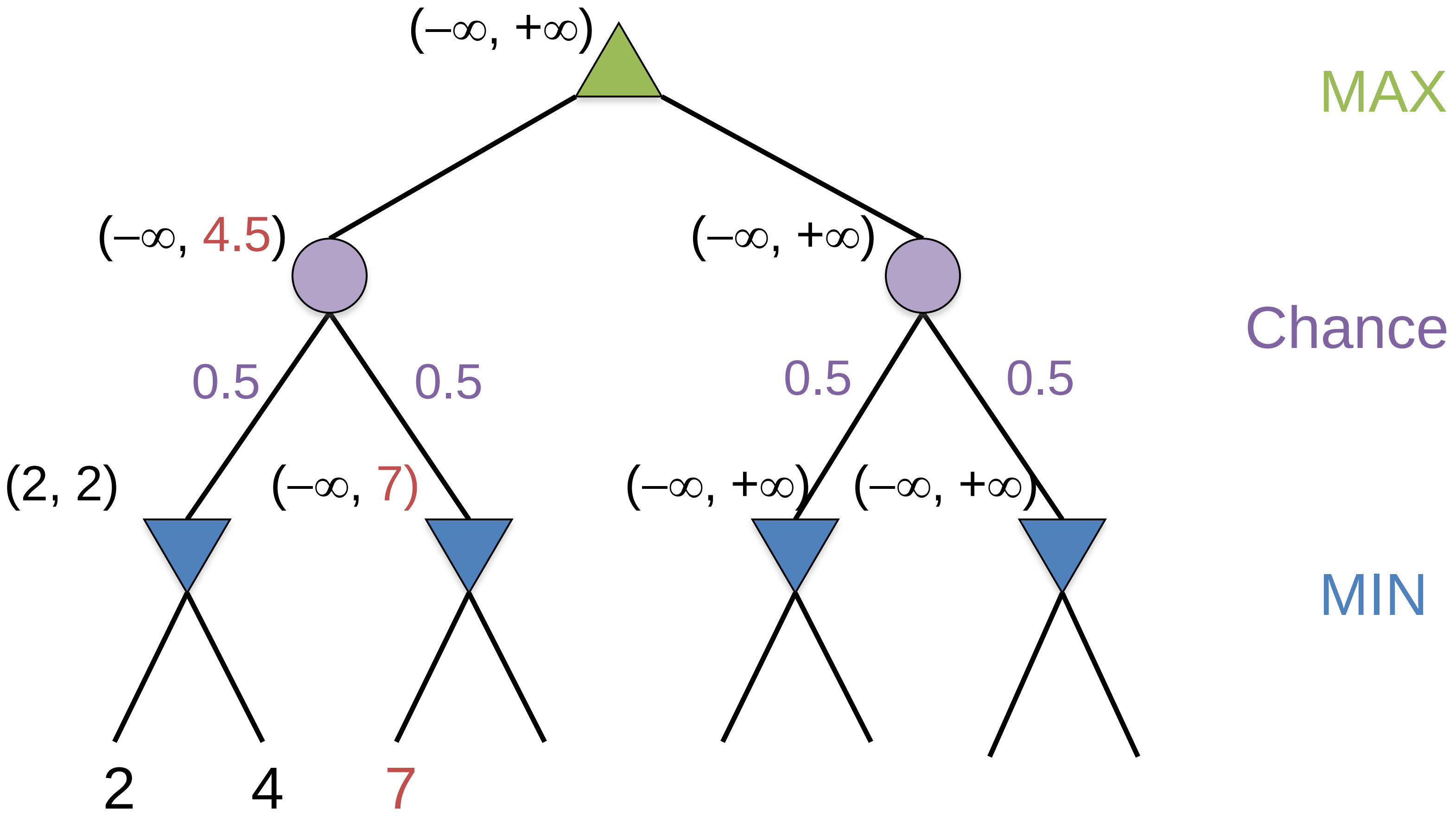

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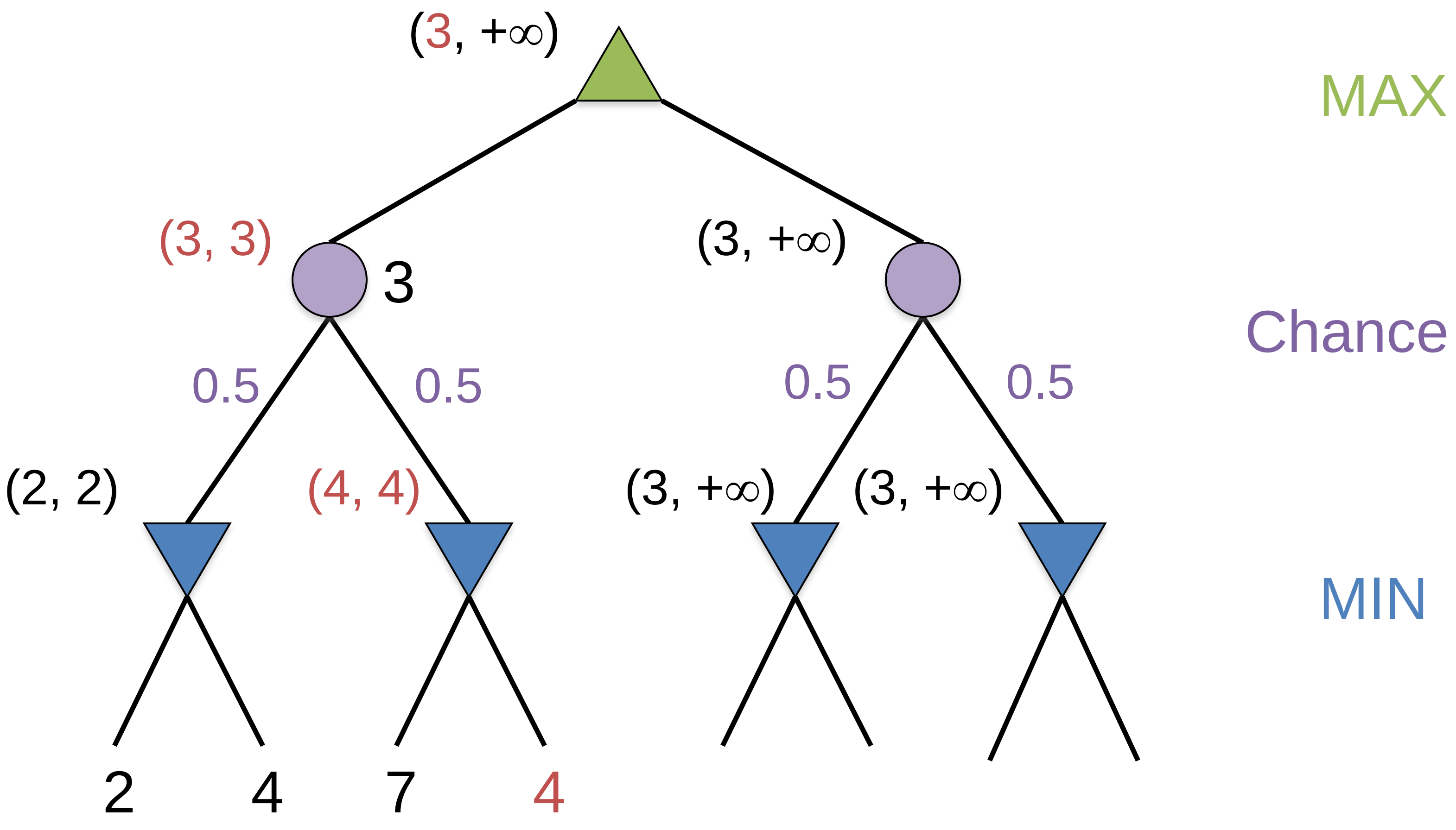

