The following number (percentage) of students scored in the following ranges:

100: 6 (18%)
95-99: 11 (33%)
90-94: 5 (15%) \[22 (67%) \text{ scored 90 or better}\]
85-89: 3 (9%)
80-84: 3 (9%) \[28 (85%) \text{ scored 80 or better}\]
70-79: 4 (12%)
<70 : 1 (3%)

The following number (percentage) of students missed points on these problems:

1. All students answered this problem correctly.
2. 2 (6%) students missed points.
3. 3 (9%) students missed points.
4. 21 (64%) students missed points.
5. 7 (21%) students missed points.
6. 2 (6%) students missed points.
7. (was mis-labeled 6, on the back) 1 (3%) students missed points.
8. (was mis-labeled 7, on the back) 12 (36%) students missed points.
9. (was mis-labeled 8, on the back) 11 (33%) students missed points.

**Problem #4: ANALYSIS OF COMMON ERRORS.** Write down an FOPC sentence such that every world in which it is true contains exactly one object.

Remember that every FOPC sentence (wff = well formed formula) separates possible worlds into two sets: (1) the set of worlds in which the wff is true, and (2) the set of worlds in which the wff is false. You were asked to produce a wff that is true in exactly those possible worlds with exactly one object, and false in all possible worlds with two or more objects. (By convention, all possible worlds must contain at least one object.)

(a) One common mistake was to quantify over \( x \) and \( y \) and treat them as if they were truth values that could be connected by the Boolean connectives, e.g.,

\[
\forall x \forall y ( x \land y ) \lor ( \neg x \land \neg y ) \\
\text{or} \\
\exists y \forall x \Rightarrow y \\
\text{or} \\
\forall x \forall y ( x \land y ) \Rightarrow ( x = y ) \\
\text{or} \\
\exists x \forall y x \land [ y \Rightarrow ( x = y ) ]
\]
This is a type error --- any variable bound to a quantifier automatically quantifies over all objects in some possible world. Thus, any variable is equivalent to a term, and may be used only as the argument to a function or predicate. An object does not have a Boolean truth value. Only predicates and complex sentences have truth values.

(b) Another common mistake was to quantify over $x$ and $y$ and treat one of them as if it were a possible world, then write a \texttt{Contains}(x,y) predicate that tries to assert that world $x$ contains object $y$, e.g.,

\[
\forall x \exists y \text{World}(x) \land \text{Contains}(x, y)
\]

This is a type error --- $x$ and $y$ quantify over objects in some possible world, but are not themselves possible worlds.

(c) Another common mistake was to introduce some arbitrary predicate, often $P(x)$, and then assert uniqueness in the quantifier using $P(x)$ as the body of the wff, e.g.,

\[
\exists! x P(x)
\]

But these wffs simply asserts that there is one unique object in the world that satisfies the predicate $P$. In particular, it fails to assert that there are no other objects in the world (e.g., there may be many other objects in the world that fail to satisfy the predicate $P$). The same thing holds when $P(x) = \text{American}(x) \land \text{President}(x)$.

(d) Some students apparently confused the universal and existential quantifiers, e.g.,

\[
\exists x \exists y \ ( x = y )
\]

This would be the correct answer with the universal quantifier. However, as written, it asserts that there exists an object that equals itself, which is true in every world.

(e) Some students inserted explicit Object($x$) predicates, e.g.,

\[
\exists x \text{Object}(x) \land \neg \exists y \text{Object}(y) \land \text{Different}(x, y)
\]

or

\[
\exists x \forall y \text{Object}(x) \land ( y = x \lor \neg \text{Object}(y))
\]
This is correct, but unnecessary, because in FOPC x and y can only be objects. Nevertheless, these students received full credit, because their wff was correct.

Problem #5: ANALYSIS OF COMMON ERRORS.

(a) One common error was to try to unify functions of variables, not the variables themselves, e.g.,

5.d. $F(x) / F(F(z))$ should be $x / F(z)$
5.d. $F(John) / F(F(John))$ should be $x / F(z)$
5.f. $F(x) / F(G(y))$ should be $x / G(y)$
5.f. $F(John) / F(G(John))$ should be $x / G(y)$

Remember that only variables can be substituted, not function calls.

(b) Another common error was to unify functions with different names, e.g.,

5.e. $F(x) / G(z)$ should be “None”

(c) Another somewhat common error was to stop after the first substitution, without providing the rest of the substitutions needed to make the unification complete.

