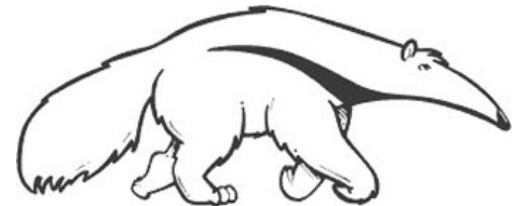


# First-Order Logic C: Knowledge Engineering

CS171, Fall Quarter, 2019  
Introduction to Artificial Intelligence  
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**Read Beforehand: R&N 8, 9.1-9.2, 9.5.1-9.5.5**

# Outline

- Review --- Syntactic Ambiguity
- Using FOL
  - Tell, Ask
- Example: Wumpus world
- Deducing Hidden Properties
  - Keeping track of change
  - Describing the results of Actions
- Set Theory in First-Order Logic
- Knowledge engineering in FOL
- The electronic circuits domain

# You will be expected to know

- Seven steps of Knowledge Engineering (R&N section 8.4.1)
- Given a simple Knowledge Engineering problem, produce a simple FOL Knowledge Base that solves the problem

# Review --- Syntactic Ambiguity

- FOPC provides many ways to represent the same thing.
- E.g., “Ball-5 is red.”
  - HasColor(Ball-5, Red)
    - Ball-5 and Red are objects related by HasColor.
  - Red(Ball-5)
    - Red is a unary predicate applied to the Ball-5 object.
  - HasProperty(Ball-5, Color, Red)
    - Ball-5, Color, and Red are objects related by HasProperty.
  - ColorOf(Ball-5) = Red
    - Ball-5 and Red are objects, and ColorOf() is a function.
  - HasColor(Ball-5(), Red())
    - Ball-5() and Red() are functions of zero arguments that both return an object, which objects are related by HasColor.
  - ...
- This can GREATLY confuse a pattern-matching reasoner.
  - Especially if multiple people collaborate to build the KB, and they all have different representational conventions.

# Review --- Syntactic Ambiguity ---

## Partial Solution

- FOL can be TOO expressive, can offer TOO MANY choices
- Likely confusion, especially for **teams** of Knowledge Engineers
- Different team members can make different representation choices
  - E.g., represent “Ball43 is Red.” as:
    - a predicate (= verb)? E.g., “Red(Ball43)” ?
    - an object (= noun)? E.g., “Red = Color(Ball43)” ?
    - a property (= adjective)? E.g., “HasProperty(Ball43, Red)” ?
- PARTIAL SOLUTION:
  - An upon-agreed **ontology** that settles these questions
  - Ontology = what exists in the world & how it is represented
  - The Knowledge Engineering teams agrees upon an ontology BEFORE they begin encoding knowledge

# Using FOL

- We want to TELL things to the KB, e.g.  
TELL(KB,  $\forall x \text{ King}(x) \Rightarrow \text{Person}(x)$  )  
TELL(KB, *King(John)* )

These sentences are assertions

- We also want to ASK things to the KB,  
ASK(KB,  $\exists x \text{ Person}(x)$  )

these are queries or goals

The KB should return the list of x's for which Person(x) is true:  
{x/John, x/Richard,...}

# Knowledge engineering in FOL

1. Identify the task
2. Assemble the relevant knowledge
3. Decide on a vocabulary of predicates, functions, and constants
4. Encode general knowledge about the domain
5. Encode a description of the specific problem instance
6. Pose queries to the inference procedure and get answers
7. Debug the knowledge base

# FOL Version of Wumpus World

- Typical percept sentence:

Percept([Stench,Breeze,Glitter,None,None],5)

- Actions:

Turn(Right), Turn(Left), Forward, Shoot, Grab, Release, Climb

- To determine best action, construct query:

$\exists a \text{ BestAction}(a,5)$

- ASK solves this and returns {a/Grab}

– And TELL about the action.