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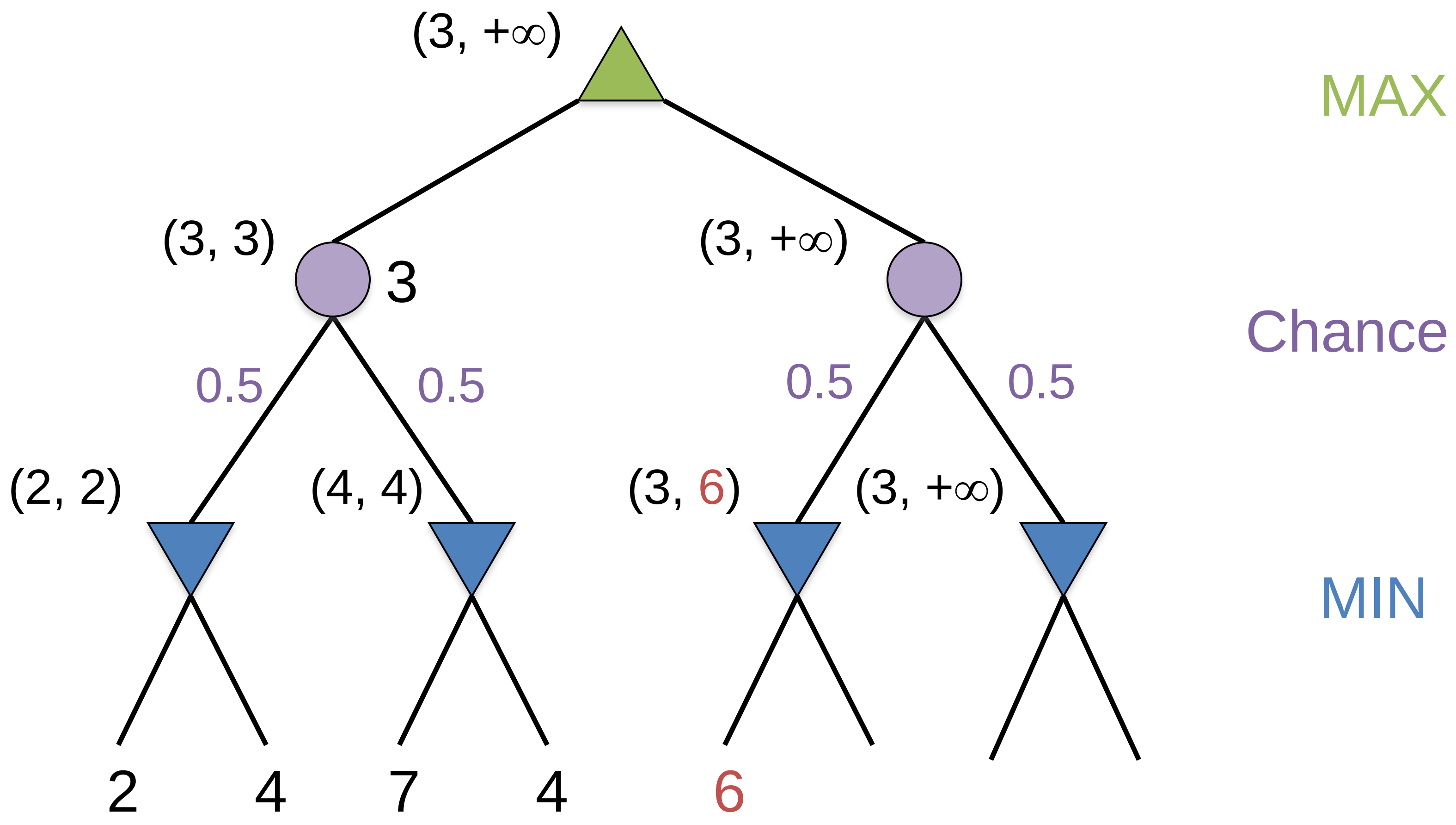