Problem #8 (was mis-labeled 7, on the back): ANALYSIS OF COMMON ERRORS.

By far the most common error occurred in substituting table values for variables that were designated as false, e.g., $m=F$ or $a=F$ or $b=F$. Many students failed to subtract the probability given in the table from 1, and instead used the positive probability. Remember that the probabilities given in the tables are for the TRUE case, and you must subtract them from 1 in order to obtain the probability of the FALSE case.

Problem #9 (was mis-labeled 8, on the back): ANALYSIS OF COMMON ERRORS.

9.b. (was 8.b.) “For every child, there is a jellybean that the child likes.”

Correct answer was:

\[
H \quad \forall x \exists y \text{Child}(x) \Rightarrow [JB(y) \land \text{Likes}(x, y)]
\]

Popular wrong answers included:

\[
E \quad \forall x \exists y \quad [\text{Child}(x) \Rightarrow JB(y)] \Rightarrow \text{Likes}(x, y)
\]

This wff is true in any world in which everything likes at least one thing, because then the consequent will always be true, and so the implication will always be true --- even if there are no jellybeans in that world. It is also true in any world in which everything is a child and at least one thing is not a jellybean, because then the antecedent will always be false, and so the implication will always be true --- even if there are no jellybeans in that world. And yet, if the world contains at least one child,
then the English sentence asserts that it must contain at least one jellybean. So, it is a mistake that the wff can be true in worlds that contain a child and no jellybeans.

It may be easiest to see the mistake by converting to CNF:
\[
\forall x \exists y [\neg \text{Child}(x) \lor \text{JB}(y)] \implies \text{Likes}(x, y) \\
\forall x \exists y \neg [\neg \text{Child}(x) \lor \text{JB}(y)] \lor \text{Likes}(x, y) \\
\forall x \exists y [\text{Child}(x) \land \neg \text{JB}(y)] \lor \text{Likes}(x, y)
\]
This wff is true in a world in which everything is a child and at least one thing is not a jellybean. It is also true in any world in which everything likes at least one thing. Both these conditions admit worlds in which there is at least one child and no jellybeans.

Another popular wrong answer:
C  \[\forall x \exists y \text{Child}(x) \land \text{JB}(y) \land \text{Likes}(x, y)\]

This wff asserts that everything is a child ("\forall x \ldots \text{Child}(x) \ldots").

Another popular wrong answer:
G  \[\exists y \forall x \text{JB}(y) \implies [\text{Child}(x) \land \text{Likes}(x, y)]\]

This wff is true in any world that contains ANY object that is NOT a jellybean, because that one object will make the antecedent always be false ("\exists y \ldots \text{JB}(y) \implies \ldots"), and so will make the implication always be true. Yet the English sentence makes an assertion about every child, not worlds that contain an object that is not a jellybean.

9.c. (was 8.c.) "For every jellybean, there is a child that likes that jellybean."
Correct answer was:
F  \[\forall y \exists x \text{JB}(y) \implies [\text{Child}(x) \land \text{Likes}(x, y)]\]

Popular wrong answers included:

D  \[\exists x \forall y \text{Child}(x) \land [\text{JB}(y) \implies \text{Likes}(x, y)]\]

This wff asserts that there exists a child ("\exists x \ldots \text{Child}(x) \ldots") and that the child likes every jellybean ("\ldots \forall y \ldots \land [\text{JB}(y) \implies \text{Likes}(x, y)]"). But this is the answer to 9.d, below. Here, we assert instead something about all jellybeans ("\forall y \ldots \text{JB}(y) \implies \ldots").

Another popular wrong answer:
G  \[\exists y \forall x \text{JB}(y) \implies [\text{Child}(x) \land \text{Likes}(x, y)]\]

This wff is true in any world that contains ANY object that is NOT a jellybean, because that one object will make the antecedent always be false ("\exists y \ldots \text{JB}(y) \implies \ldots"), and so will make the implication always be true. Yet the English sentence makes an assertion about jellybeans, not worlds that contain an object that is not a jellybean.

9.d. (was 8.d.) "There is a child that likes every jellybean."
Correct answer was:
D  \[\exists x \forall y \text{Child}(x) \land [\text{JB}(y) \implies \text{Likes}(x, y)]\]
Popular wrong answers included:

C $\forall x \exists y \text{Child}(x) \land \text{JB}(y) \land \text{Likes}(x, y)$

This wff asserts that everything is a child (“$\forall x \ ... \ \text{Child}(x) \ ...$”).