# Knowledge Base for Wumpus World

- Perception

- $\forall s,g,x,y,t \text{ Percept}([s,\text{Breeze},g,x,y],t) \Rightarrow \text{Breeze}(t)$
- $\forall s,b,x,y,t \text{ Percept}([s,b,\text{Glitter},x,y],t) \Rightarrow \text{Glitter}(t)$

- Reflex action

- $\forall t \text{ Glitter}(t) \Rightarrow \text{BestAction}(\text{Grab},t)$

- Reflex action with internal state

- $\forall t \text{ Glitter}(t) \wedge \neg \text{Holding}(\text{Gold},t) \Rightarrow \text{BestAction}(\text{Grab},t)$

$\text{Holding}(\text{Gold},t)$  can not be observed: keep track of change.

# Deducing hidden properties

Environment definition:

$$\forall x,y,a,b \text{ Adjacent}([x,y],[a,b]) \Leftrightarrow \\ [a,b] \in \{[x+1,y], [x-1,y],[x,y+1],[x,y-1]\}$$

Properties of locations:

$$\forall s,t \text{ At}(\text{Agent},s,t) \wedge \text{Breeze}(t) \Rightarrow \text{Breezy}(s)$$

Squares are breezy near a pit:

- **Diagnostic** rule---infer cause from effect  
 $\forall s \text{ Breezy}(s) \Leftrightarrow \exists r \text{ Adjacent}(r,s) \wedge \text{Pit}(r)$
- **Causal** rule---infer effect from cause (model based reasoning)  
 $\forall r \text{ Pit}(r) \Rightarrow [\forall s \text{ Adjacent}(r,s) \Rightarrow \text{Breezy}(s)]$

## Keeping track of change

Facts hold in **situations**, rather than eternally

E.g.,  $Holding(Gold, Now)$  rather than just  $Holding(Gold)$

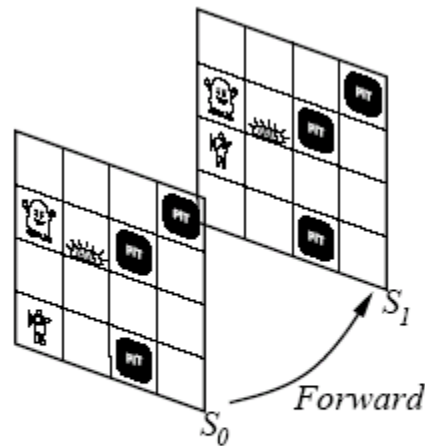
**Situation calculus** is one way to represent change in FOL:

Adds a situation argument to each non-eternal predicate

E.g.,  $Now$  in  $Holding(Gold, Now)$  denotes a situation

Situations are connected by the *Result* function

$Result(a, s)$  is the situation that results from doing  $a$  in  $s$



# Yale shooting problem

- The **Yale shooting problem** illustrates the frame problem. (Its inventors were working at Yale University when they proposed it.)
- Fred (a turkey) is initially alive and a gun is initially unloaded. Loading the gun, waiting for a moment, and then shooting the gun at Fred is expected to kill Fred.
- However, in one solution, Fred indeed dies; in another (also logically correct) solution, the gun becomes mysteriously unloaded and Fred survives.
- By Hanks and McDermott, adapted from Wikipedia

## Describing actions I

“Effect” axiom—describe changes due to action

$$\forall s \text{ AtGold}(s) \Rightarrow \text{Holding}(\text{Gold}, \text{Result}(\text{Grab}, s))$$

“Frame” axiom—describe **non-changes** due to action

$$\forall s \text{ HaveArrow}(s) \Rightarrow \text{HaveArrow}(\text{Result}(\text{Grab}, s))$$

**Frame problem**: find an elegant way to handle non-change

- (a) representation—avoid frame axioms
- (b) inference—avoid repeated “copy-overs” to keep track of state

**Qualification problem**: true descriptions of real actions require endless caveats—what if gold is slippery or nailed down or . . .

**Ramification problem**: real actions have many secondary consequences—what about the dust on the gold, wear and tear on gloves, . . .

## Describing actions II

Successor-state axioms solve the representational frame problem

Each axiom is “about” a **predicate** (not an action per se):

$$\begin{aligned} P \text{ true afterwards} &\Leftrightarrow [\text{an action made } P \text{ true} \\ &\vee P \text{ true already and no action made } P \text{ false}] \end{aligned}$$

For holding the gold:

$$\begin{aligned} \forall a, s \text{ Holding}(\text{Gold}, \text{Result}(a, s)) &\Leftrightarrow \\ &[(a = \text{Grab} \wedge \text{AtGold}(s)) \\ &\vee (\text{Holding}(\text{Gold}, s) \wedge a \neq \text{Release})] \end{aligned}$$

# Set Theory in First-Order Logic

Can we define set theory using FOL?