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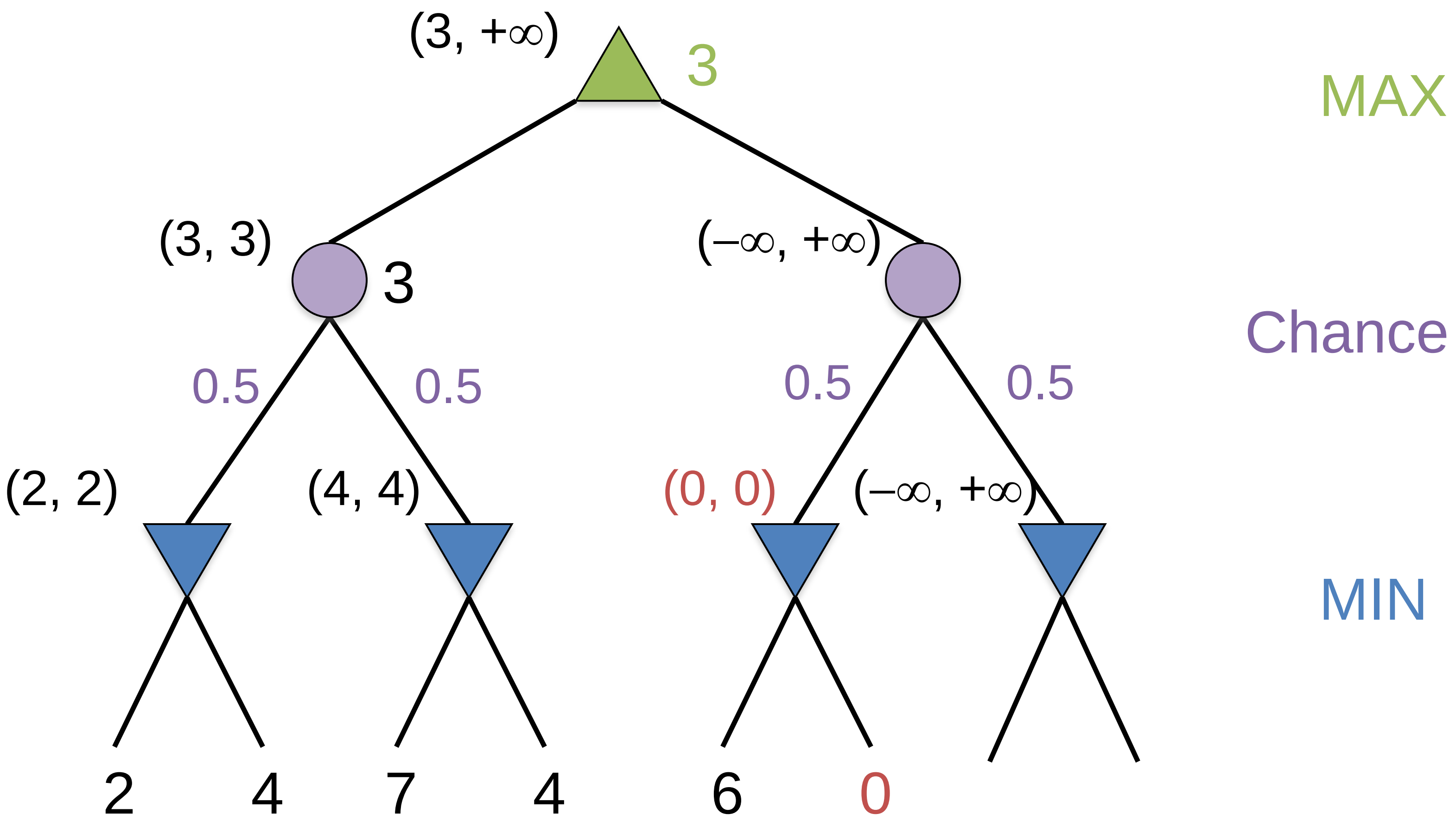