Another popular wrong answer:

F $\forall y \exists x \text{JB}(y) \Rightarrow [ \text{Child}(x) \land \text{Likes}(x, y) ]$

This wff was the correct answer to 9.c., above. It asserts something about all jellybeans (“$\forall y \ ... \ \text{JB}(y) \Rightarrow \ ...$”). Please compare the two logical forms carefully.
1. (5 pts) **Definition of conditional probability.** Write the definition of $P(H | D)$ in terms of $P(H)$, $P(D)$, $P(H \land D)$, and $P(H \lor D)$.

\[
P(H | D) = \frac{P(H \land D)}{P(D)}
\]

2. (5 pts) **Bayes' Rule.** Write the result of applying Bayes' Rule to $P(H | D)$.

\[
P(H | D) = \frac{P(D | H) P(H)}{P(D)}
\]

3. (5 pts) **Conjunction.** Write the definition of $P(H \land D)$ in terms of $P(D)$, $P(H)$, and $P(H \lor D)$.

\[
P(H \land D) = P(D) + P(H) - P(H \lor D)
\]

4. (5 points) **Logic and possible worlds.** Write down an FOPC sentence such that every world in which it is true contains exactly one object.

\[\exists x \forall y, x=y \quad (\forall x \forall y, x=y) \text{ is also OK because all worlds must contain at least one object.} \]
\[([\exists! x, x=x] \text{ is also OK (iff there’s an appropriate predicate after “}\exists! x,\text{”); it’s just syntactic sugar}]

5. (20 pts total, 4 pts each) **Unifiers and Unification.** Write the most general unifier (or MGU) of the two terms given, or “None” if no unification is possible. Write your answer in the form of a substitution as given in your book, e.g., \{x / John, y / Mary, z / Bill\}. The first one is done for you as an example.

5a. UNIFY( Knows( John, x ), Knows( John, Jane )) \quad \{x / Jane\}

5b. UNIFY( Knows( y, x ), Knows( John, Jane )) \quad \{x / Jane, y / John\}

5c. UNIFY( Knows( John, x ), Knows( y, Father(y) )) \quad \{y / John, x / Father(John)\}

5d. UNIFY( Knows( John, F(x) ), Knows( y, F(F(z)) )) \quad \{y / John, x / F(z)\}

5e. UNIFY( Knows( John, F(x) ), Knows( y, G(z) )) \quad \text{None}

5f. UNIFY( Knows( John, F(x) ), Knows( y, F(G(y)) )) \quad \{y / John, x / G(John)\}

6. (15 pts total, -5 for each error, but not negative) **Bayesian Networks.** Write down the factored conditional probability expression corresponding to this Bayesian Network.

\[
P(A | B) \quad P(B | C, D) \quad P(C | D, F) \quad P(D | E, F) \quad P(E) \quad P(F)
\]

**** TURN QUIZ OVER. QUIZ CONTINUES ON THE REVERSE. ****
6. (15 pts total, -5 for each error, but not negative) Bayesian Networks. Draw the Bayesian Network corresponding to this factored conditional probability expression. Draw left-to-right, as in Problem 5.

\[
P(A | C) P(B | C, E) P(C | D) P(D | F) P(E | D, F) P(F)
\]

7. (15 pts total, -5 for each error, but not negative) Bayesian Networks. Shown below is the Bayesian network corresponding to the Burglar Alarm problem, \(P(J | A) P(M | A) P(A | B, E) P(B) P(E)\).

Write down an expression that will evaluate to \(P(j = T \land m = F \land a = F \land b = F \land e = T)\). Express your answer as a series of numbers (numerical probabilities) separated by multiplication symbols. You do not need to carry out the multiplication to produce a single number (probability). **SHOW YOUR WORK.**

\[
P(j = T \land m = F \land a = F \land b = F \land e = T)
= P(j = T | a = F) * P(m = F | a = F) * P(a = F | b = F \land e = T) * P(b = F) * P(e = T)
= .05 * .99 * .71 * .999 * .002
\]

8. (15 pts total, 5 pts each) FOPC translation. Write on the left the letter for the best corresponding wff on the right. Use Child(x) for “x is a child,” JB (x) for “x is a JellyBean,” and Likes(x, y) for “x likes y.” The first one is done for you as an example. **Note:** There are more FOPC than English sentences.

**A.**  
Every child likes every jellybean.

**B.**  
For every child, there is a jellybean that the child likes.

**C.**  
For every jellybean, there is a child that likes that jellybean.

**D.**  
There is a child that likes every jellybean