

1. (5 pts each, 30 pts total) Mark the following reasoning patterns as S (= sound, carries true premises to true conclusions) or U (= unsound, may carry true premises to false conclusions). Premises are shown above the line, conclusions below the line. Here, " $\Rightarrow$ " means "implies" and " $\neg$ " means "not." The first one is done for you as an example.

a. S

$P \Rightarrow Q$
$P$
$Q$

b. U

$P \Rightarrow Q$
$Q$
$P$

c. U

$P \Rightarrow Q$
$P \text{ or } \neg Q$

d. S

$P \Rightarrow \neg Q$
$Q$
$\neg P$

e. S

$P \Rightarrow Q$
$\neg Q$
$\neg P$

f. S

$P \Rightarrow Q$
$\neg P \text{ or } Q$

g. U

$\neg P \Rightarrow Q$
$P$
$\neg Q$

2. (5 pts each, 40 pts total) In each of the following,  $KB$  is a set of sentences,  $\{\}$  is the empty set of sentences, and  $S$  is a single sentence. Recall  $\models$  means “entails” and  $\vdash$  means “derives,” where  $\vdash_i$  means “inference procedure  $i$  derives.” Use these keys:

Snd = Sound.

Unsnd = Unsound.

C = Complete.

I = Incomplete.

V = Valid.

Sat = Satisfiable.

Unsat = Unsatisfiable.

N = None of the above.

For each blank below, write in the key above that best corresponds to the correct term.

(a) Suppose some inference procedure  $i$  has the property, that for some  $KB$  and some  $S$ ,  $KB \models S$  but not  $KB \vdash_i S$ . Then the inference procedure  $i$  is I.

(b) Let  $S$  be given in advance. Suppose that for some  $KB_1$ ,  $KB_1 \models S$ ; but that for some other  $KB_2$ ,  $KB_2 \models \neg S$ . Then  $S$  is Sat.

(c) Suppose some inference procedure  $i$  has the property, that for any  $KB$  and any  $S$ , whenever  $KB \models S$  then  $KB \vdash_i S$ . Then the inference procedure  $i$  is C.

(d) Suppose inference procedure  $i$  has the property, that for some  $KB$  and some  $S$ ,  $KB \vdash_i S$  but not  $KB \models S$ . Then the inference procedure  $i$  is Unsnd.

(e) Let  $S$  be given in advance. Suppose that  $\{\} \models S$ . Then  $S$  is V.

(f) Suppose some inference procedure  $i$  has the property, that for any  $KB$  and any  $S$ , whenever  $KB \vdash_i S$  then  $KB \models S$ . Then the inference procedure  $i$  is Snd.

(g) Suppose that  $KB \models S$ , then the sentence  $(KB \Rightarrow S)$  is V.

(h) Suppose that  $KB \models S$ , then the sentence  $(KB \text{ and } \neg S)$  is Unsat.

3. Consider the KB shown below.

a. (5 pts each, 15 pts total) Translate the following *KB* into Conjunctive Normal Form. The first one is done for you as an example (it was already in Conjunctive Normal Form ;-)).

A.  $P \vee R$ .                    $P \vee R$                   

B.  $Q \Rightarrow S$ .                    $\neg Q \vee S$                   

C.  $P \Rightarrow Q$ .                    $\neg P \vee Q$                   

D.  $R \Rightarrow S$ .                    $\neg R \vee S$                   

b. (15 pts total, -5 for each wrong step, but not negative. The order may vary, if proof is correct.) Write a complete resolution proof that  $KB \models S$ . Show the two clauses that you resolve in front of the symbol  $\vdash$ , and the resulting clause after  $\vdash$ . You may not require all of the lines provided. The sentence labeled "E." adds the negated goal. The first one is done for you as an example.

E.  $\neg S$

(a)            $\neg S$                   ,            $\neg Q \vee S$                   ,  $\vdash$             $\neg Q$                   .

(b)            $\neg Q$                   ,            $\neg P \vee Q$                   ,  $\vdash$             $\neg P$                   .

(c)            $\neg P$                   ,            $P \vee R$                   ,  $\vdash$             $R$                   .

(d)            $R$                   ,            $\neg R \vee S$                   ,  $\vdash$             $S$                   .

(e)            $S$                   ,            $\neg S$                   ,  $\vdash$             $\square$                   .

Other proofs are fine if correct. For example, at step (d) above you could have resolved with  $\neg S$ :

(d)            $\neg S$                   ,            $\neg R \vee S$                   ,  $\vdash$             $\neg R$                   .

(e)            $\neg R$                   ,            $R$                   ,  $\vdash$             $\square$                   .