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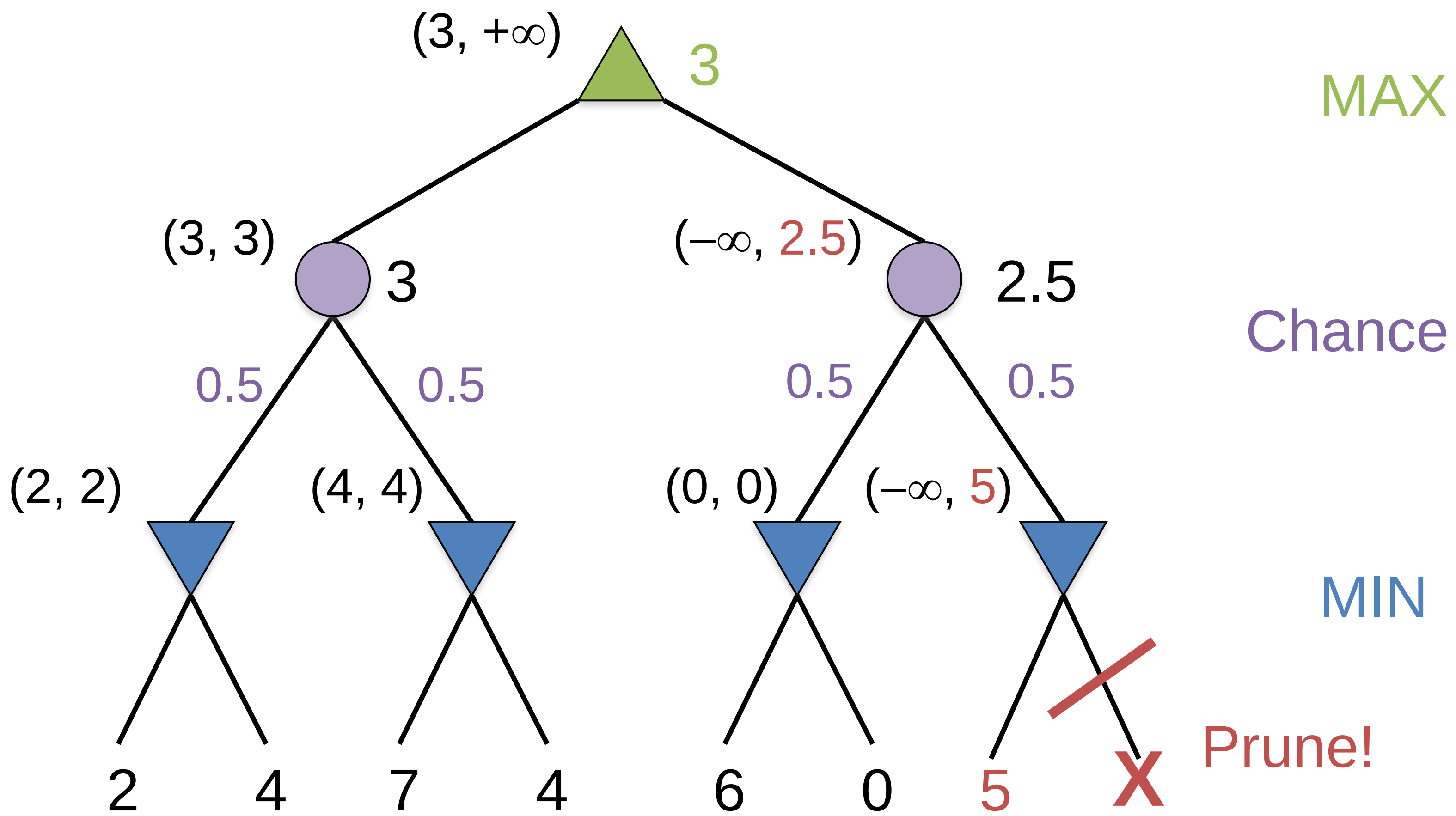
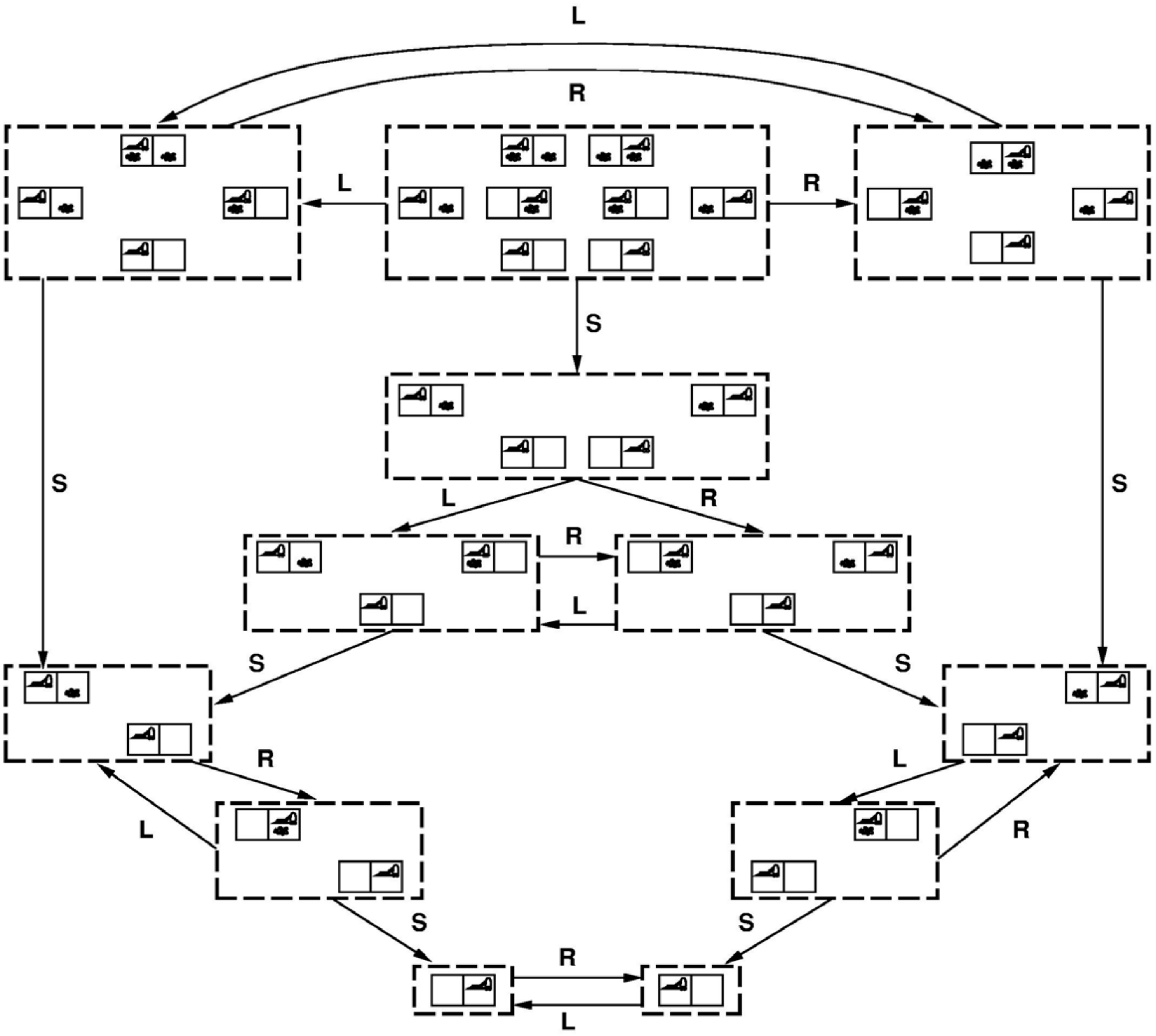