- individual sets, union, intersection, etc

Answer is yes.

Basics:

- empty set = constant =  $\{ \}$

- unary predicate  $\text{Set}(\ )$ , true for sets

- binary predicates:

$x \in S$  (true if  $x$  is a member of the set  $s$ )

$S_1 \subseteq S_2$  (true if  $s_1$  is a subset of  $s_2$ )

- binary functions:

intersection  $S_1 \cap S_2$ , union  $S_1 \cup S_2$ , adjoining  $\{x|s\}$

# A Possible Set of FOL Axioms for Set Theory

The only sets are the empty set and sets made by adjoining an element to a set

$$\forall s \text{ Set}(s) \Leftrightarrow (s = \{\}) \vee (\exists x, s_2 \text{ Set}(s_2) \wedge s = \{x | s_2\})$$

The empty set has no elements adjoined to it

$$\neg \exists x, s \{x | s\} = \{\}$$

Adjoining an element already in the set has no effect

$$\forall x, s \ x \in s \Leftrightarrow s = \{x | s\}$$

The only elements of a set are those that were adjoined into it. Expressed recursively:

$$\forall x, s \ x \in s \Leftrightarrow [ \exists y, s_2 (s = \{y | s_2\} \wedge (x = y \vee x \in s_2)) ]$$



# A Possible Set of FOL Axioms for Set Theory

A set is a subset of another set iff all the first set's members are members of the 2<sup>nd</sup> set

$$\forall s_1, s_2 \quad s_1 \subseteq s_2 \Leftrightarrow (\forall x \quad x \in s_1 \Rightarrow x \in s_2)$$

Two sets are equal iff each is a subset of the other

$$\forall s_1, s_2 \quad (s_1 = s_2) \Leftrightarrow (s_1 \subseteq s_2 \wedge s_2 \subseteq s_1)$$

An object is in the intersection of 2 sets only if a member of both

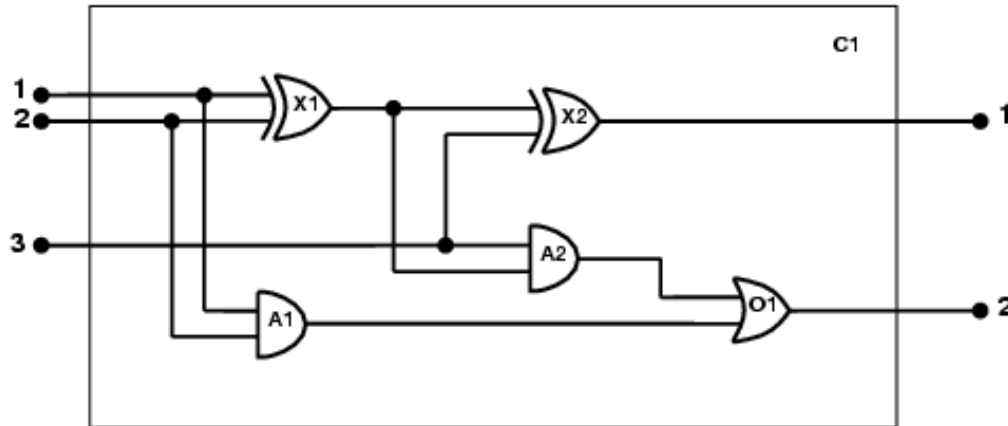
$$\forall x, s_1, s_2 \quad x \in (s_1 \cap s_2) \Leftrightarrow (x \in s_1 \wedge x \in s_2)$$

An object is in the union of 2 sets only if a member of either

$$\forall x, s_1, s_2 \quad x \in (s_1 \cup s_2) \Leftrightarrow (x \in s_1 \vee x \in s_2)$$

# The electronic circuits domain

One-bit full adder



Possible queries:

- does the circuit function properly?
  - what gates are connected to the first input terminal?
  - what would happen if one of the gates is broken?
- and so on

# The electronic circuits domain

## 1. Identify the task

- Does the circuit actually add properly?

