1. (10 points total, 5 pts off for each wrong answer, but not negative)
a. (5 pts) Write down the definition of $\mathrm{P}(\mathrm{H} \mid \mathrm{D})$ in terms of $\mathrm{P}(\mathrm{H}), \mathrm{P}(\mathrm{D}), \mathrm{P}(\mathrm{H} \wedge \mathrm{D})$, and $\mathrm{P}(\mathrm{H} \vee \mathrm{D})$.
$\mathrm{P}(\mathrm{H} \mid \mathrm{D})=$
b. (5 pts) Write down the expression that results from applying Bayes' Rule to $\mathrm{P}(\mathrm{H} \mid \mathrm{D})$.
$\mathrm{P}(\mathrm{H} \mid \mathrm{D})=$
c. (5 pts) Write down the expression for $\mathrm{P}(\mathrm{H} \wedge \mathrm{D})$ in terms of $\mathrm{P}(\mathrm{H}), \mathrm{P}(\mathrm{D})$, and $\mathrm{P}(\mathrm{H} \vee \mathrm{D})$.
$P(H \wedge D)=$
d. (5 pts) Write down the expression for $\mathrm{P}(\mathrm{H} \wedge \mathrm{D})$ in terms of $\mathrm{P}(\mathrm{H}), \mathrm{P}(\mathrm{D})$, and $\mathrm{P}(\mathrm{H} \mid \mathrm{D})$.
$P(H \wedge D)=$
2. (10 pts total, 5 pts each) We have a database describing 100 examples of printer failures. Of these, 75 examples are hardware failures, and 25 examples are driver failures. Of the hardware failures, 15 had Windows operating system. Of the driver failures, 15 had Windows operating system. Show your work.
a. ( 5 pts ) Calculate P (windows | hardware) using the information in the problem.
b. ( 5 pts ) Calculate P (driver | windows) using Bayes' rule and the information in the problem.
3. ( 5 pts ) After your yearly checkup, the doctor has bad news and good news. The bad news is that you tested positive for a serious disease and that the test is $99 \%$ accurate (i.e., the probability of testing positive when you do have the disease is 0.99 , as is the probability of testing negative when you don't have the disease). The good news is that it is a rare disease, striking only 1 in 10,000 people of your age. What is the probability that you actually have the disease? Show your work.
4. ( 15 pts total, 5 pts each) Suppose you are given a bag containing $n$ unbiased coins. You are told that $n-1$ of these coins are normal, with heads on one side and tails on the other, whereas one coin is a fake, with heads on both sides. Show your work for the questions below.
a. Suppose you reach into the bag, pick out a coin uniformly at random, flip it, and get a head. What is the conditional probability that the coin you chose is the fake coin?
b. Suppose you continue flipping the coin for a total of $k$ times after picking it and see $k$ heads. Now what is the conditional probability that you picked the fake coin?
c. Suppose you wanted to decide whether a chosen coin was fake by flipping it $k$ times. The decision procedure returns FAKE if all $k$ flips come up heads, otherwise it returns NORMAL. What is the (unconditional) probability that this procedure makes an error on coins drawn from the bag?
5. ( 10 pts total, 5 pts each) Consider the learning data shown in Figure 18.3 of your book (both $2^{\text {nd }} \& 3^{\text {rd }}$ ed.). Your book (Section 18.3, "Choosing Attribute Tests") shows that Gain(Patrons) $\approx 0.541$ while Gain $($ Type $)=0$. Calculate Gain (Alternate) and Gain(Hungry) .
a. $(5 \mathrm{pts})$ Gain $($ Alternate $)=$
b. $(5 \mathrm{pts})$ Gain(Hungry) $=$
6. (15 pts total, 5 pts each) Consider an ensemble learning algorithm that uses simple majority voting among M learned hypotheses. Suppose that each hypothesis has error $\varepsilon>0$ and that the errors made by each hypothesis are independent of the others'. Show your work.
a. ( 5 pts ) Calculate a formula for the error of the ensemble algorithm in terms of M and $\varepsilon$.
b. ( 5 pts ) Evaluate it for the cases where $\mathrm{M}=5,10$, and 20 and $\varepsilon=0.1,0.2$, and 0.4 .
c. ( 5 pts ) If the independence assumption is removed, is it possible for the ensemble error to be worse than $\varepsilon$ ? Produce either an example or a proof that it is not possible.
7. (35 pts total, 5 pts off for each wrong answer, but not negative) Label as TRUE/YES or FALSE/NO.
a. ( 5 pts ) Suppose that you are given two weight vectors for a perceptron. Both vectors, w1 and w2, correctly recognize a particular class of examples. Does the vector w $3=w 1-w 2$ ALWAYS correctly recognize that same class?
b. (5 pts) Does the vector $\mathrm{w} 4=\mathrm{w} 1+\mathrm{w} 2$ ALWAYS correctly recognize that same class?
c. $(5 \mathrm{pts})$ Does the vector $\mathrm{w} 5=\mathrm{cw} 1$ where $\mathrm{c}=42$ ALWAYS correctly recognize the same class?
d. $(5 \mathrm{pts})$ Does the vector $\mathrm{w} 6=\mathrm{dw} 2$ where $\mathrm{d}=-117$ ALWAYS correctly recognize the same class?
e. ( 5 pts ) Now suppose that you are given two examples of the same class $A, x 1$ and $x 2$, where $x 1 \neq x 2$. Suppose the example $\mathrm{x} 3=0.5 \mathrm{x} 1+0.5 \mathrm{x} 2$ is of a different class B. Is there ANY perceptron that can correctly classify x 1 and x 2 into class A and x 3 into class B?
f. (5 pts) Suppose that you are given a set of examples, some from one class A and some from another class B. You are told that there exists a perceptron that can correctly classify the examples into the correct classes. Is the perceptron learning algorithm ALWAYS guaranteed to find a perceptron that will correctly classify these examples?
g. (5 pts) An artificial neural network can learn and represent only linearly separable classes.
h. ( 5 pts ) Learning in an artificial neural network is done by adjusting the weights to minimize the error, and is a form of gradient descent.
i. (5 pts) An artificial neural network is not suitable for learning continuous functions (function approximation or regression) because its transfer function outputs only 1 or 0 depending on the threshold.