Partially observable games

- R&N Chapter 5.6 – “The fog of war”
- Background: R&N, Chapter 4.3-4
  - Searching with Nondeterministic Actions/Partial Observations

- Search through Belief States (see Fig. 4.14)
  - Agent’s current belief about which states it might be in, given the sequence of actions & percepts to that point
- Actions(b) = ?? Union? Intersection?
  - Tricky: an action legal in one state may be illegal in another
  - Is an illegal action a NO-OP? or the end of the world?
- Transition Model:
  - Result(b,a) = { s’ : s’ = Result(s, a) and s is a state in b }
- Goaltest(b) = every state in b is a goal state

Belief States for Unobservable Vacuum World



Partially observable games

- R&N Chapter 5.6
- Player’s current node is a belief state
- Player’s move (action) generates child belief state
- Opponent’s move is replaced by Percepts(s)
  - Each possible percept leads to the belief state that is consistent with that percept
- Strategy = a move for every possible percept sequence
- Minimax returns the worst state in the belief state

- <u>Many more complications and possibilities!!</u>
  - Opponent may select a move that is not optimal, but instead minimizes the information transmitted, or confuses the opponent
  - May not be reasonable to consider ALL moves; open P-QR3??
- **See R&N, Chapter 5.6, for more info**

The State of Play

- Checkers:
  - Chinook ended 40-year-reign of human world champion Marion Tinsley in 1994.
- Chess:
  - Deep Blue defeated human world champion Garry Kasparov in a six-game match in 1997.
- Othello:
  - human champions refuse to compete against computers:
    they are too good.
- Go:
  - AlphaGo recently (3/2016) beat 9th dan Lee Sedol
  - b > 300 (!); full game tree has > 10^760 leaf nodes (!!)
- See (e.g.) http://www.cs.ualberta.ca/~games/ for more info

High branching factors

- What can we do when the search tree is too large?
  - Example: Go (b = 50 to 300+ moves per state)
  - Heuristic state evaluation (score a partial game)
- Where does this heuristic come from?
  - Hand designed
  - Machine learning on historical game patterns
  - Monte Carlo methods – play random games

Monte Carlo heuristic scoring

- Idea: play out the game randomly, and use the results as a score
  - Easy to generate & score lots of random games
  - May use 1000s of games for a node
- The basis of Monte Carlo tree search algorithms…

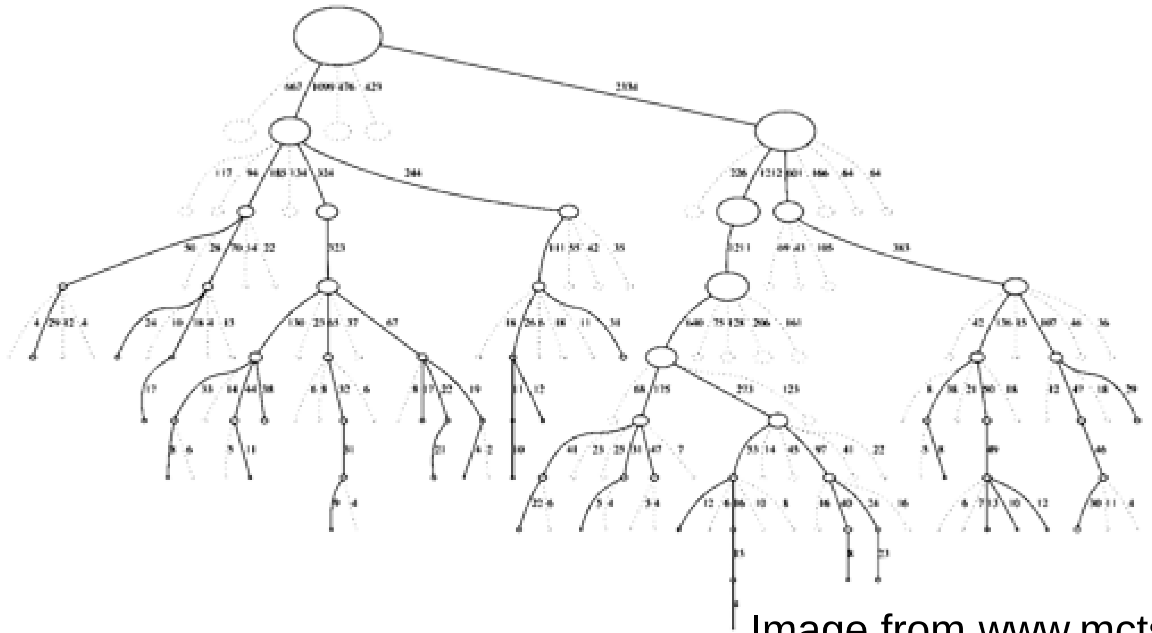


Image from www.mcts.ai

Monte Carlo Tree Search

- Should we explore the whole (top of) the tree?
  - Some moves are obviously not good…
  - Should spend time exploring / scoring promising ones
- This is a <u>*multi-armed bandit*</u> (MAB) problem:
- Want to spend our time on good moves
- Which moves have high payout?
  - Hard to tell – random…
- *Explore* vs. *exploit* tradeoff