## 2. Assemble the relevant knowledge

- Composed of wires and gates; Types of gates (AND, OR, XOR, NOT)
- 
- Irrelevant: size, shape, color, cost of gates

## 3. Decide on a vocabulary

- Many alternative ways to say X1 is an OR gate:
- 
- $\text{Type}(X_1) = \text{XOR}$  (function)  
 $\text{Type}(X_1, \text{XOR})$  (binary predicate)  
 $\text{XOR}(X_1)$  (unary predicate)  
etc.

# The electronic circuits domain

## 4. Encode general knowledge of the domain

- $\forall t_1, t_2 \text{ Connected}(t_1, t_2) \Rightarrow \text{Signal}(t_1) = \text{Signal}(t_2)$
- $\forall t \text{ Signal}(t) = 1 \vee \text{Signal}(t) = 0$
- $1 \neq 0$
- $\forall t_1, t_2 \text{ Connected}(t_1, t_2) \Rightarrow \text{Connected}(t_2, t_1)$
- $\forall g \text{ Type}(g) = \text{OR} \Rightarrow \text{Signal}(\text{Out}(1, g)) = 1 \Leftrightarrow \exists n \text{ Signal}(\text{In}(n, g)) = 1$
- $\forall g \text{ Type}(g) = \text{AND} \Rightarrow \text{Signal}(\text{Out}(1, g)) = 0 \Leftrightarrow \exists n \text{ Signal}(\text{In}(n, g)) = 0$
- $\forall g \text{ Type}(g) = \text{XOR} \Rightarrow \text{Signal}(\text{Out}(1, g)) = 1 \Leftrightarrow \text{Signal}(\text{In}(1, g)) \neq \text{Signal}(\text{In}(2, g))$
- $\forall g \text{ Type}(g) = \text{NOT} \Rightarrow \text{Signal}(\text{Out}(1, g)) \neq \text{Signal}(\text{In}(1, g))$

# The electronic circuits domain

## 5. Encode the specific problem instance

Type( $X_1$ ) = XOR

Type( $A_1$ ) = AND

Type( $O_1$ ) = OR

Connected(Out(1, $X_1$ ),In(1, $X_2$ ))

Connected(Out(1, $X_1$ ),In(2, $A_2$ ))

Connected(Out(1, $A_2$ ),In(1, $O_1$ ))

Connected(Out(1, $A_1$ ),In(2, $O_1$ ))

Connected(Out(1, $X_2$ ),Out(1, $C_1$ ))

Connected(Out(1, $O_1$ ),Out(2, $C_1$ ))

Type( $X_2$ ) = XOR

Type( $A_2$ ) = AND

Connected(In(1, $C_1$ ),In(1, $X_1$ ))

Connected(In(1, $C_1$ ),In(1, $A_1$ ))

Connected(In(2, $C_1$ ),In(2, $X_1$ ))

Connected(In(2, $C_1$ ),In(2, $A_1$ ))

Connected(In(3, $C_1$ ),In(2, $X_2$ ))

Connected(In(3, $C_1$ ),In(1, $A_2$ ))

# The electronic circuits domain

## 6. Pose queries to the inference procedure:

What are the possible sets of values of all the terminals for the adder circuit?

$$\exists i_1, i_2, i_3, o_1, o_2 \text{ Signal(In}(1, C_1)) = i_1 \wedge \text{Signal(In}(2, C_1)) = i_2 \wedge \text{Signal(In}(3, C_1)) = i_3 \\ \wedge \text{Signal(Out}(1, C_1)) = o_1 \wedge \text{Signal(Out}(2, C_1)) = o_2$$

## 7. Debug the knowledge base

May have omitted assertions like  $1 \neq 0$

# Review --- Knowledge engineering in FOL

1. Identify the task
2. Assemble the relevant knowledge
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# Summary

- First-order logic:
  - Much more expressive than propositional logic
  - Allows objects and relations as semantic primitives
  - Universal and existential quantifiers
  - syntax: constants, functions, predicates, equality, quantifiers
- Knowledge engineering using FOL
  - Capturing domain knowledge in logical form