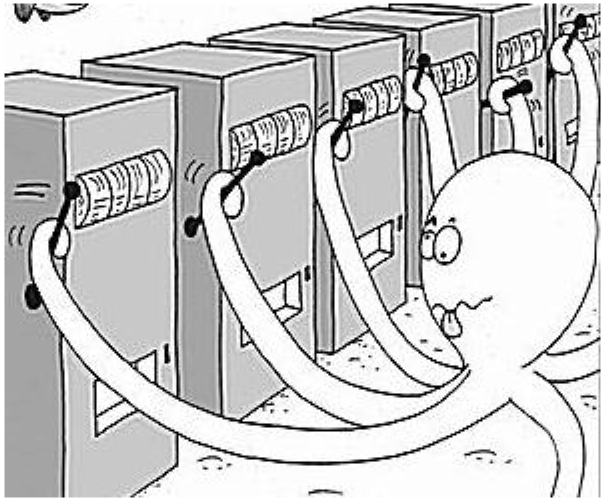
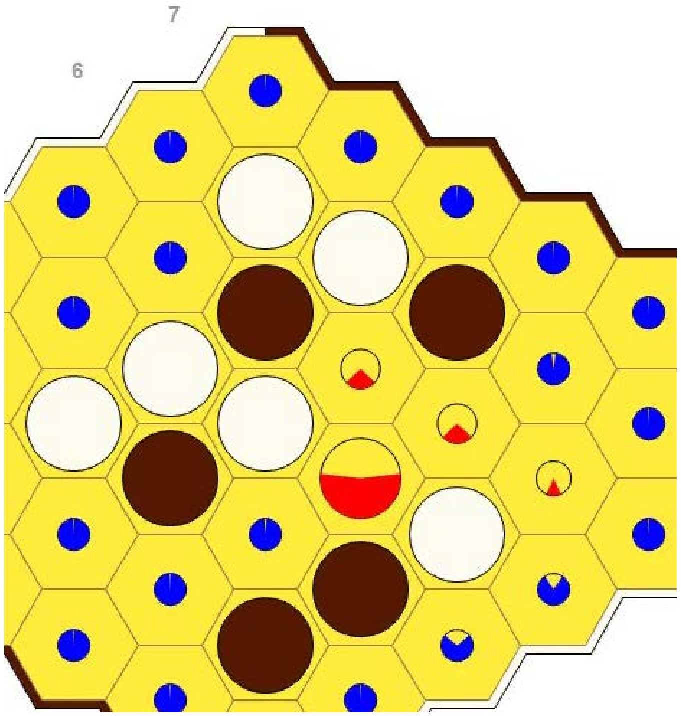
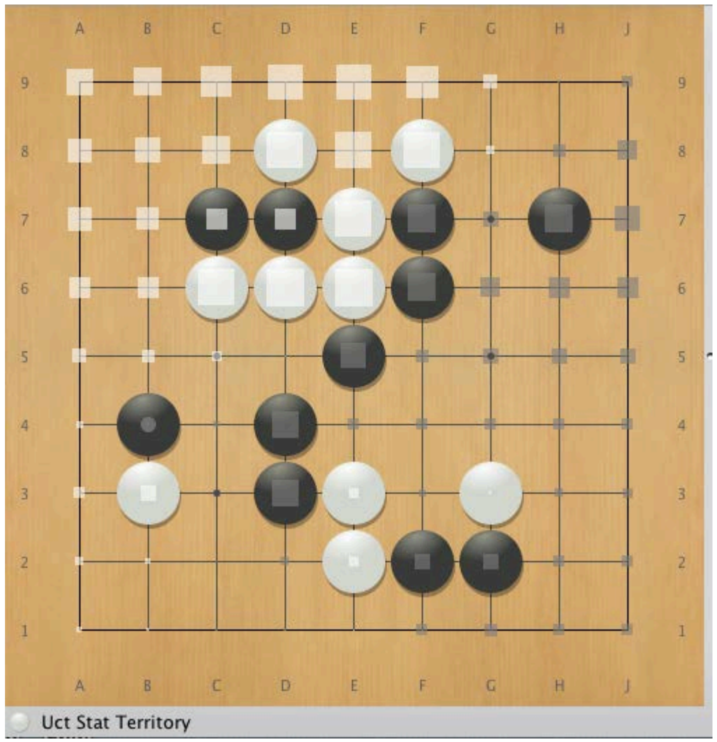


Image from Microsoft Research

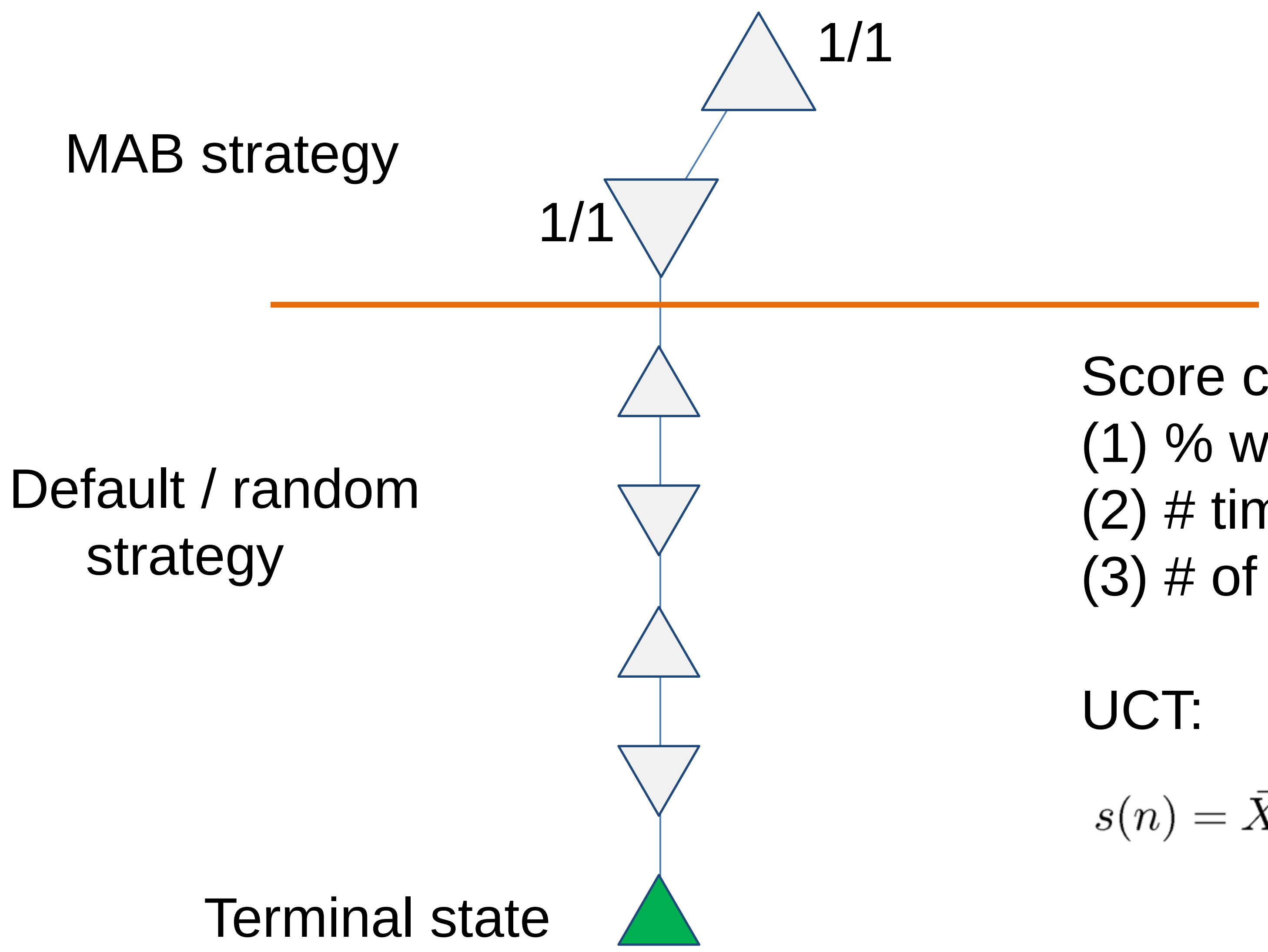
Visualizing MCTS

- At each level of the tree, keep track of
  - Number of times we’ve explored a path
  - Number of times we won
- Follow winning (from max/min perspective) strategies more often, but also explore others





MCTS

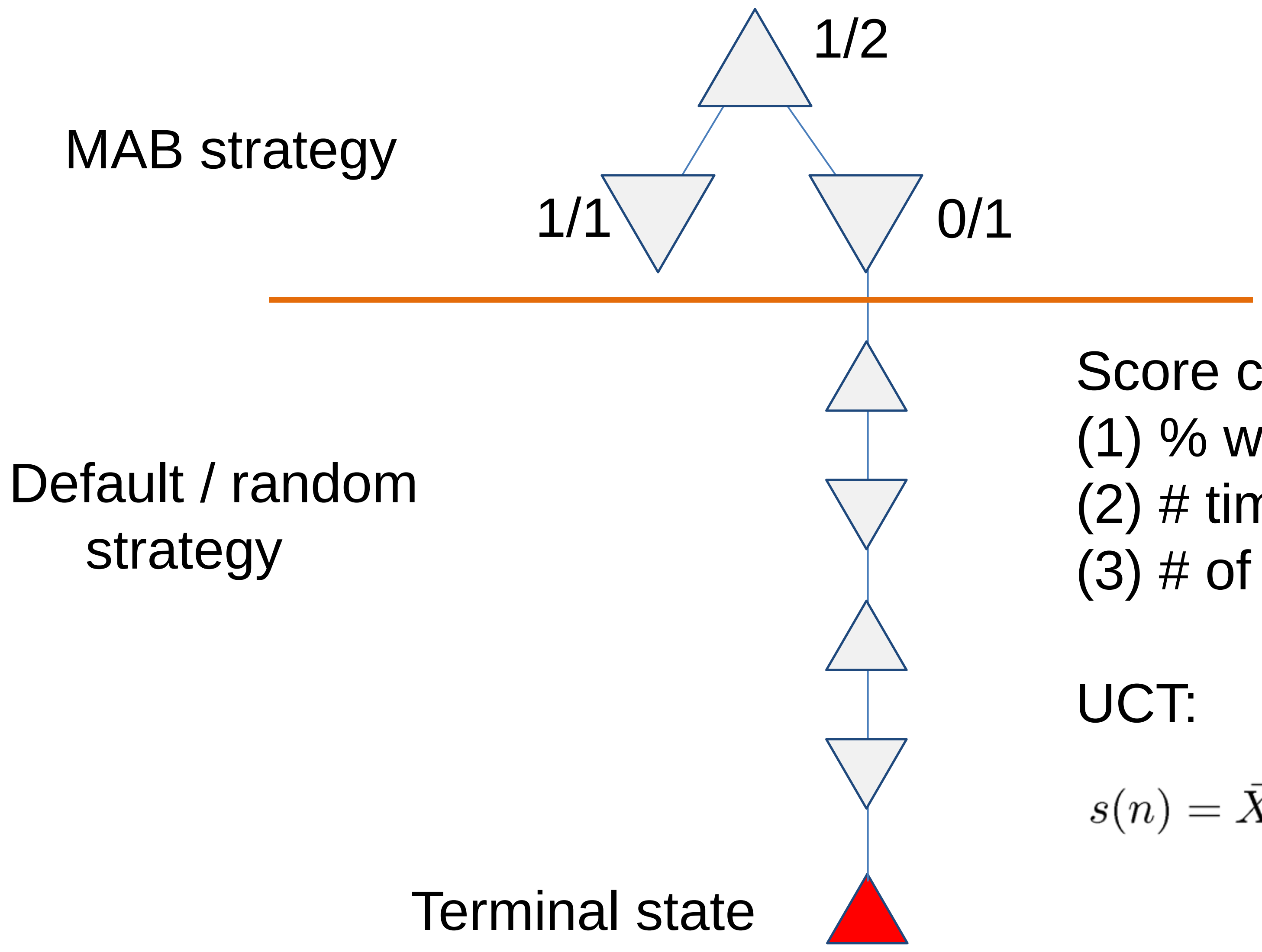


Score consists of
(1) % wins
(2) # times tried
(3) # of steps total

UCT:

$$s(n) = \bar{X}_n \pm \sqrt{\frac{\log n}{t(n)}}$$

MCTS



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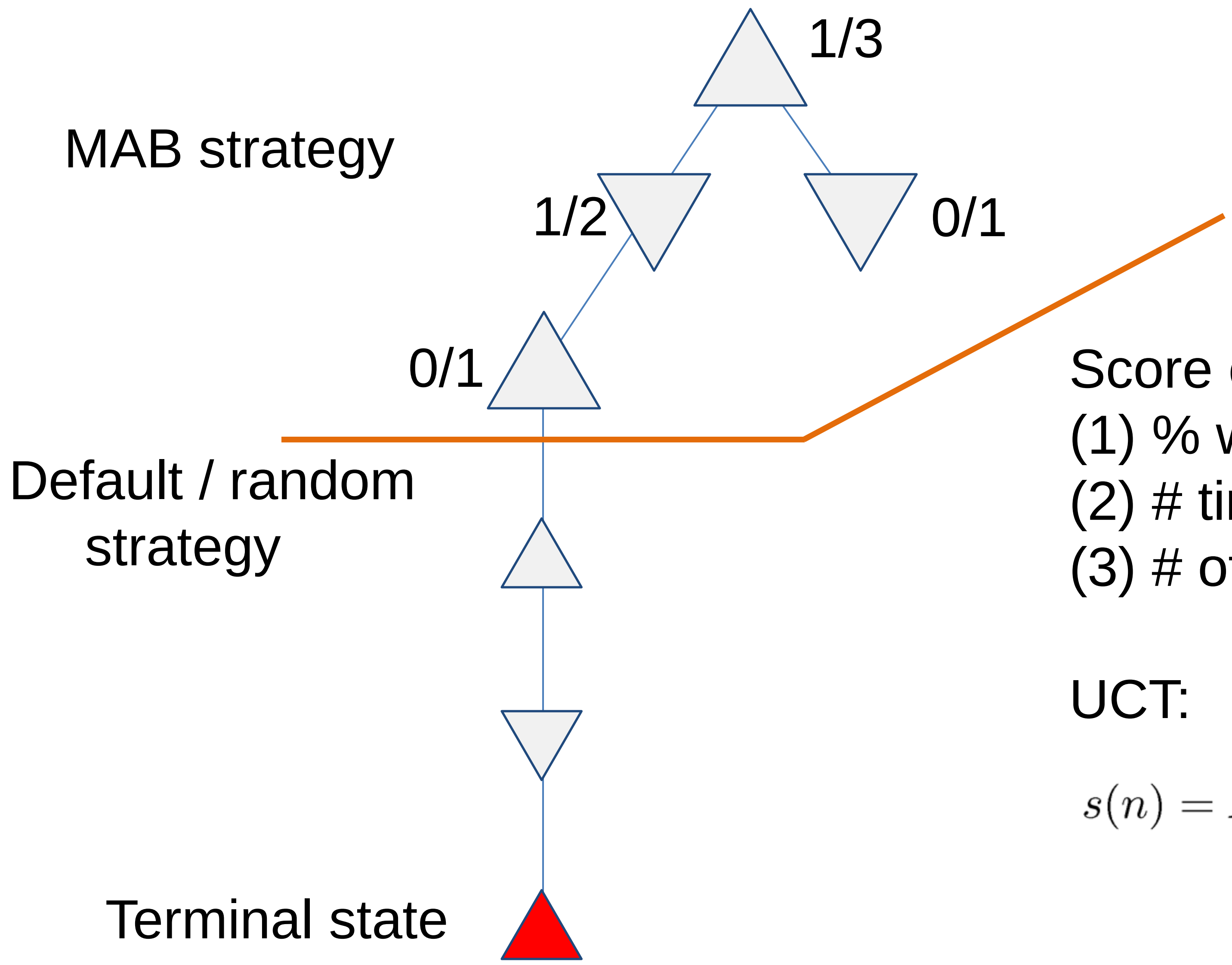

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Summary

- Game playing is best modeled as a search problem
- Game trees represent alternate computer/opponent moves
- Evaluation functions estimate the quality of a given board configuration for the Max player.
- Minimax is a procedure which chooses moves by assuming that the opponent will always choose the move which is best for them
- Alpha-Beta is a procedure which can prune large parts of the search tree and allow search to go deeper
- For many well-known games, computer algorithms based on heuristic search match or out-perform human world experts.